EMI Calculation Formula

The EMI to be paid for a loan can be calculated as:

$$E = P \times r \times \frac{(1+r)^n}{(1+r)^n - 1}$$

Assume that a loan of P principal amount is granted to you with an annual interest rate of $R\%$ for a duration of n months.

Then, the rate of interest applicable every month on the outstanding principal amount can be calculated as, $r = \frac{R}{12 \times 100}$

Let P_i denote the amount that is due at the end of the i^{th} month.

Let E denote the EMI to be deducted every month so as to pay off P_i at the end of i^{th} month

At the end of the first month, the amount that you owe to the bank can be calculated as:

P_1 = principal amount + monthly interest on the principal amount - EMI

Or, $P_1 = P + (r \times P) - E$

Or, $P_1 = P \times (1 + r) - E$

Similarly, at the end of the second month, the amount you owe the bank becomes:

$P_2 = P_1 \times (1 + r) - E$

On using the value of P_1 derived earlier, the expression becomes:

$P_2 = (P \times (1 + r) - E) \times (1 + r) - E$

Or, $P_2 = P \times (1 + r)^2 - E((1 + r) + 1)$

Similarly, P_3 can be written as:

$P_3 = P \times (1 + r)^3 - E((1 + r)^2 + (1 + r) + 1)$

To simplify things, let us substitute $(1 + r)$ with t such that P_3 can be written as:

$P_3 = Pt^3 - E(t^2 + t + 1)$

At the end of n months, the principal amount outstanding can be calculated as:

$P_n = Pt^n - E(t^{n-1} + ... + t^2 + t + 1)$

Or, $P_n = Pt^n - \frac{E(t^n - 1)}{(t-1)}$ [note that $t^{n-1} + ... + t^2 + t + 1$ is a series with a common ration t.]

The principal amount of loan at the end of n months should become zero, hence we can write:

$$P t^n - \frac{E(t^n - 1)}{(t-1)} = 0$$

Or, $P t^n = \dfrac{E(t^n - 1)}{(t-1)}$

Or, $E = P t^n \times \dfrac{(t-1)}{(t^n-1)}$

On substituting the value of t as $(r+1)$, we get:

$$E = P(1+r)^n \times \frac{(1+r-1)}{((1+r)^n-1)}$$

Rearranging the above equation we get:

$$E = P \times r \times \frac{(1+r)^n}{(1+r)^n - 1}$$