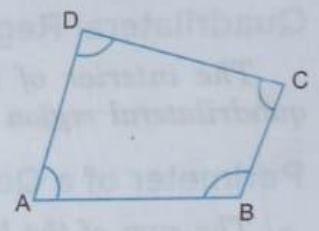
QUADRILATERALS 17

QUADRILATERAL

A simple closed figure bounded by four line segments is called a **quadrilateral.**

In the adjoining figure, ABCD is a quadrilateral.



A quadrilateral ABCD has :

(i) four **sides**, namely **AB, BC, CD** and **DA.**

(ii) four **angles**, namely **∠DAB**, **∠ABC**, **∠BCD** and **∠CDA**.
also denoted by ∠A, ∠B, ∠C and ∠D respectively.

(iii) four **vertices**, namely **A**, **B**, **C** and **D**.

Adjacent Sides : *Two sides of a quadrilateral which have a common end point are called its* **adjacent sides.**

So, we may say that two adjacent sides of a quadrilateral meet at a common vertex.

Thus, (AB, BC), (BC, CD), (CD, DA) and (DA, AB) are four pairs of adjacent sides of the quadrilateral ABCD, meeting at vertices B, C, D and A respectively.

Opposite Sides : *Two sides of a quadrilateral which do not have a common end point are called its* **opposite sides.**

Thus, (AB, CD) and (AD, BC) are two pairs of opposite sides of the quadrilateral ABCD.

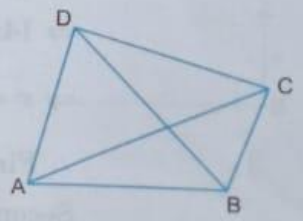
Adjacent Angles : *Two angles of a quadrilateral having a common side are called its* **adjacent angles.**

Thus, (∠A, ∠B), (∠B, ∠C), (∠C, ∠D) and (∠D, ∠A) are four pairs of adjacent angles of the quadrilateral ABCD.

Opposite Angles : *Two angles of a quadrilateral which are not adjacent angles are known as the* **opposite angles** *of the quadrilateral.*

Thus, (∠A, ∠C) and (∠B, ∠D) are two pairs of opposite angles of the quadrilateral ABCD.

Diagonals of a Quadrilateral : *The line segment joining the opposite vertices of a quadrilateral is called a* **diagonal** *of the quadrilateral.*



Thus, in the adjoining figure, AC and BD are the two diagonals of the quadrilateral ABCD.

Interior and Exterior of a Quadrilateral

Consider a quadrilateral PQRS. It divides the whole plane into three parts.

(i) The part of the plane lying inside the boundary PQRS is called the interior of the quadrilateral PQRS. Each point of this part is called an interior point of the quadrilateral. In the adjoining figure, the points A, B, C are the interior points of the quadrilateral PQRS.

(ii) The part of the plane lying outside the boundary PQRS is called the exterior of the quadrilateral PQRS. Each point of this part is called an exterior point of the quadrilateral. In the given figure, the points D, E and F are the exterior points of the quadrilateral PQRS.

(iii) The boundary PQRS
Clearly, the points K and T lie on the quadrilateral PQRS.

Quadrilateral Region

The interior of the quadrilateral PQRS together with its boundary is called the quadrilateral region PQRS.

Perimeter of a Quadrilateral

The sum of the lengths of all the four sides of a quadrilateral is called its perimeter.

Perimeter of quad. ABCD = AB + BC + CD + DA.

Angle Sum Property of a Quadrilateral

The sum of all the four angles of a quadrilateral is 360°.

Example 1. *Three angles of a quadrilateral ABCD are* $\angle A = 70°$, $\angle B = 50°$ *and* $\angle C = 82°$. *Find* $\angle D$.

Solution : We know that the sum of the four angles of a quadrilateral is 360°.

$\therefore \angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow 70° + 50° + 82° + \angle D = 360°$

$\Rightarrow 202° + \angle D = 360°$

$\Rightarrow \angle D = (360° - 202°) = 158°$

Hence, $\angle D = 158°$.

Example 2. *The angles of a quadrilateral are* $4(x - 1)°$, $(3x - 5)°$, $(4x + 21)°$ *and* $3(x + 4)°$ *respectively. Find (i) the value of x and (ii) each angle of the quadrilateral.*

Solution : We know that the sum of the angles of a quadrilateral is 360°.

$\therefore 4(x - 1) + (3x - 5) + (4x + 21) + 3(x + 4) = 360$

$\Rightarrow 4x - 4 + 3x - 5 + 4x + 21 + 3x + 12 = 360$

$\Rightarrow 14x + 24 = 360 \Rightarrow 14x = 336$

$\Rightarrow x = \frac{336}{14} = 24.$

$\therefore$ First angle $= 4(x - 1)° = 4(24 - 1)° = 4 \times 23° = 92°$;

Second angle $= (3x - 5)° = (3 \times 24 - 5)° = 67°$;

Third angle $= (4x + 21)° = (4 \times 24 + 21)° = 117°$;

Fourth angle $= 3(x + 4)° = 3(24 + 4)° = (3 \times 28)° = 84°$.

