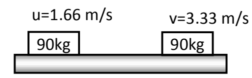
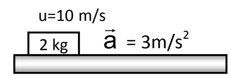
SOLUTIONS TO CONCEPTS
CHAPTER – 8

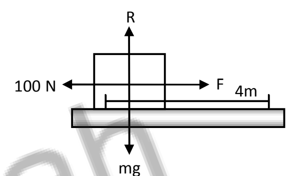
1. $M = m_c + m_b = 90kg$
   u = 6 km/h = 1.666 m/sec
   v = 12 km/h = 3.333 m/sec
   Increase in K.E. = ½ Mv^2 – ½ Mu^2
   = ½ 90 × $(3.333)^2$ – ½ × 90 × $(1.66)^2$ = 494.5 – 124.6 = 374.8 ≈ 375 J



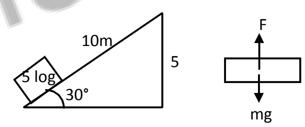
2. m_b = 2 kg.
   u = 10 m/sec
   a = 3 m/aec^2
   t = 5 sec
   v = u + at = 10 + 3 I 5 = 25 m/sec.
   ∴ F.K.E = ½ mv^2 = ½ × 2 × 625 = 625 J.



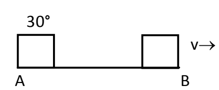
3. F = 100 N
   S = 4m, θ = 0°
   $\omega = \vec{F}.\vec{S} = 100 \times 4 = 400$ J



4. m = 5 kg
   θ = 30°
   S = 10 m
   F = mg
   So, work done by the force of gravity
   ω = mgh = 5 × 9.8 × 5 = 245 J

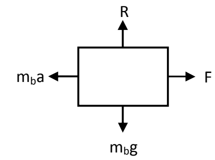


5. F= 2.50N, S = 2.5m, m =15g = 0.015kg.
   So, w = F × S ⇒ $a = \frac{F}{m} = \frac{2.5}{0.015} = \frac{500}{3} m/s^2$
   =F × S cos 0° (acting along the same line)
   = 2.5 × 2.5 = 6.25J
   Let the velocity of the body at b = U. Applying work-energy principle ½ mv^2 – 0 = 6.25
   $\Rightarrow V = \sqrt{\frac{6.25 \times 2}{0.015}}$ = 28.86 m/sec.
   So, time taken to travel from A to B.
   $\Rightarrow t = \frac{v-u}{a} = \frac{28.86 \times 3}{500}$
   ∴ Average power = $\frac{W}{t} = \frac{6.25 \times 500}{(28.86) \times 3} = 36.1$



6. Given
   $\vec{r}_1 = 2\hat{i} + 3\hat{j}$
   $r_2 = 3\hat{i} + 2\hat{j}$
   So, displacement vector is given by,
   $\vec{r} = \vec{r}_1 - \vec{r}_2 \Rightarrow \vec{r} = (3\hat{i} + 2\hat{j}) - (2\hat{i} + 3\hat{j}) = \hat{i} - \hat{j}$

So, work done = $\vec{F} \times \vec{s}$ = 5 × 1 + 5(-1) = 0



7. m_b = 2kg, s = 40m, a = 0.5m/sec^2

So, force applied by the man on the box

F = $m_b a$ = 2 × (0.5) = 1 N

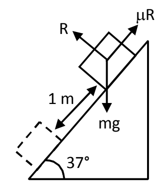
ω = FS = 1 × 40 = 40 J

8. Given that F= a + bx

Where a and b are constants.

So, work done by this force during this force during the displacement x = 0 and x = d is given by

W = $\int_0^d F\,dx = \int_0^d (a + bx)dx$ = ax + (bx^2/2) = [a + ½ bd] d



9. m_b = 250g = .250 kg

θ = 37°, S = 1m.

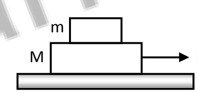
Frictional force f = μR

mg sin θ = μ R ..(1)

mg cos θ ..(2)

so, work done against μR = μRS cos 0° = mg sin θ S = 0.250 × 9.8 × 0.60 × 1 = 1.5 J

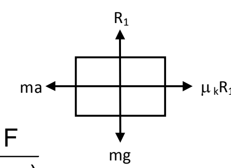
10. a = $\frac{F}{2(M+m)}$ (given)



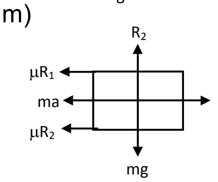
a) from fig (1)

ma = μ_k R_1 and R_1 = mg

$\Rightarrow \mu = \frac{ma}{R_1} = \frac{F}{2(M+m)g}$



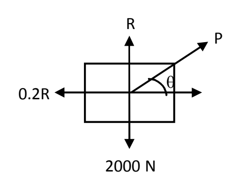
b) Frictional force acting on the smaller block f = μR = $\frac{F}{2(M+m)g} \times mg = \frac{m \times F}{2(M+m)}$



c) Work done w = fs s = d

$$w = \frac{mF}{2(M+m)} \times d = \frac{mFd}{2(M+m)}$$

11. Weight = 2000 N, S = 20m, μ = 0.2



a) R + Psinθ - 2000 = 0 ..(1)

P cosθ - 0.2 R =0 ..(2)

From (1) and (2) P cosθ – 0.2 (2000 – P sinθ)=0

P = $\frac{400}{\cos\theta + 0.2\sin\theta}$ ..(3)

So, work done by the person, W = PS cosθ = $\frac{8000\cos\theta}{\cos\theta + 0.2\sin\theta} = \frac{8000}{1 + 0.2\sin\theta} = \frac{40000}{5 + \tan\theta}$

