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F - 3861

M.A./M. Sc. (Final) Examination, 2022
Mathematics
(Optional)
Paper Fourth (*ii*)
(Wavelets)

Time : Three Hours] *[Maximum Marks:100*

Note: Attempt any two parts from each question. All questions carry equal marks.

Unit - I

1. (A) If ψ be a function defined by $\hat{\psi}(\xi)=x_1(\xi)$ where: $I=[-2\pi,-\pi]U[\pi,2\pi]$ then prove that ψ is an orthonormal wavelet for $L^2(¡)$.

(B) Define Shannon wavelet. Show that the modulus of the Fourier Transform of a scaling function can be expressed in terms of the modulus of the Fourier transform of the wavelet.

(C) Let f be a function in $L^2(¡)$ and $\alpha > 0;$ suppose.

$$\int_{2^i\pi\le|\xi|\le 2^{j+1}\pi} |\hat{f}(\xi)|\, d\xi \le C^{2-\alpha j}$$

j=0,1,2,....... then show that, if $\propto\notin N, f\in\wedge_{\propto}(¡)$ and if $\propto\in N, f\in_{\wedge-\in}(¡)$ for all $0 < \in < \propto$

Unit - II

2. (A) If ψ is an orthonormal wavelet and $\left|\frac{a}{\psi}\right|$ is continuous at zero then prove that $\hat{\psi}(0)=0$

(B) Show that a function $f\in L^2(¡)$ belongs to V_0 if and only if $\xi^2\hat{f}(\xi)$ is a 2π periodic function on ¡ .

(C) Show that $f\in C_j$ if and only if

$f \in B_{j+1}, F[f](0) = 0$ and $\sum_{l \in Z} \frac{F[f](n+2^j l)}{(n+2^j l)^2} = 0$

for $n = 1, 2, ...N (= 2^j - 1)$

Unit - III

3. (A) Let H be a Hilbert space and $\{e_j : j = 1, 2, ...\}$ be a family of elements of H. Then show that

$\|f\|^2 = \sum_{j=1}^{\infty} |< f, e_j >|^2$ holds for all $f \in H$ if and

only if $f = \sum_{j=1}^{\infty} < f, e_j > e_j$, with convergence in H, for all $f \in H$

(B) If ψ is an orthonormal wavelet and

$$G_n(\xi) = \sum_{k \in z} \hat{\psi}(2^n(\xi + 2k\pi) \overline{\hat{\psi}(2^j(\xi + 2k\pi)} \hat{\psi}(2^j \xi)$$

a.e. for all n >=1 then show that $G_n(\xi) = G_{n-1}(2\xi)$

(C) Show that there is no orthonormed wavelet for $H^2(¡)$ satisfying. The regularity condition (R^0) given by: $(i)\ |\hat{\psi}(\xi)|$

$(ii)\ |\hat{\psi}(\xi)| = 0(c1 + |\xi|)^{-<-1/2}$ at ∞, for same $\alpha > 0$

Unit - IV

4. (A) Define frame. Give an example to even when all zero elements are removal from a frame, then show that the new frame is not necessary a basis.

OR

(B) For any $h \in L^2(¡)$, if $Qh \in L^2(¡)$ and $Ph \in L^2(¡)$, then show that

$$(i)\ R(Qh)\ (S,t) = S(Rh)\ (S,t) + \frac{1}{2\pi i}\frac{\partial}{\partial t}(Rh)(S,t)$$

and

$$(ii)\ R(Ph)\ (S,t) = -i\frac{\partial}{\partial s}(Rh)(S,t)$$

(C) Suppose that $\{e_j : j = 1, 2, ...\}$ be a family in Hilbert space H satisfying:

$$A\|f\|^2 \leq \sum_{j=i}^{\infty} \left| < f, e_j > \right|^2 \leq B\|f\|^2$$ for all f belongs to dense subject D of H. Then prove that $\{e_j : j = 1, 2, 3, ...\}$ is a frame for H.

Unit - V

5. (A) Write and derive decomposition and reconstruction algorithms for wavelets.

(B) If N = 2^q then prove that C_N=E_1, E_2,.........E_q where each E_j is an N×N matrix such that each row has precisely two non - Zero entries.

(C) How the Haar - wavelet works for doing the decomposition algorithm?