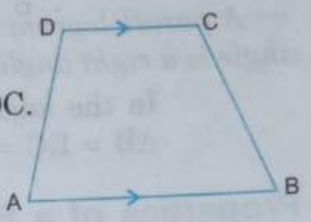
VARIOUS TYPES OF QUADRILATERALS

The quadrilaterals may broadly be classified into three types :

I. Trapezium II. Parallelogram III. Kite.

I. TRAPEZIUM

A quadrilateral having one and only one pair of opposite sides parallel is called a **trapezium.**



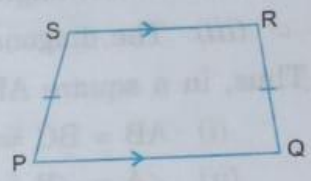
In the adjoining figure, ABCD is a quadrilateral in which AB ∥ DC.

∴ ABCD is a trapezium.

The non-parallel sides of a trapezium are called ***oblique sides.*** In the trapezium ABCD, the sides AD and BC are oblique sides.

Isosceles Trapezium

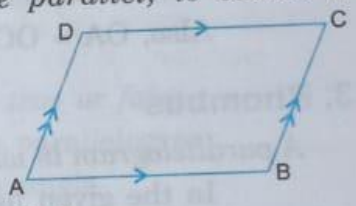
If the two non-parallel sides of a trapezium are equal, it is known as an isosceles trapezium.



In the adjoining figure, PQRS is an isosceles trapezium in which PQ ∥ SR and PS = QR.

II. PARALLELOGRAM

A quadrilateral in which both pairs of opposite sides are parallel, is called a **parallelogram.**



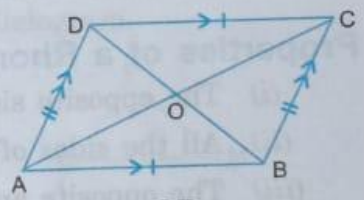
In the adjoining figure, ABCD is a parallelogram in which AB ∥ DC and AD ∥ BC.

Properties of a Parallelogram

(i) The opposite sides of a parallelogram are equal and parallel.

(ii) The opposite angle of a parallelogram are equal.

(iii) The diagonals of a parallelogram bisect each other.



Thus, in a parallelogram ABCD, we have

(i) AB = DC, AD = BC and AB ∥ DC, AD ∥ BC.

(ii) ∠BAD = ∠BCD and ∠ABC = ∠ADC.

(iii) If the diagonals AC and BD intersect at O, then OA = OC and OB = OD.

SPECIAL TYPES OF PARALLELOGRAMS

1. Rectangle

A *parallelogram* in *which each angle is a right angle is called a rectangle.*

In the given figure, ABCD is a rectangle in which AB ∥ DC, AD ∥ BC and ∠A = ∠B = ∠C = ∠D = 90°.

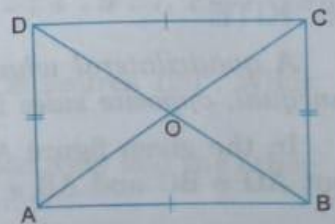
Therefore, ABCD is a rectangle.

Properties of a Rectangle

(i) Opposite sides of a rectangle are equal and parallel.

(ii) Each angle of a rectangle is 90°.

(iii) Diagonals of a rectangle are equal.



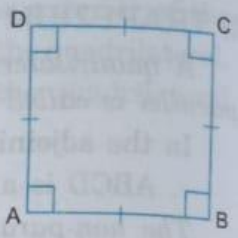
Thus, in a rectangle ABCD, we have:

(i) AB = DC, AD = BC and AB ∥ DC and AD ∥ BC.

(ii) ∠A = ∠B = ∠C = ∠D = 90°.

(iii) diagonal AC = diagonal BD.

2. Square



A parallelogram in which all the sides are equal and each angle is a right angle, is called a **square.**

In the adjoining figure, ABCD is a square in which AB = BC = CD = DA and $\angle A = \angle B = \angle C = \angle D = 90°$.

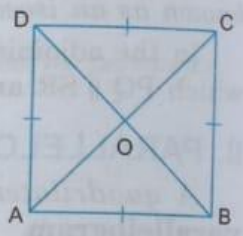
Properties of a Square

(i) All the sides of square are equal.

(ii) Each angle of a square is 90°.

(iii) The diagonals of a square are equal and bisect each other at right angles.

Thus, in a square ABCD, we have:



(i) AB = BC = CD = DA.

