

X CLASS MATHEMATICS

Imp formulas and definitions
AT GLANCE

1. REAL NUMBERS

Rational numbers: The numbers which can be written in the form of $\frac{p}{q}$ ($q \neq 0$) where p and q are integers.

Example: 2, -1, 0.5, $\frac{22}{7}$,

Irrational numbers: The numbers which cannot be written in the form of $\frac{p}{q}$ are called irrational numbers.

Example: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{15}$, π , $-\frac{\sqrt{2}}{\sqrt{3}}$, 0.1011011101110..... etc.

Note: An irrational number between a and b is \sqrt{ab}

\sqrt{p} , \sqrt{q} are two irrational numbers then $\sqrt{p+q}$ is an irrational number and $\sqrt{p-q}$ is also irrational number.

Real numbers: The set of rational and irrational numbers together are called real numbers.

Example: 2, 0, -5, $\sqrt{27}$, π , 0.101001000.....

Euclid's division lemma: Given positive integers a and b, there exists unique positive integers q and r satisfying $a = bq + r$, $0 \leq r < b$. Where a = Dividend, b = Divisor, q = Quotient, r = remainder, so that,

Dividend = (Divisor \times Quotient) + remainder.

It is a technique to calculate the Height Common Factor (HCF) of two given positive integers.

Fundamental Theorem of Arithmetic: Every composite

number can be expressed (factorized) as a product of its primes, and this factorization is unique, apart from the order in which the prime factors occur.

Let $x = \frac{p}{q}$ be a rational number,

such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

Let $x = \frac{p}{q}$ be rational number, such that the prime factorization of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating (recurring).

Relationship between L.C.M., and H.C.F of two numbers.

For any two positive integers a and b given by

HCF (a and b) \times LCM (a and b) = a \times b (Exponential form) $a^x = N \Leftrightarrow \log_x^N = x$ (logarithmic form).

where a and N are positive real numbers, $a \neq 1$

Note: 1. \sqrt{a} is rational, if a is a not perfect square.

2. $a \pm \sqrt{b}$ is irrational, if 'b' is not perfect square.

3. $\sqrt{a} \pm \sqrt{b}$ is irrational, if 'a' and 'b' are not perfect squares.



4. 'p' is prime number, p is a divisor of $a^2 \Leftrightarrow$ p is a divisor of 'a'.

Properties of Logarithms:

1. $\log_a^x = \log_a^x + \log_a^y$ (product rule)

2. $\log \frac{x}{y} = \log_a^x - \log_a^y$ (quotient rule)

3. $\log_a^m = m \log_a^x$ (power rule)

4. $\log_a^a = 1$

5. $\log_a^1 = 0$

6. $a^{\log_a^N} = N$

The last digit of 6^{100} is 6.

Let p be a prime number. If p divides a^2 , (where a is a positive integer) then p divides a.

2. SETS

A collection of well-defined objects is called a Set. Set theory was developed by 'George Cantor'.

The symbol for belongs to is "∈" and does not belong to is "∉".

A set which does not contain any element in it is called empty set or null set or a void set. i.e. $\phi = \{ \}$ but $\phi \neq \{0\}$.

A set is called a finite set, if it is possible to count the number of elements of that set.

The universal set is denoted by μ .

The universal set is usually drawn by the shape of rectangles.

$A \subset B$ & $B \subset A \Leftrightarrow A=B$.

$A \cap B$ is the set containing only those elements that are common in A & B. $A \cap B = \{x: x \in A \text{ and } x \in B\}$,

$A \cup B$ = contains the elements that are either in A or in B or in both.

$A \cup B = \{x: x \in A \text{ or } x \in B\}$,

if, $A \cap B = \phi$ then A & B are two disjoint sets then $n(A \cap B) = 0$.

If A and B are two non-zero sets then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

If A & B are disjoint then

$n(A \cup B) = n(A) + n(B)$

$A-B = \{x: x \in A \text{ and } x \notin B\}$,

$B-A = \{x: x \in B \text{ and } x \notin A\}$

Every set is a subset of itself.

Null set is subset of every set.

If $A \subset B$, $B \subset C$ then $A \subset C$.

If $A \subset B$ then $A \cup B = B$ and $A \cap B = A$.

Cardinal number: The number of elements in a set is called cardinal number of the set.

Example: $A = \{x/x \text{ is a letter of the word 'MATHEMATICS'}\} \Rightarrow$

$A = \{M, A, T, H, E, I, C, S\}$ then cardinal number of the set $n(A) = 8$

3. POLYNOMIALS

Let x be a variable, n be a positive integer and $a_0, a_1, a_2, \dots, a_n$ be constants. Then $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$ is called a polynomial in variable. (or)

An algebraic expression becomes a polynomial if the powers of the variable(s) are whole numbers.

A polynomial does not contain the terms like \sqrt{x} , x^2 , $\frac{1}{x}$, $x^{3/2}$

Degree of a polynomial: The highest power of the variable of the all terms of the given polynomial is that the degree term in a polynomial.

Example: $p(x) = x^3 + 3x^2 - 4x$ the degree is 3.

If $f(x)$ is a polynomial and k is any real number, then the real number obtained by replacing x by k in $f(x)$ at $x = k$ and is denoted by $f(k)$.

Zero of a polynomial: For a polynomial, if p(x), if $p(k) = 0$, then k is called zero of the polynomial p(x). A polynomial of degree n can have at most n real zeroes.

Geometrically, the zeroes of a polynomial f(x) are the x-coordinates of the points where the graph $y = f(x)$ intersects x-axis.

For any quadratic polynomial $ax^2 + bx + c$, $a \neq 0$, the graph of the corresponding equation $y = ax^2 + bx + c$ has one of the two shapes either open upwards like U or downwards like \cap , depending on

whether $a > 0$ or $a < 0$, these curves are called *Parabolas*.

Relation between the zeros and coefficients:

For the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$ If α and β are the zeroes, then

(i) sum of the zeros $\alpha + \beta = -\frac{b}{a}$ (ii) product of the zeros $\alpha\beta = \frac{c}{a}$

For the cubic polynomial $p(x) = ax^3 + bx^2 + cx + d$, $a \neq 0$

If α, β, γ are the zeroes

then $\alpha + \beta + \gamma = -\frac{b}{a}$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$,

$\alpha\beta\gamma = -\frac{d}{a}$.

If α, β, γ are the zeroes, the polynomial can be written as

$(x-\alpha)(x-\beta)(x-\gamma) = x^3 - x^2(\alpha + \beta + \gamma) + x(\alpha\beta + \beta\gamma + \gamma\alpha) - \alpha\beta\gamma = 0$. **The graph of the quadratic polynomial is a parabola.**

(i) If the graph cuts x-axis at $(x_1, 0)$ and $(x_2, 0)$ then x_1 and x_2 are the zeros of the polynomial. These roots are real and distinct.

(ii) If the graph touches x-axis at only one point $(x_1, 0)$ then two zeros are x_1 and x_1 are real and equal.

(iii) If the graph does not cuts (touches) the x-axis then the parabola has no real zeros.

4. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

An equation which can be put in the form $ax+by+c=0$, where a, b, c are real numbers and a, b $\neq 0$, is

called a linear equation in two variables x and y.

Nature of the lines:

1. If the lines intersect at a point, then the point gives the unique solution of the two equations. In this case, the pair of equations is consistent.

2. If the lines coincide, then there are infinitely many solutions-each point on the line being a solution. In this case, the pair of equations is dependent and consistent.

3. If the lines are parallel then the pair of equations has no solution. In this case, the pair of equations is inconsistent.

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