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F-3853

M.A./M.Sc. (Previous) Examination, 2022

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours] *[Maximum Marks : 100*

Note : All questions are compulsory. Attempt any two parts from each question. All questions carry equal marks.

Unit - 1

1. (a) Define countable set. Prove that a finite product of countable sets is countable.

   (b) Define accumulation point. Prove that a subset A of a topological space (X, τ) is closed if and only if A contains all its limit points.

   (c) Define base for a topology. Let (X, τ) be a topo logical space and $\beta c \tau$. Then prove that β is a base for τ if and only if for any $x \in X$ and any open set G containing x, there exists $B \in \beta$ such that $x \in B$ and $B \subset G$.

Unit-II

2. (a) Define continuous function in topological space. Let X and Y be topological spaces. Show that a mapping $f : X \rightarrow Y$ is continous if only if the inverse image under *f* of every open set in Y is open in X.

   (b) State and prove Urysohn's Lemma.

   (c) Define Normal space. Show that a closed. Subspace of a Normal space is normal.

Unit-III

3. (a) Show that a subspace of a real line is connected if and only if it is an interval.

   (b) Define Locally compact space. Show that any open subspace of a locally compact space is locally compact.

(c) State and prove the Stone-Cech compactification theorem.

Unit-IV

4. (a) Define projection map. Prove that the projection functions are open.

(b) Show that the product space $X_1 \times X_2$ are connected iff both X_1 *and* X_2 *are connected.*

(c) Prove that every second countable normal space is metrizable.

Unit-V

5. (a) Show that a filter *F* on a set *X* is an ultrafilter if and only if *F* contains all those subsets of *X* which intersect every member of *F*.

(b) Define covering map. Prove that a covering map is a local homeomorphism.

(c) Let (X, τ) be a topological space and $Y \subset X$. Then a point $x_0 \in X$ is a limit point of Y if and only if a net in Y-$\{x_0\}$ converges to $\{x_0\}$.