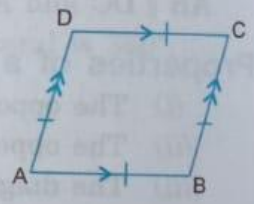
(ii) $\angle A = \angle B = \angle C = \angle D = 90°$.

(iii) AC = BD and AC ⊥ BD

i.e., $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$.

Also, OA = OC = OB = OD.

3. Rhombus



A parallelogram in which all sides are equal is called a rhombus.

In the given figure, ABCD is a parallelogram in which AB = BC = CD = DA.

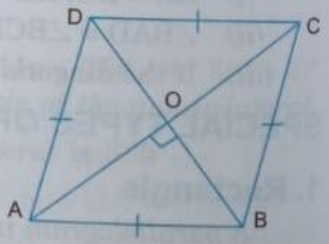
Properties of a Rhombus

(i) The opposite sides of a rhombus are parallel.

(ii) All the sides of a rhombus are equal.

(iii) The opposite angles of a rhombus are equal.

(iv) The diagonals of a rhombus bisect each other at right angles.



Thus, in a rhombus ABCD, we have :

(i) AB ∥ DC and AD ∥ BC

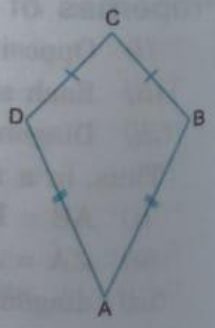
(ii) AB = BC = CD = DA.

(iii) $\angle DAB = \angle BCD$ and $\angle ABC = \angle CDA$.

(iv) If diagonals AC and BD intersect at O, then OA = OC, OB = OD

Also, $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$.

III. KITE



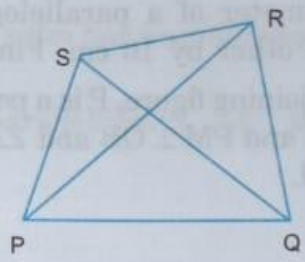
A quadrilateral which has two pairs of equal adjacent sides but unequal, opposite sides is called a **kite.**

In the given figure ABCD is a kite in which CB = CD and AB = AD but AD ≠ BC and AB ≠ CD.

EXERCISE 17 A

1. Is the figure shown here a quadrilateral? State, giving reasons.

2. *In the adjoining figure, PQRS is a quadrilateral.*

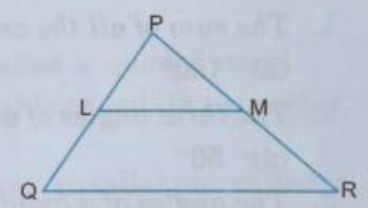


   (i) Name a pair of its adjacent sides .
   (ii) Name a pair of its opposite sides.
   (iii) Name a pair of its adjacent angles.
   (iv) Name a pair of its opposite angles.
   (v) Name its diagonals.

3. (i) How many pairs of adjacent sides does a quadrilateral have?
   (ii) How many pairs of opposite sides does a quadrilateral have?
   (iii) How many pairs of adjacent angles does a quadrilateral have?
   (iv) How many pairs of opposite angles does a quadrilateral have?
   (v) How many diagonals does a quadrilateral have?

4. *For each of the statements given below, indicate whether it is true or false :*
   (i) A rectangle is a parallelogram. (ii) A trapezium is a parallelogram.
   (iii) A parallelogram is a trapezium. (iv) A square is a rectangle.
   (v) A square is a rhombus. (vi) A rhombus is a square.
   (vii) A parallelogram is a rhombus. (viii) A kite is a parallelogram.

5. In the adjoining figure, L and M are points on the sides PQ and PR respectively, of ΔPQR such that $LM \parallel QR$. What special name would you give to the quadrilateral LQRM?



6. *Let ABCD be a parallelogram. What special name would you give it, when :*
   (i) AB = AD? (ii) $\angle ABC = 90°$?
   (iii) AB = AD and $\angle ABC = 90°$?

7. (i) How does a trapezium differ from a parallelogram?
   (ii) How does a rhombus differ from a square?
   (iii) How does a kite differ from a parallelogram?

8. Three angles of a quadrilateral measure 36°, 78° and 116° respectively. Find the measure of the fourth angle.

9. The measures of the angles of a quadrilateral are in the ratio 2 : 4 : 5 : 7. Find the measure of each of its angles.

10. Three angles of a quadrilateral are equal and the fourth angle measures 120°. What is the measure of each of the equal angles?

11. Two angles of a quadrilateral are of measures 75° and 117° respectively and the other two angles are equal. Find the measure of each of the equal angles.

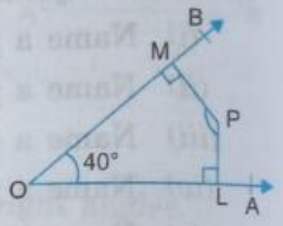
12. A quadrilateral has three acute angles, each measuring 75°. What is the measure of its fourth angle?

