Roll No. $\qquad$

## F - 1000

## M.A./M.Sc. (Fourth Semester)

EXAMINATION, May-June, 2022
MATHEMATICS
Paper Fifth
(Optional-B)
(Graph Theory-II)

Time : Three Hours]
[Maximum Marks: 80
Note-Attempt all sections as directed.

## Section-A

(Objective/Multiple Choice Questions)

## (1 mark each)

## Note-Attempt all questions.

Choose the correct answer :

1. A graph is collection of
(A) Row and column
(B) Vertices and edges
(C) Equations
(D) None of these
2. The maximum degree of any vertex in a simple graph with $n$ vertices is
(A) $n$
(B) $\mathrm{n}+1$
(C) $n-1$
(D) $2 \mathrm{n}-1$
3. A graph with no edges is known as empty graph. Empty graph is also known as
(A) Trivial graph
(B) Regular graph
(C) Biparticle graph
(D) None of these
4. The degree of any vertex of graph is
(A) The number of edges in a graph
(B) Number of vertex in a graph
(C) Number of vertices adjacent to that vertex
(D) The number of edges incident with vertex
5. A graph $G$ is called----------if it is a connected acyclic graph.
(A) Cyclic graph
(B) Tree
(C) Regular graph
(D) Not a graph
6. The expression $\mathrm{a}+\mathrm{ac}$ is equivalent to
(A) $a$
(B) c
(C) $a+c$
(D) 1
7. The graph is tree if and only if
(A) Is minimally
(B) Contains a circuit
(C) Is plan as
(D) None of these
8. The degree of $v$ if $v$ is an isolated vertex in a graph
(A) 3
(B) 2
(C) 1
(D) 0
9. A graph with one vertex and no edges is called
(A) Multi graph
(B) Trivial graph
(C) Isolated graph
(D) Digraph
10. The number of various word can be taken out of the letters of the word VARANASI
(A) 64
(B) 120
(C) 720
(D) 40320
11. The size of a simple graph of order $n$ can not exceed
(A) $n_{c_{2}}$
(B) $n_{c_{1}}$
(C) $n-1$
(D) $\mathrm{n}-2$
12. Degree of a graph with 12 vertices is
(A) 212
(B) 56
(C) 25
(D) 24
13. A graph representing universal relation is called
(A) Partial digraph
(B) Complete digraph
(C) Empty graph
(D) Partial subgraph
14. Disconnected components can be created in case of
(A) Undirected graph
(B) Partial subgraph
(C) Disconnected graph
(D) Complete graph
15. A vertex with zero degree is called
(A) Source
(B) Sink
(C) Both (A) and (B)
(D) None of these
16. A simple graph can have
(A) Multiple edges
(B) Self loops
(C) Parallel edges
(D) None of these
17. Every digraph without odd cycles has a
(A) No basis
(B) 1-basis
(C) 2-basis
(D) None of these
18. A graph is strong if every vertex is
(A) Source
(B) Sink
(C) Both source and sink
(D) None of these
19. Which of the following is not true
(A) Any digraph has a basis
(B) Every basis is a dependent set
(C) Both (A) \& (B)
(D) None of these
20. Which of the following is true
(A) Every weak isograph is strong
(B) Every acyclic digraph has a unique I-basis
(C) A strong digraph is bipartite if it has no odd cycles
(D) All of above

## Section - B <br> (Very Short Answer Type Questions)

(2 marks each)
Note: -Attempt all questions.
Explain following terms.

1. Ramsey graph
2. Symmetric concepts
3. Pseudo-similarity
4. Chromatic polynomial
5. Graph enumeration
6. Forbidden subgraph
7. Flows in networks
8. Degree sequences.

## Section-C

(Short Answer Type Questions)
(3 marks each)

## Note- Attempt any eight questions.

## Describe following.

1. Perfectness- preserving operations
2. Ramsey numbers
3. The bivariate colouring polynomials
4. Co-di chromatic
5. Covers and basis
6. Types of connectedness
7. Unilateral component
8. Strictly weak digraph
9. Automorphism groups

## Section-D

## (Long Answer Type Questions)

(5 marks each)

## Note-Attempt all questions.

1. Prove that every traingulated graph is perfect.

OR
Prove that a graph is traingulated if every minimal vertex separator induces a complete subgraph.
2. Prove that every vertex of a composite connected graph lies on a 4-cycle.

OR
Prove that an edge-transitive graph without isolated vertices is either vertex-transitive or bipartite.
3. Prove that each cycle $C_{n}, n \geq 3$ is chromatically unique.

OR

Prove that for any edge e of a graph $\mathrm{G}, \phi(\mathrm{G}, \mathrm{x})=\theta(\mathrm{G}-\mathrm{e}, \mathrm{x})$ $\phi(\mathrm{Gle} \mathrm{e}$ )
4. Prove that every di graph without odd cycles has I-basis.

OR
State and prove menger's theorem for digraph- (vertex form)

