Roll No. .....

E–3882

B. C. A. (Part I) EXAMINATION, 2021

(Old Course)

Paper Second

CALCULUS AND STATISTICAL METHODS

(101)

Time : Three Hours] [*Maximum Marks : 50*

Note : Attempt any *two* parts from each Unit/question. All questions carry equal marks.

Unit—I

1. (a) Prove that :

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$$

(b) Test for continuity of the following function at $x = 0$:

$$f(x) = \begin{cases} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(c) If :

$$f(x) = \begin{cases} \dfrac{x}{1 + e^{1/x}} ; & x \neq 0 \\ 0 \quad ; & x = 0 \end{cases}$$

then show that $f(x)$ is continuous but not differentiable at $x = 0$.

Unit—II

2. (a) Find $\dfrac{dy}{dx}$; when

$$x = a(t + \sin t)$$
$$y = a(1 - \cos t)$$

(b) Find the value of $\dfrac{dy}{dx}$; when $x^{\tan y} = y^{\tan x}$.

(c) If $y = \log\left\{\dfrac{ax + b}{cx + d}\right\}$, then find $\dfrac{dy}{dx}$.

Unit—III

3. (a) Find the equation of the normal to the curve $ay^2 = x^3$ at the point (am^2, am^3).

(b) Prove that in the curve $y^2 = ax^3$ the square of subtangent is proportional to the subnormal.

(c) Find the maximum and minimum values, if any, of the following function :

$$f(x) = \sin x + \frac{1}{2}\cos 2x, \; x \in \left[0, \frac{\pi}{2}\right].$$

Unit—IV

4. (a) The first twelve letters of the alphabet are written at random. Find the probability that there are exactly four letters between A and B.

   (b) Explain the following terms with examples :

   (i) Random variable

   (ii) Conditional probability

   (iii) Compound probability

   (iv) Equally like events

   (c) A bag contains 3 white and 2 black balls, an another bag contains 5 white and 3 black balls. If a bag is selected at random and a ball is drawn from it, find the probability that it is white.

Unit—V

5. (a) For a Poisson distribution with mean m, show that :

$$\mu_{r+1} = mr.\mu_{r-1} + \frac{md\mu_r}{dm},$$

where $\mu_r = \sum_{x=0}^{\infty} (x - m)^r \frac{e^{-m} m_x}{x!}$.

   (b) Fit a straight line to the following data regarding x as the independent variables :

x	y
0	1
1	1.8
2	3.3
3	4.5
4	6.3

(c) Find the coefficient of correlation for the following table :

x	y
10	18
14	12
18	24
22	6
26	30
30	36