Roll No. ....

E–3830

M. Sc./M. A. (Final) EXAMINATION, 2021

MATHEMATICS

(Optional)

Paper Fourth (*i*)

(Operations Research)

Time : Three Hours] [*Maximum Marks : 100*

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) For the following linear programming problem :

Maximize :

$$z = 3x_1 + 5x_2$$

subject to the constraints :

$$x_1 + x_2 \le 1$$

$$2x_1 + 3x_2 \le 1$$

$$x_1, x_2 \ge 0.$$

obtain the variations in c_j (j = 1, 2) which are permitted without changing the optimal solutions.

(b) Apply the principle of duality to solve the following linear programming problem :

Maximize :

$$z = 2x_1 + x_2$$

Subject to the constraints :

$$x_1 + 2x_2 \le 10$$

$$x_1 + x_2 \le 6$$

$$x_1 - x_2 \le 2$$

$$x_1 - 2x_2 \le 1$$

$$x_1, x_2 \ge 0 .$$

(c) For the following parametric linear programming problem :

Maximize :

$$z = (3 - 6\lambda)\, x_1 + (2 - 2\lambda)\, x_2 + (5 + 5\lambda)\, x_3$$

Subject to the constraints :

$$x_1 + 2x_2 + x_3 \le 430$$

$$3x_1 + 2x_3 \le 460$$

$$x_1 + 4x_2 \le 420$$

$$x_1, x_2, x_3 \ge 0 .$$

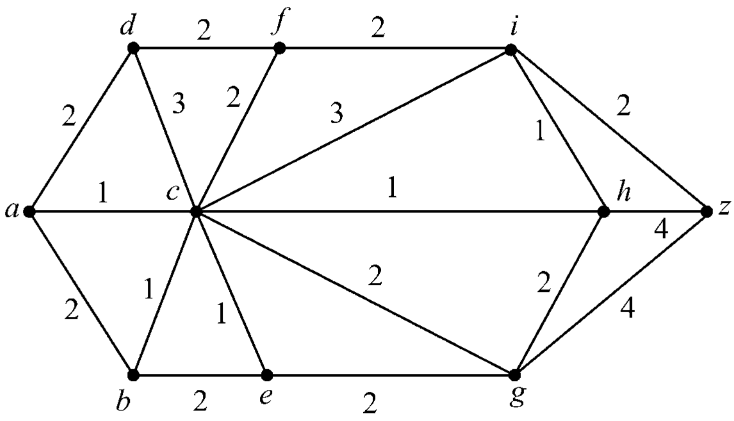
Find the range of λ over which the solution remains basic feasible and optimal.

Unit—II

2. (a) Solve the transportation problem with the cost coefficients, demands and supplies as given in the following table :

Origin	W_1	W_2	W_3	W_4	Supply
O_1	1	2	–2	3	70
O_2	2	4	0	1	38
O_3	1	2	–2	5	32
Demand	40	28	30	42	

(b) Find the shortest path from *a* to *z* in the following weighted graph :



(c) A small assembly plant assembles PCs through 9 interlinked stages according to the following precedence/ process :

Stage from to	**Duration (hours)**
1—2	4
1—3	12
1—4	10
2—4	8
2—5	6
3—6	8
4—6	10
5—7	10
6—7	0
6—8	8
7—8	10
8—9	6

(i) Draw an arrow diagram (network) representing above assembly work.

(ii) Tabulate earliest start, earliest finish, latest start and latest finish time for all the stages.

(iii) Find the critical path and the assembly duration.

(iv) Tabulate total float, free float and independent float.

Unit—III

3. (a) Solve the following L. P. P. by dynamic programming approach :

Minimize :

$$z = x_1^2 + 2x_2^2 + 4x_3$$

Subject to the constraints :

$$x_1 + 2x_2 + x_3 \geq 8$$

$$x_1, x_2, x_3 \geq 0 .$$

(b) Two companies A and B are competing for the same product. Their different strategies are given in the following payoff matrix :

$$\begin{array}{cc} & \text{Company A} \\ \text{Company B} & \begin{bmatrix} 2 & -2 & 3 \\ -3 & 5 & -1 \end{bmatrix} \end{array}$$

Use linear programming to determine the best strategies for both the players.

(c) Describe the branch and bound method for the solution of integer programming problem.

Unit—IV

4. (a) Explain blending problem.

(b) Formulate petroleum refinery operations as a linear programming problem.

(c) Explain Leontief system.

Unit—V

5. (a) Use Beale's method to solve the N. L. P. P. :

Maximize :

$$z = 10x_1 + 25x_2 - 10x_1^2 - x_2^2 + 4x_1x_2$$

Subject to the constraints :

$$x_1 + 2x_2 \le 10$$

$$x_1 + x_2 \le 9$$

$$x_1, x_2 \ge 0.$$

(b) Using Wolfe's method to solve the following Q. P. P. :

Maximize :

$$z = 4x_1 + 6x_2 - 2x_1^2 - 2x_2^2 - 2x_1x_2$$

Subject to the constraints :

$$x_1 + 2x_2 \le 2$$

$$x_1, x_2 \ge 0.$$

(c) What do you mean by quadratic programming problem ? How does quadratic programming problem differ from linear programming problem ?