E–995

M. A./M. Sc. (Fourth Semester) (Main/ATKT) EXAMINATION, May-June, 2021

MATHEMATICS

(Optional)

Paper Third (B)

(Fuzzy Sets Theory and Its Application—II)

Time : Three Hours]   [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A   1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. Which one of the following propositions is unconditional and qualified proposition ?

    (a) p : υ is F.

    (b) p : υ is F is S.

    (c) p : If x is A, then y is B is S.

    (d)   None of the above

P. T. O.

2. The truth value of the fuzzy proposition :

    p : There are Q i's in I such that $v_1(i)$ is F_1 and $v_2(i)$ is F_2

    is defined by :

    (a) $T(p) = Q(|E|)$; where $w = |E|$

    (b) $T(p) = Q(w)$; where $w = \dfrac{|E_1 \cap E_2|}{|E_1|}$

    (c) $T(p) = Q(w)$, where $w = |E_1 \cap E_2|$

    (d)   None of the above

3. For a modifier h; which one is correct ?

    (a) $h(0) = 0$ and $h(1) = 1$

    (b) h is continuous function

    (c)   If h is a strong, then h^{-1} is weak and vice-versa

    (d)   All of the above

4. Quantifier of first kind is also known as :

    (a)   Relative quantifier

    (b)   Absolute quantifier

    (c)   Linguistic hedge

    (d)   None of the above

5. R-implication is defined by :

    (a) $T(a, b) = \sup\{x \in [0,1] : i(a, x) \le b\}$

    (b) $T(a, b) = \inf\{x \in [0,1] : i(a, x) \le b\}$

    (c) $T(a, b) = u(c(a), b)$

    (d)   None of the above

6. The kernel of any expert system consists of :

    (a) a knowledge base

    (b) a database

    (c) an inference engine

    (d) All of the above

7. If $B_1' = A' \circ \left(\bigcup\limits_{\rho \in N_n} R_j \right)$; $B_2' = A' \circ \left(\bigcap\limits_{\rho \in N_n} R_j \right)$;

    $B_3' = \bigcup\limits_{\rho \in N_n} A' \circ R_j$; $B_4' = \bigcap\limits_{\rho \in N_n} A' \circ R_j$, then which one is

    correct ?

    (a) $B_1' \subseteq B_2' \subseteq B_3' \subseteq B_4'$

    (b) $B_2' \subseteq B_4' \subseteq B_1' = B_3'$

    (c) $B_2' \subseteq B_4' \subseteq B_3' \subseteq B_1'$

    (d)   None of the above

8. Which one of the following fuzzy implication satisfy the generalized modus ponens, modus tollens and hypothetical syllogism ?

    (a)   Gaines Rescher implication $\mathbf{T}_s$

    (b) W_u implication $\mathbf{T}_{wu}$

    (c)   Both (a) and (b)

    (d)   None of the above

9. The membership function forced of uncertainty is expressed by :

    (a)   Triangular form

    (b)   Trapezoidal form

    (c)   Gaussian form

    (d)   None of the above

10. The fuzzification function has the form :

   (a) $f_e : [a, -a] \to R$

   (b) $f_e : [0, 1] \to R$

   (c) $f_e : [-a, a] \to R$

   (d)   None of the above

11. In any fuzzy neural networks :

   (a)   Inputs are fuzzy numbers.

   (b)   Outputs are fuzzy numbers.

   (c)   Weights are fuzzy numbers.

   (d)   All of the above

12. A finite fuzzy automaton A = (X, Y, Z, R, S) defined by X is non-empty finite set of input states , Y is output state, Z is internal states and R is relation on Z × Y and S is fuzzy relation on X × Z × Z, then R is called :

   (a)   Response relation

   (b)   State transition relation

   (c)   Both (a) and (b)

   (d)   None of the above

13. Which one is an example of fuzzy controllers ?

   (a)   Inverted pendulum

   (b)   Air conditioner

   (c)   Blood pressure during anesthesia

   (d)   All of the above

14. The method of defuzzification given by
$d_{\mathrm{cm}}(\mu) = \dfrac{\inf M + \sup M}{2}$ is given by :

   (a) Center of area method

   (b) Center of maxima method

   (c) Weighted average method

   (d) None of the above

15. Center of area method is also known as :

   (a) Centre of gravity method

   (b) Centre of sum method

   (c) Centre of maxima method

   (d) None of the above

16. If e, $\dot{e}$ and υ are the variables of fuzzy controller, the inference rules have the canonical form. If $e = A$ and $\dot{e} = B$, then $\upsilon = C$, where A, B and C are fuzzy numbers chosen from the set of fuzzy numbers that represent the linguistic states NL, NM, NS, AZ, PS, PM and PL, then the total number of possible non-conflicting fuzzy inference rules is :

   (a) $7^2 = 49$

   (b) $2^7 = 128$

   (c) $7^3 = 343$

   (d) None of the above

17. Which of the following is true ?

    (a) $u_i(x)$ should be zero if the constraints are strongly violated.

    (b) $u_i(x)$ should be 1 of all the constraints are fixed.

    (c)   Both (a) and (b) are true.

