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F - 3969

B. C. A. (Part III) Examination, 2022
(Old Course)
Paper First
(Calculus and Geometry)
(301)

Time : Three Hours] *[Maximum Marks:50*

Note: Attempt any two parts from each question. All questions carry equal marks.

Unit - I

1. (a) Let $f(x) = x$, on [0,1], then show that f is R - integrable on [0,1] and $\int_0^1 f(x)\,dx = \frac{1}{2}$

(b) If $f \in R\,[a,b]$, then show that:

$$m(b-a) \le \int_a^b f dx \le M(b-a)$$

Where m and M are infimum and supremum of f on [a,b].

(c) If $f \in R\,[a,b]$, then prove that $|f| \in R[a,b]$ and

$$\left|\int_a^b f\right| \le \int_a^b |f|$$

Unit - II

2. (a) Find the maximum and minimum value of the function $u = x^3 y^2 (1 - x - y)$

(b) In a plane triangle, find the maximum value of $u = \sin A.\sin B.\sin C$ by Lagrange's method.

(c) Find the maximum and minimum values of function $w = x^2 + y^2 + z^2$, given that

$$ax^2 + by^2 + cz^2 = 1$$

Unit - III

3. (a) Prove that $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ converges.

(b) Discuss the convergence of the following improper integral: $I=\int_0^2 \frac{\log x}{\sqrt{2-x}}dx$

(c) Test the convergence of $\int_a^\infty \frac{e^{-x}\cos x}{x^2}dx,$ where $a>1$.

Unit - IV

4. (a) Find the equation of the right - circular cone whose vertex is the origin, axis is the z - axis and semi vertical angle is $\propto$.

(b) Find the equation of the cone whose vertex is (0,0,3) and base is the circle

$$x^2+y^2=4, z=0$$

(c) Find the equation of right - circular cylinder whose guiding circle is

$$x^2+y^2+z^2=9; \quad x-y+z=3$$

Unit - V

5. (a) Explain the relation between Cartesian and Polar coordinates.

(b) Show that the two conics $\frac{l_1}{r}=1+e_1\cos\theta$ and $\frac{l_2}{r}=1+e_2\cos(\theta-\alpha)$ will touch one another if:

$$l_1^2(1-e_2^2)+l_2^2(1-e_1^2)=2l_1l_2(1-e_1e_2\cos\alpha)$$

(c) If PSP' and QSQ' are two perpendicular focal chords of a conic, then show that

$$\frac{1}{PS.SP^1}+\frac{1}{SQ.SQ^1}$$ is constant.