Roll No. ....

E–3821

M. A./M. Sc. (Previous) EXAMINATION, 2021

MATHEMATICS

Paper First

(Advanced Abstract Algebra)

Time : Three Hours] [*Maximum Marks : 100*

Note : Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) If G is a commutative group having a composition series, then G is a finite group. Prove it.

   (b) Prove that any two composition series of a finite group are equivalent.

   (c) If E is a finite separable extension of a field F, then prove that E is a simple extension of F.

Unit—II

2. (a) Define group of F-automorphisms of E. If E is a finite extension of a field F, then prove that :

$$|G(E|F)| \leq [E:F] .$$

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(b) Find the Galois group of $x^4 + 1 \in Q[x]$.

(c) Show that the polynomial $x^7 - 10x^5 + 15x + 5$ is not solvable by radicals over Q.

Unit—III

3. (a) State and prove the fundamental theorem on homomorphism of modules.

(b) Prove that every homomorphic image of a Noetherian module is Noetherian.

(c) State and prove Wedderburn-Artin theorem.

Unit—IV

4. (a) For any non-zero $T \in A(V)$ there exists a unique monic polynomial $m(x) \in F(x)$ such that :

(i) $m(T) = 0$

(ii) For any $g(x) \in F(x)$, $g(T) = 0$, if and only if $m(x) \mid g(x)$

(iii) $F(T) = \dfrac{F(x)}{(m(x))}$. Prove.

(b) Prove that, two nilpotent linear transformations S, $T \in A(V)$ are similar if and only if they have the same invariants.

(c) Find the Jordan canonical form of the matrix :

$$A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}.$$

Unit—V

5. (a) Find the invariant factors of the matrix :

$$\begin{bmatrix} -x & 4 & -2 \\ -3 & 8-x & 3 \\ 4 & -8 & -2-x \end{bmatrix}.$$

(b) Reduce the following matrix A to a rational canonical form :

$$A = \begin{bmatrix} -3 & 2 & 0 \\ 1 & 0 & 1 \\ 1 & -3 & -2 \end{bmatrix}.$$

(c) Find invariant factors, elementary divisors and the Jordan canonical form of the following matrix :

$$\begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}.$$