Roll No. ....

E–997

M. A./M. Sc. (Fourth Semester) (Main/ATKT) EXAMINATION, May-June, 2021

MATHEMATICS

(Optional—A)

Paper Fourth

(Operations Research—II)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 2 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. "Markovian property" is used in :
   (a) Integer Programming
   (b) Dynamic Programming
   (c) Non-linear Programming
   (d) None of the above

2. "Principle of optimality" searched by :
   (a) Benjamin
   (b) Witcher
   (c) Bellman
   (d) Chung
3. Two person zero sum game is also called :
   (a) Rectangular game
   (b) Square game
   (c) Cubic game
   (d) Octahedron game
4. Participants in game theory is called :
   (a) Sportsman
   (b) Gamer
   (c) Pay-off
   (d) Player
5. If all variables are required to take real values, then problem is called :
   (a) Pure integer
   (b) Mixed integer
   (c) Fractional integer
   (d) Gomorian integer
6. Program related to tree :
   (a) Dymamic programming
   (b) Game theory
   (c) Branch and bound technique
   (d) Queuing theory

7. Queuing system is called :
   (a) Zero-server model
   (b) One-server model
   (c) K-server model
   (d) Another model
8. In model only arrivals are connected and no departure takes place is called :
   (a) Pure birth models
   (b) Pure death models
   (c) Steady state
   (d) Poisson model
9. The relative minimum of the function :

$$f(x_1, x_2) = x_1^3 + x_2^3 - 3x_1 - 12x_2 + 25$$

   is at the point :
   (a) $x_1 = -1, x_2 = -2$
   (b) $x_1 = -1, x_2 = 2$
   (c) $x_1 = 1, x_2 = 2$
   (d) $x_1 = 1, x_2 = -2$
10. The quadratic form $x^T Q x$ is said to be negative semi-definite, if :
   (a) $x^T Q x > 0$
   (b) $x^T Q x < 0$
   (c) $x^T Q x \geq 0$
   (d) $x^T Q x \leq 0$

Section—B 2 each

(Very Short Answer Type Questions)

Note : Attempt all questions.

1. Write *one* characteristic of dynamic programming.
2. Write principle of optimality.
3. What is Saddle point ?
4. What is mixed strategies in game theory ?
5. Which method is based on modified simplex method ?
6. What is queue discipline ?
7. What is transient state ?
8. What is 0 – 1 integer programming ?

Section—C 3 each

(Short Answer Type Questions)

Note : Attempt all questions.

1. Write *two* applications of dynamic programming.
2. Write *two* differences between linear programming and dynamic programming.
3. What is symmetric game ?
4. Define strategy and optimal strategy with reference to game theory.
5. Write the advances of branch and bound method.
6. Write *two* limitations of queuing theory.
7. What is traffic intensity ?
8. Write the significance of Lagrange multipliers.

Section—D 4 each

(Long Answer Type Questions)

Note : Attempt all questions.

1. Use dynamic programming to show that :

$$z = p_1 \log p_1 + p_2 \log p_2 + ... + p_n \log p_n$$

subject to the constraints

$$p_1 + p_2 + ... + p_n = 1$$

and $$p_j \geq 0$$

where $j = 1, 2, 3, ..., n$ is a minimum, when $p_1 = p_2 = ... = p_n = \frac{1}{n}$.

Or

Use dynamic programming to solve the following LPP :

Maximize :

$$z = 3x_1 + 5x_2$$

subject to the constraints :

$$x_1 \leq 4$$

$$x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 18$$

and $$x_1, x_2 \geq 0.$$

2. Solve the following 2 × 2 game graphically :

Player B

$$\text{Player A} \begin{array}{c} A_1 \\ A_2 \end{array} \overset{\begin{array}{cccc} B_1 & B_2 & B_3 & B_4 \end{array}}{\begin{bmatrix} 2 & 1 & 0 & -2 \\ 1 & 0 & 3 & 2 \end{bmatrix}}$$

Or

Solve the following game by linear programming technique :

Player B

$$\text{Player A} \begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \\ 6 & 2 & -2 \end{pmatrix}$$

3. Solve the following mixed integer programming problem :

Maximize :

$$z = 4x_1 + 6x_2 + 2x_3$$

subject to the constraints :

$$4x_1 - 4x_2 \leq 5$$

$$-x_1 + 6x_2 \leq 5$$

$$-x_1 + x_2 + x_3 \leq 5$$

$$x_1, x_2, x_3 \geq 0$$

x_1 and x_3 are integers.

Or

Use Branch and Bound method to solve the following LPP :

Maximize :

$$z = 7x_1 + 9x_2$$

subject to the constraints :

$$-x_1 + 3x_2 \leq 6$$

$$7x_1 + x_2 \leq 35$$

$$x_2 \leq 7$$

$$x_1, x_2 \geq 0.$$

4. Assume that the goods trains are coming in a yard at the rate of 30 trains per day and suppose that the inter-arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 minutes. If the yard can admit a train at a time (there being 10 lines, one of which is reserved for shunting purposes). Calculate the probability that the yard is empty and find the average queue length.

Or

A bank has 2 tellers working on savings accounts. The 1st teller handles withdrawals only. The 2nd teller handles deposits only. It has been found the service time distribution for both deposits and withdrawals is exponential with mean service time 3 minutes per customer. Deposits are found to arrive in Poisson fashion with mean arrival rate of 14 per hour. What would be the effect on the average waiting time

for depositors and withdrawals if each teller could handle both withdrawal and deposits ? What could be the effect if this could be accomplished by increasing the mean service time to 3.5 minutes ?

5. Use Wolfe's method to solve the QPP :

Maximize :

$$z = 2x_1 + 3x_2 - 2x_1^2$$

subject to the constraints :

$$x_1 + 4x_2 \leq 4$$

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$

Or

Use Beal's method to solve the following NLPP :

Minimize :

$$z = 6 - 6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2$$

subject to the constraints :

$$x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0.$$