13. The lengths of two adjacent sides of a parallelogram are 7 cm and 5 cm respectively. Find the perimeter of the parallelogram.

14. Two sides of a parallelogram are in the ratio 5 : 3 and its perimeter is 48 cm. Find the length of each of its sides.

15. The perimeter of a parallelogram is 88 cm and one of its adjacent sides is longer than the other by 10 cm. Find the length of each of its sides.

16. In the adjoining figure, P is a point in the interior of $\angle AOB$. If $PL \perp OA$ and $PM \perp OB$ and $\angle AOB = 40°$, find the measure of $\angle LPM$.



17. *Give reasons for the following:*
   (i) A square can be thought of as a special rectangle.
   (ii) A square can be thought of as a special rhombus.
   (iii) A rectangle can be thought of as a special parallelogram.

18. A figure is said to be regular if its sides are equal in length and angles are equal in measure. What do you mean by a regular quadrilateral?

EXERCISE 17 B

MULTIPLE CHOICE QUESTIONS

Choose the correct answer in each of the following :

1. *The sum of all the angles of a quadrilateral is*
   (a) 180° (b) 270° (c) 300° (d) 360°

2. *The three angles of a quadrilateral are* 65°, 75°, 80°. *The fourth angle is*
   (a) 50° (b) 90° (c) 140° (d) 160°

3. *The angles of a quadrilateral are in the ratio* 3 : 4 : 5 : 6. *The largest of these angles is*
   (a) 90° (b) 120° (c) 150° (d) 168°

4. *A quadrilateral having one and only one pair of parallel sides, is called*
   (a) a rhombus (b) a trapezium (c) a kite (d) none of these

5. *A quadrilateral whose opposite sides are parallel, is called*
   (a) a trapezium (b) a kite (c) a parallelogram (d) none of these

6. *An isosceles trapezium has*
   (a) equal parallel sides (b) equal non-parallel sides
   (c) equal opposite sides (d) none of these°

7. *If the diagonals of a quadrilateral bisect each other at right angles, then this quadrilateral is*
   (a) a rectangle (b) a rhombus (c) a kite (d) none of these

8. *A rhombus has*
   (a) all sides equal and diagonals unequal
   (b) all sides equal and diagonals equal
   (c) all sides unequal and diagonals unequal
   (d) none of these

9. *A square has*
   (a) all sides equal and diagonals
   (b) all sides equal and diagonals equal
   (c) all sides unequal and diagonals *unequal*
   (d) none of these

10. *A quadrilateral having two pairs of equal adjacent sides but unequal opposite sides, is called*
   (a) a trapezium (b) a kite (c) a parallelogram (d) none of these

MENTAL MATHS

1. *Fill in the blanks :*
   (i) The opposite sides of a parallelogram are _____ and _____ .
   (ii) The diagonals of a _____ bisect each other at right angles.
   (iii) A quadrilateral which has two pairs of equal adjacent sides but unequal opposite sides is called a _____
   (iv) If one angle of a rhombus is 90°, then it becomes a _____
   (v) A parallelogram having all sides equal is called a _____ .

2. *Write (T) for true and (F) for false statement given below :*
   (i) Both the diagonals of a rhombus are equal. _____
   (ii) The diagonals of a parallelogram are equal. _____
   (iii) In a kite, two pairs of adjacent sides are equal. _____
   (iv) Every rectangle is a parallelogram. _____
   (v) The diagonals of a square are equal. _____

THINGS TO REMEMBER

1. A simple closed figure bounded by four line segments is called a quadrilateral.
2. The sum of the lengths of all the four sides of a quadrilateral is called its perimeter.
3. The sum of all the four angles of a quadrilateral is 360°.
4. A quadrilateral in which both pairs of opposite sides are parallel, is called a parallelogram.
5. In a parallelogram :
   (i) Opposite sides are parallel and equal;
   (ii) Opposite angles are equal;
   (iii) Adjacent angles are supplementary;
   (iv) The diagonals bisect each other.
6. A parallelogram in which each angle is a right angle, is called a rectangle.
7. The diagonals of a rectangle are equal.
8. A parallelogram in which all the sides are equal, is called a rhombus.
9. The diagonals of a rhombus bisect each other at right angles.
10. A parallelogram in which all the sides are equal and each angle is a right angle, is called a square.
11. The diagonals of a square are equal and bisect each other at right angles.
12. A quadrilateral having one pair of opposite parallel sides is called a trapezium.
13. A trapezium in which non-parallel sides are equal, is called an isosceles trapezium.
14. A quadrilateral having two pairs of equal adjacent sides, but opposite sides unequal, is called a kite.