Roll No. .....

E–355

M. Sc. (IT) (First Semester)

EXAMINATION, Dec.-Jan., 2020-21

Paper Third

MATHEMATICAL FOUNDATION OF

COMPUTER SCIENCE

[MSC (IT)—103]

Time : Three Hours] [*Maximum Marks : 100*

[*Minimum Pass Marks : 40*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. Determine which of the following sets are finite :
   (a) The set of lines parallel to the *x*-axis
   (b) The set of numbers which are multiples of 5
   (c) The set of letters in the English alphabet
   (d) The set of circles through the origin (0, 0)

2. Suppose A is finite set with n elements. The number of elements in the largest equivalence relation of A is :

   (a) 1

   (b) n

   (c) $n + 1$

   (d) n^2

3. The number of functions from a 6-element set to a 4-element set is :

   (a) $6 + 4$

   (b) 6^4

   (c) 4^6

   (d) 6×4

4. What is Existential quantifier ?

   (a) Symbol ($\forall$) for all

   (b) Symbol ($\exists$) for some

   (c) Symbol $\forall$ and ($\exists$)

   (d) Symbol (~) Negation

5. A self complemented, distributive lattice is called :

   (a) Boolean Algebra

   (b) Modular Lattice

   (c) Complete Lattice

   (d) Self Dual Lattice

6. The Idempotent law is defined as :

   (a) $a * o = o$

   (b) $a * a + b = a$

   (c) $a + a * b = a$

   (d) $a * a = a$

7. The Boolean expression A + BC equals :

   (a) $\bar{A} + B \quad \bar{A} + C$

   (b) (A + B) (A + C)

   (c) (A + B) ($\bar{A}$ + C)

   (d) None of the above

8. For any *a*, *b* in a Boolean algebra (B, $\vee$, $\wedge$, '), if $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$, then this law is called :

   (a) Associative law

   (b) Absorption law

   (c) Involution law

   (d) De-Morgan's law

9. In the group (I, $+_5$), where I = { 0, 1, 2, 3, 4} the inverse of 1 is :

   (a) 4

   (b) 3

   (c) 2

   (d) 0

10. Which of the following is TRUE ?

   (a) The set of all rational negative numbers forms a group under multiplication

   (b) The set of all matrices forms a group under multiplication

   (c) The set of all non-sigular matrices forms a group under multiplication.

   (d) Both (b) and (c) are true

11. Let G be a cyclic group of order 4. Then number of generators of G is :

    (a) 1

    (b) 2

    (c) 3

    (d) 4

12. Which of the following statements is true ?

    (a) Every group is field

    (b) Every ring is a field

    (c) Every integral domain is a field

    (d) Every finite integral domain is a field

13. The generators of the multiplicative cyclic group $\{1, \omega, \omega^2\}$ are :

    (a) ω and ω^2

    (b) 1 and ω^2

    (c) 1 and ω

    (d) 1, ω and ω^2

14. In a directed graph :

    (a) Underlying graph is fixed

    (b) Directions are fixed

    (c) Both (a) and (b)

    (d) None of these

15. The minimum number of edges in a connected graph with n vertices :

    (a) $n - 1$

    (b) $n + 1$

    (c) n

    (d) None of these

16. Maximum number of edges in an n-node undirected graph without self loops is :

    (a) n^2

    (b) $\frac{n \quad n + 1}{2}$

    (c) $n - 1$

    (d) $\frac{n \quad n - 1}{2}$

17. Preorder traversal is nothing but :

    (a) Linear order

    (b) Breadth first order

    (c) Depth of first order

    (d) Topological order

18. If height of a tree is 10, the highest level of the tree is :

    (a) 10

    (b) 9

    (c) 5

    (d) 1

19. A binary tree T has n leaf nodes. The number of nodes of degree 2 in T is :
    (a) $n - 1$
    (b) 2^n
    (c) n
    (d) $\log_2 n$

20. In a preorder traversal, the ..... processed first.
    (a) left subtree
    (b) right subtree
    (c) root
    (d) Both (a) and (b)

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions in 2-3 sentences.

1. Define Cartesian product with example.
2. What are Quantifiers ?
3. Define sublattice.
4. Write uses of switching circuits.
5. Define Boolean Algebra.
6. Define subgraph.
7. Define planar graph.
8. Define binary tree.
9. Define co-set.
10. Define centre of a tree.

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions in less than 75 words.

1. Let the functions f and g be defined by $f(x) = 2x + 1$ and $g(x) = x^2 - 2$. Find the formula defining the composition function g o f.

2. Prove that :

$$\left[p \Rightarrow q \wedge p \right] \Rightarrow q$$

is a tautology.

3. Define phase-structure grammar.
4. Define lattices and isomorphic lattices.
5. Explain logic gates.
6. Explain the path and connectivity in a directed graph.
7. Explain complete and extended binary trees.
8. Define Ring with example.
9. Explain traversing binary tree.
10. Define bipartite graphs.

Section—D 6 each

(Long Answer Type Questions)

Note : Attempt all questions in 150 words.

1. Consider the Z of integers and an integer $m > 1$. We say that x is congruent to y modulo m, written $x \equiv y \pmod{m}$ if $x - y$ is divisible by m. Show that this defines an equivalence relation on Z.

Or

Is the following argument valid ?

If two sides of a triangle are equal then the opposite are equal,

Two side of a triangle are not equal,

$\therefore$ The opposite angles are not equal.

2. Let a, b be elements of a Boolean algebra, then $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$.

Or

Let L be a bounded distributive lattice. It a complement of any element exists, it is unique.

3. Show that every finite integral domain is a field.

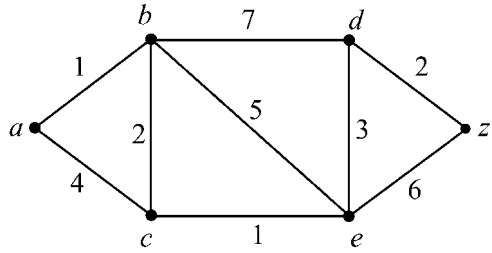
Or

Let (H, *) and (K, *) are subgroups of group (G, *). Then show that (H $\cap$ K, *) is also a subgroup.

4. Prove that if G is a connected graph and every vertex of G has even degree, then G has a Euler circuit.

Or

Compute the shortest distance between source *a* and destination *z* using Dijkstra's algorithm for the following graphs :



5. Write the applications of trees in Computer Science.

Or

Prove that every connected graph has at least *one* spanning tree.