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F - 3852

M.A./M.Sc.(Previous) Examination, 2022

Mathematics

Paper Second

(Real Analysis)

Time : Three Hours] *[Maximum Marks : 100*

Note: Attempt any two parts from each question. All questions carry equal marks.

Unit - I

1. (a) Define Riemann - Stieltjes integral in brief. If P* is a refinement of P, then show that

$L(P,f,\alpha)\le L(P^*,f,\alpha)$ and

$U(P^*,f,\alpha)\le U(P,f,\alpha)$

(b) State and prove the fundamental theorem of calculus.

(c) If $Y:[a,b]\to R^K$ be a curve and if $c\in(a,b)$, then prove that

$$\wedge_y(a,b)=\wedge_y(a,c)+\wedge_y(c,b).$$

Unit - II

2. (a) Show that by a rearrangement of the terms of a conditionally convergent series, the rearranged series can be made to converge, diverge or oscillate.

(b) State and prove Weierstrass approximation theorem.

(c) Define radius of convergence of a power series. Show that the series obtained by integrating or differentiating a power series term by term has the same radius of convergence as the original series.

Unit - III

3. (a) State and prove the generalized version of chain rule.

(b) If $u_1, u_2, u_3,.........u_n$ are functions of $y_1, y_2, y_3,.........y_n$, and

$y_1, y_2, y_3, ...y_n$, are functions of

$x_1, x_2, x_3, ... x_n$ then show that

$$\frac{\partial(u_1,u_2,u_3,...,u_n)}{\partial(x_1,x_2,x_3,...,x_n)} = \frac{\partial(u_1,u_2,...,u_n)}{\partial(y_1,y_2,y_3,...,y_n)} \times \frac{\partial(y_1,y_2,...,y_n)}{\partial(x_1,x_2,...,x_n)}$$

(c) Define stationary point of a function. Give Lagrange's multiplier method for an extremum for any function F.

Unit - IV

4. (a) Define Lebesgue measurable set. Show that a set A is measurable if and only if its complement A' is measurable.

(b) Define Borel measurable set and prove that a Borel measurable set is Lebesgue measurable.

(c) Prove that a function f is of bounded variation on [a,b] if and only if f is the difference of two monotone real - valued functions of [a,b].

Unit - V

5. (a) If (X,B,μ) be a measure space, $E_i \in B$, $\mu(E_i) < \infty$ and $E_i \supset E_{i+1}$ then

$$\mu\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{x\to\infty} \mu(E_n)$$

(b) State and prove Minkowski's inequality.

(c) State and prove Egoroff's theorem.