    (d)   None of the above

18. The component(s) of individual decision-making is/are :

    (a)   a set of possible events/action

    (b)   a set of goal

    (c)   a set of constraints

    (d)   All of the above

19. In a fuzzy linear programming problem, the set of vectors which satisfy all the constraints is called :

    (a)   a fuzzy set

    (b)   a convex set

    (c)   a feasible set

    (d)   a finite set

20. For each $x_i \in X$, the overall relation preference grades $p(x_i)$ of x_i with respect to all other alternatives in X is given by the formula :

    (a) $p(x_i) = \min\limits_{x_j \in X}(x_i, x_j)$

    (b) $p(x_i) = \max\limits_{x_j \in X}(x_i, x_j)$

    (c) $p(x_i) = \sup\limits_{x_i \in X}(x_i, x_j)$

    (d)   None of the above

Section—B   2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Write any *two* differences between classical logic and fuzzy logic.

2. Define generalized modus ponens.

3. What do you mean by metaknowledge base in fuzzy expert system ?

4. Which is a suitable fuzzy implication for approximate reasoning based upon the generalized modus tollens ?

5. Write *four* modules of a general fuzzy controller.

6. Write center of maxima method.

7. If goal $G = \dfrac{.11}{a_1} + \dfrac{.3}{a_2} + \dfrac{.48}{a_3} + \dfrac{.8}{a_4}$ and subject to constraints :

$$C_1 = \frac{.4}{a_1} + \frac{.6}{a_2} + \frac{.2}{a_3} + \frac{.2}{a_4}$$

$$C_2 = \frac{.1}{a_1} + \frac{.9}{a_2} + \frac{.7}{a_3} + \frac{1}{a_4}$$

then obtain fuzzy decision.

8. Write any *two* methods for total ordering of fuzzy numbers A and B to be compare.

Section—C   3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Let sets of values of variable x, y and z be $X = \{x_1, x_2, x_3\}$, $Y = \{y_1, y_2\}$ and $Z = \{z_1, z_2\}$ respectively. Assume that a proposition "If x is A, then y is B" and "y is B, then z is C" is given, where :

$$A = \frac{.5}{x_1} + \frac{1}{x_2} + \frac{.6}{x_3}$$

$$B = \frac{1}{y_1} + \frac{.4}{y_2}$$

$$C = \frac{.2}{z_1} + \frac{1}{z_2}$$

Derive a conclusion in the form

"If x is A, then z is C" by using the method of generalized hypothetical syllogism.

2. Define unconditional and qualified proposition.

3. Let $i\,(a, b) = \max\,(0, a + b - 1)$ for all $a, b \in [0, 1]$ and let $\mathbf{T}_a$ be Lukaisewicz implication that i.e. $\omega_i(a, b) = \mathbf{T}_a(a, b) = \min\,(1, 1 - a + b)$ for all $a, b \in [0, 1]$, then $A \overset{\omega_i}{\circ} B = [C(B) \overset{\omega_j}{\circ} C(A)]^{-1}$ holds for any fuzzy sets A, B and consequently $R = A \overset{w_j}{\circ} B$ is the greatest approximate solution to $B = A \overset{i}{\circ} R$ and $C\,(A) = C\,(B) \overset{i}{\circ} R^{-1}$ for the standard fuzzy complement C.

4. Explain forced uncertainty and opted uncertainty.

5. What do you mean by Defuzzification ?

6. Let A_1, A_2 and A_3 are three fuzzy sets as shown in the following figures (1), (2) and (3). Figure (4) illustrates the aggregate of fuzzy sets using the centre of sums method to defuzzified A_1, A_2 and A_3.
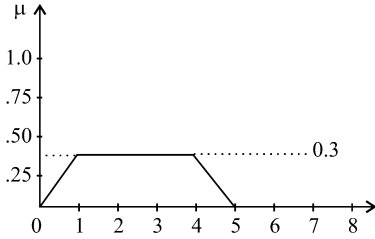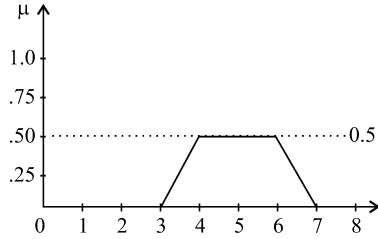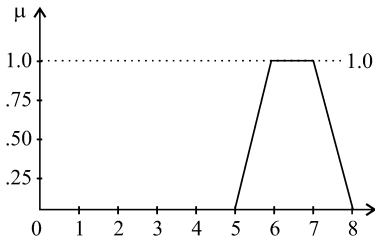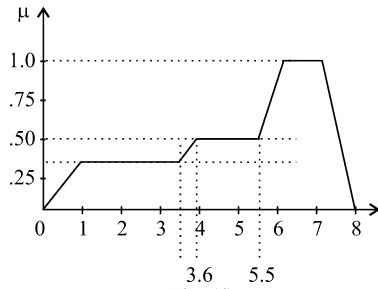
Fig. (1) : Fuzzy Set A_1

Fig. (2) : Fuzzy Set A_2

Fig. (3) : Fuzzy Set A_3

Fig. (4)

