Roll No. ....

E–521

M. A./M. Sc. (Second Semester)

EXAMINATION, May-June, 2021

MATHEMATICS

Paper Third

(General and Algebraic Topology)

Time : Three Hours] [*Maximum Marks : 80*

Note : Attempt all Sections as directed.

Section—A 1 each

(Objective/Multiple Choice Questions)

Note : Attempt all questions.

Choose the correct answer :

1. Let $\pi_1 : R \times R \to R$ be projection on to first coordinate. Then which one is not true ?
   (a) π_1 is continuous
   (b) π_1 is subjective
   (c) π_1 is open
   (d) π_1 is closed

2. Product of Hausdorff space is :
   (a) Hausdorff
   (b) Disjoint
   (c) Normal
   (d) None of the above

3. A topological space is said to be T_3-space if it is ....
   (a) Normal and T_1
   (b) Regular and T_1
   (c) Completely regular and T_1
   (d) None of the above

4. Product of normal space is :
   (a) Need not normal
   (b) Hausdorff
   (c) Normal
   (d) Need not Hausdorff

5. Tychonoff theorem states :
   (a) Any arbitrary product of compact spaces is compact in the product topology.
   (b) A topological space for which every open covering contains a countable subcovering.
   (c) Path connected topological space is connected but converse is not true.
   (d) None of the above

6. A Hilbert cube is a space of the form :

   (a) $(0,1)^I$

   (b) $[-1,1]^I$

   (c) $[0,1]^I$

   (d) $(-1,1)^I$

   where I is enumerable set.

7. Let $\{(X_\alpha\ T_\alpha) : \alpha \in \wedge\}$ be arbitrary collection of topological spaces and let X = product of $\{X_\alpha : \alpha \in \wedge\}$. Then the topology T on X which has subbase the collection $B = \{\pi_\alpha^{-1}(G_\alpha) : G_\alpha \in T_\alpha \forall \alpha \in \Lambda\}$ is called :

   (a) Usual topology

   (b) Discrete topology

   (c) Tychonoff topology

   (d) Co-countable topology

8. A subset of R^n is closed and bounded iff it is closed. This theorem is known as :

   (a) Generalised Heine-Borel theorem

   (b) Tychonoff theorem

   (c) Alexander sub-base theorem

   (d) Tietze extension theorem

9. A compact Hausdorff space is separable and metrizable if it is :

   (a) First countable

   (b) Second countable

   (c) Normal

   (d) Regular

10. Every regular Lindeloff space is :

    (a) Normal

    (b) Compact

    (c) Paracompact

    (d) None of the above

11. A space X is metrizable iff it is paracompact and locally metrizable. This theorem is known as :

    (a) Locally metrizable theorem

    (b) Smirnov metrization theorem

    (c) Special van Kampen theorem

    (d) None of the above

12. Which of the following is not an example of locally finite ?

    (a) $u = \{(n, n+z) : n \in z\}$

(b) $\beta = \{(n, 2n) : n \in z\}$

(c) $u_1 = \left\{\left(0\ \frac{1}{n}\right) : n \in z\right\}$

(d) None of the above

13. Let $X = \{a\,b\,c\}$. Then which of following is a filter ?

(a) $F_1 = \{\phi, \{a\}, \{a, b\} X\}$

(b) $F_2 = \{\{a\}, \{b\} \{ab\} X\}$

(c) $F_3 = \{\{ab\}, \{ac\} X\}$

(d) $F_4 = \{\{a\}, \{ab\}, \{ac\} X\}$

14. Which is not true ?

(a) Intersection of all filter on X is the filter {X} which is coarsest filter on X.

(b) Union of two filters on set is filter.

(c) The collection of all sets of natural numbers whose complements are finite is a filter.

(d) The collection of all sets of real numbers whose complements are countable is a filter on R.

15. Which is true ?

(a) A subset A of topological space is closed iff each net in A converges to a point in X – A.

(b) A topological space (X T) is Hausdorff iff every net in X can converge at least one point of X.

