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F-3948**B.C.A. (Part-II) EXAMINATION, 2022****(NEW COURSE)****PAPER FIRST****CALCULUS AND DIFFERENTIAL EQUATIONS****(BCA-201)***Time : Three Hours]**[Maximum Marks:80*

Note : Attempt any two parts from each question. All questions carry equal marks. Only simple calculator is allowed.

Unit - I

1. (a) Let $f(x) = x \frac{e^{1/x} - e^{-1/x}}{e^{1/x} + e^{-1/x}}$ for $x \neq 0$, $f(0) = 0$

Show that f is continuous but not differentiable at $x=0$.

(b) State and prove Intermediate value theorem.

P.T.O.

(c) If $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

Then test the continuity of $f(x)$ at $x = 0$.

Unit - II

2. (a) If $y = \log(\log(\log x))$ then find $\frac{dy}{dx}$.
- (b) Investigate for what value of x , $5x^6 - 18x^5 + 15x^4 - 10$ is a maximum or minimum.
- (c) If $y = Ae^{-kt} \cos(pt + c)$, then prove that

$$\frac{d^2y}{dx^2} + 2k \frac{dy}{dx} + n^2y = 0$$

where $n^2 = p^2 + k^2$

Unit - III

3. (a) Show that $\int_0^{1/2} \frac{dx}{4+5\cos x} = \frac{\log 2}{3}$
- (b) Integrate $\int \frac{x^2}{(a+bx)^2} dx$
- (c) Integrate $\int \sin^{-1}x dx$

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Unit - IV

4. (a) Prove that $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$

(b) Find the value of $\int_0^1 \frac{x \tan^{-1} x}{(1+x^2)^{3/2}} dx$.

(c) Show that $\int_0^1 \frac{\log(1+x)}{1+x^2} dx = \frac{\pi}{8} \log 2$

Unit - V

5. (a) Solve $(x + y) (dx - dy) = dx + dy$.

(b) Solve the differential equation

$$\sqrt{a+x} \frac{dy}{dx} + x = 0$$

(c) Form the differential equation from $y = Ae^{2x} + Be^x + C$ where A , B and C are constant.