7. Explain fuzzy systems and neural networks.

8. Let us consider an automaton with $X = \{x_1, x_2\}$, $Z = \{z_1, z_2, z_3\}$ and the state transition function expressed by the matrix :

$$
\begin{array}{cc}
 & x_1 \quad x_2 \\
\begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array}
\left[\begin{array}{cc} z_1 & z_2 \\ z_3 & z_1 \\ z_1 & z_3 \end{array}\right]
\end{array}
$$

whose entries are next internal states for any given present internal and output states. The fuzzy goal at $t = 2$ is $C^2 = \dfrac{.3}{z_1} + \dfrac{1}{z_2} + \dfrac{.8}{z_3}$ and the fuzzy constraints :

$$A^0 = \frac{.7}{x_1} + \frac{1}{x_2}$$

$$A^1 = \frac{1}{x_1} + \frac{.6}{x_2}$$

Find the best decision.

Section—D   5 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Define truth value restriction method for conditional and qualified proposition. Suppose we have fuzzy proposition of the form p : "If x is A, then y is B is very true", where :

$$A = \frac{1}{x_1} + \frac{.5}{x_2} + \frac{.7}{x_3}$$

$$B = \frac{.6}{y_1} + \frac{1}{y_2}$$

and S stands for every true. Let $S(a) = a^2$ for all $a \in [0, 1]$.
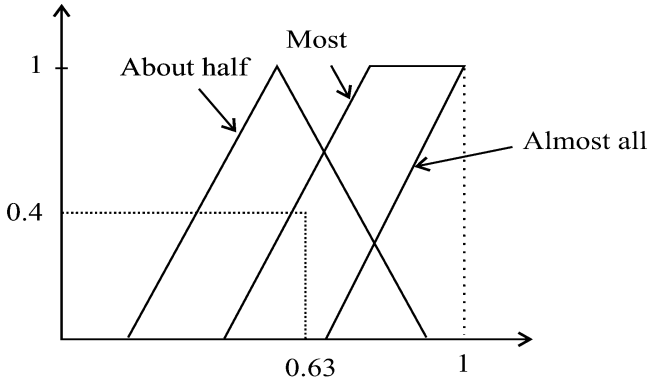
Given a fact "x is A^1" where :

$$A^1 = \frac{.9}{x_1} + \frac{.6}{x_2} + \frac{.7}{x_3}$$

Derive a conclusion in the form "y is B'".

Or

Assume that there are ten students in a class about half of them are young girls. Most of those young girls are good

students. Using the quantifiers specific shown in figure below, answer the question. Approximately how many young girls in the class are good students ?

2. Explain multiconditional approximate reasoning.

Or

Let the range of the membership function A in B $(y) = \sup_{x \in X} i[A(x), \mathbf{T}(A(x), B(y)]$ cover the whole interval [0, 1]. Then show that :

(a) Gaines Rescher implication $\mathbf{T}_s$

(b) Godel Implication $\mathbf{T}_g$

(c) W_u Implication $\mathbf{T}_{wu}$

satisfy given equation for any *t*-norm *i*.

3. Explain fuzzy controllers.

Or

Consider a fuzzy automaton with $X = \{x_1, x_2\}$; $Y = \{y_1, y_2, y_3\}$; $Z = \{z_1, z_2, z_3, z_4\}$ whose output relation

R and state transition relations S are defined respectively by the matrix :

$$
\begin{array}{c}
\quad\quad\quad y_1 \; y_2 \; y_3 \\
R = \begin{array}{c} z_1 \\ z_2 \\ z_3 \\ z_4 \end{array}
\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
.5 & 1 & .3
\end{array}\right]
\end{array}
$$

and the three-dimensional array.

$$
\begin{array}{c}
\quad\quad x_1 \quad\quad\quad\quad\quad\quad\quad x_2 \\
\quad z_1 \; z_2 \; z_3 \; z_4 \quad\quad\quad z_1 \; z_2 \; z_3 \; z_4 \\
S = \begin{array}{c} z_1 \\ z_2 \\ z_3 \\ z_4 \end{array}
\left[\begin{array}{cccc|cccc}
0 & .4 & .2 & 1 & 0 & 0 & 1 & 0 \\
.3 & 1 & 0 & .2 & .2 & 0 & 0 & 1 \\
.5 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & .3 & 0 & .6
\end{array}\right]
\end{array}
$$

Generate sequence of three fuzzy internal and output states under the following condition : The initial fuzzy state is $C^1 = \begin{bmatrix} 1 & .8 & .6 & .4 \end{bmatrix}$ the input fuzzy states are $A^1 = \begin{bmatrix} 1 & .4 \end{bmatrix}$, $A^2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$.

4. Solve the following fuzzy linear programming problem :

Max. :

$$Z = 6x_1 + 5x_2$$

s. t. :

$$(5, 3, 2)\, x_1 + (6, 4, 2)\, x_2 \le (25, 6, 9)$$
$$(5, 3, 2)\, x_1 + (2, 1.5, 1)\, x_2 \le (13, 7, 4)$$
$$x_1, x_2 > 0.$$

Or

Explain multi-criteria decision-making.