(c) Every convergent net in Hausdorff space X has unique cluster point which is not a unique limit point of the net.

(d) A point *x* in the topological space is a cluster point of net F iff some subnet of F converges to *x*.

16. Let X be a non-empty set and B be non-empty family of subsets of X such that :

(i) $\phi \notin B$

(ii) $P, Q \in B \Rightarrow R \in B \text{ s.t. } R \subset R \cap Q$

Then B is called :

(a) Ultra filter

(b) Filter base

(c) Free filter

(d) Fixed filter

17. The relation of being homotopic relative to any set A is a an :

(a) partial ordered relation

(b) equivalence relation

(c) anti-symmetric relation

(d) None of the above

18. A continuous map $f : X \to Y$ is said to be null homotopic it is homotopic to some ....

    (a) Identity mapping

    (b) Constant mapping

    (c) Both are true

    (d) Both are false

19. Let $x_0, x_1 \in X$. If there is a path in X from x_0 to x_1, then the group $\pi_1(X, x_0)$ and $\pi_1(x, x_1)$ are :

    (a) Homomorphic

    (b) Isomorphic

    (c) Endomorphism

    (d) Homotopy

20. A covering map is a/an ....

    (a) local homeomorphism

    (b) homeomorphism

    (c) isomorphism

    (d) None of the above

Section—B $1\frac{1}{2}$ each

(Very Short Answer Type Questions)

Note : Attempt all questions using 2-3 sentences.

1. Define projection function.

2. Define evaluation function.
3. State Alexander sub-base theorem.
4. Define path connected space.
5. Define paracompact space.
6. Define locally finite space.
7. Define directed set.
8. Define filter.
9. What is fundamental group ?
10. Define covering space.

Section—C $2\frac{1}{2}$ each

(Short Answer Type Questions)

Note : Attempt all questions precisely using less than 75 words.

1. Prove that the intersection of any family of boxes is box and intersection of finite number of large boxes is large box.
2. If f_i be a mapping of a topological space X into a topological space $Y_i \ \forall\, i \in I$, then the evaluation mapping e of X into $\Pi\{Y_i : i \in I\}$ is one one iff the collection $\{f_i : i \in I\}$ distinguishes points of X.
3. If X_1 and X_2 are connected space, then show that product space $X_1 \times X_2$ is also connected.
4. Let J be any set of cardinality not exceeding C. Then for any separable space Y, the power Y^J is separable.

5. Prove that every Tychonoff space X can be embedded as a subspace of cube.

6. Prove that every paracompact space X is normal.

7. Every filter F on set X is the intersection of all the ultrafilters finer than F.

8. Let (X T) be a topological and $Y \subset X$. Then a point $x_0 \in X$ is a limit point of Y iff there exists a net in $Y\text{-}\{x_0\}$ converges to x_0.

9. Let f_1, f_2, g_1 and g_2 be paths such that $f_1 \sim g_1$ and $f_2 \sim g_2$. If $f_1 * f_2$ exist, then $g_1 * g_2$ exist and $f_1 * f_2 \sim g_1 * g_2$.

10. Prove that any covering map $P : \overline{X} \to X$ is open.

Section—D 4 each

(Long Answer Type Questions)

Note : Attempt all questions precisely using 150 words.

1. The product space $X = \pi\{X_i : i \in I\}$ is a T_1-space iff each coordinate space is T_1-space.

Or

The product of completely regular spaces is completely regular.

2. Show that metrisability is countably productive property.

Or

A product space is locally connected iff each coordinate space is locally connected and all except finitely many of them are connected.

3. State and prove Embedding lemma.

Or

State and prove Urysohn metrization theorem.

4. A topological space (X T) is Hausdorff iff every net in X can converge to at most one point.

Or

A filter F on set X is an ultrafilter iff F contains all those subset of X which intersect every member of F.

5. Prove that the fundamental group of circles is infinite cyclic.

Or

State and prove fundamental theorem of algebra.