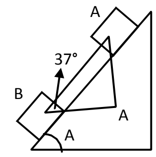
b) For minimum magnitude of force from equn(1)

d/dθ (cos θ + 0.2 sinθ) = 0 $\Rightarrow$ tan θ = 0.2

putting the value in equn (3)

W = $\frac{40000}{5 + \tan\theta} = \frac{40000}{(5.2)}$ = 7690 J

12. w = 100 N, θ = 37°, s = 2m



Force F= mg sin 37° = 100 × 0.60 = 60 N

So, work done, when the force is parallel to incline.

w = Fs cos θ = 60 × 2 × cos θ = 120 J

In ΔABC AB= 2m

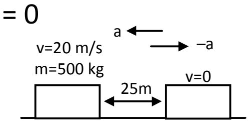
CB = 37°

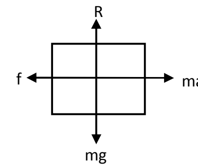
so, h = C = 1m

$\therefore$ work done when the force in horizontal direction

W = mgh = 100 × 1.2 = 120 J

13. m = 500 kg, s = 25m, u = 72km/h= 20 m/s, v = 0

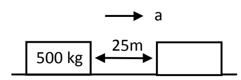


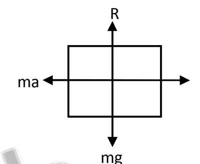


$(-a) = \frac{v^2 - u^2}{2S} \Rightarrow a = \frac{400}{50} = 8 m/sec^2$

Frictional force f = ma = 500 × 8 = 4000 N

14. m = 500 kg, u = 0, v = 72 km/h = 20m/s





$a = \frac{v^2 - u^2}{2s} = \frac{400}{50} = 8 m/sec^2$

force needed to accelerate the car F = ma = 500 × 8 = 4000 N

15. Given, v = a$\sqrt{x}$ (uniformly accelerated motion)

displacement s = d – 0 = d

putting x = 0, v_1= 0

putting x = d, v_2= a$\sqrt{d}$

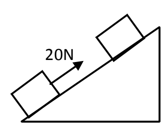
$a = \frac{{v_2}^2 - {u_2}^2}{2s} = \frac{a^2 d}{2d} = \frac{a^2}{2}$

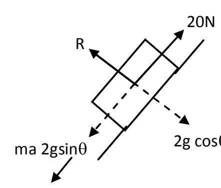
force f = ma = $\frac{ma^2}{2}$

work done w = FS cos θ = $\frac{ma^2}{2} \times d = \frac{ma^2 d}{2}$

16. a) m = 2kg, θ = 37°, F = 20 N

From the free body diagram





F = (2g sin θ) + ma $\Rightarrow$ a = (20 – 20 sinθ)/s = 4m/sec^2

S = ut + ½ at^2 (u = 0, t = 1s, a = 1.66)

= 2m

So, work, done w = Fs = 20 × 2 = 40 J

b) If W = 40 J

$S = \frac{W}{F} = \frac{40}{20}$

h = 2 sin 37° = 1.2 m

So, work done W = –mgh = – 20 × 1.2 = –24 J

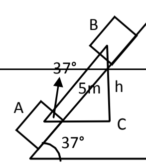
c) v = u + at = 4 × 10 = 40 m/sec

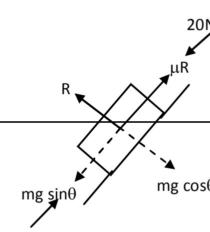
So, K.E. = ½ mv^2 = ½ × 2 × 16 = 16 J

17. m = 2kg, θ = 37°, F = 20 N, a = 10 m/sec^2

a) t = 1sec

So, s= ut + ½ at^2 = 5m





Work done by the applied force w = FS cos 0° = 20 × 5 = 100 J

b) BC (h) = 5 sin 37° = 3m

So, work done by the weight W = mgh = 2 × 10 × 3 = 60 J

c) So, frictional force f = mg sinθ

work done by the frictional forces w = fs cos0° = (mg sinθ) s = 20 × 0.60 × 5 = 60 J

18. Given, m = 25o g = 0.250kg,

u = 40 cm/sec = 0.4m/sec

μ = 0.1, v=0

Here, μ R = ma {where, a = deceleration}

a = $\frac{\mu R}{m} = \frac{\mu mg}{m}$ = μg = 0.1 × 9.8 = 0.98 m/sec^2

S = $\frac{v^2 - u^2}{2a}$ = 0.082m = 8.2 cm

Again, work done against friction is given by

– w = μ RS cos θ

= 0.1 × 2.5 × 0.082 × 1 (θ = 0°) = 0.02 J

$\Rightarrow$ W = – 0.02 J

19. h = 50m, m = 1.8 × 10^5 kg/hr, P = 100 watt,

P.E. = mgh = 1.8 × 10^5 × 9.8 × 50 = 882 × 10^5 J/hr

Because, half the potential energy is converted into electricity,

Electrical energy ½ P.E. = 441 × 10^5 J/hr

So, power in watt (J/sec) is given by = $\frac{441 \times 10^5}{3600}$

$\therefore$ number of 100 W lamps, that can be lit $\frac{441 \times 10^5}{3600 \times 100}$ = 122.5 $\approx$122

20. m = 6kg, h = 2m

P.E. at a height '2m' = mgh = 6 × (9.8) × 2 = 117.6 J

P.E. at floor = 0

Loss in P.E. = 117.6 – 0 = 117. 6 J $\approx$ 118 J

21. h = 40m, u = 50 m/sec

Let the speed be 'v' when it strikes the ground.

Applying law of conservation of energy

mgh + ½ mu^2 = ½ mv^2

$\Rightarrow$ 10 × 40 + (1/2) × 2500 = ½ v^2 $\Rightarrow$ v^2 = 3300 $\Rightarrow$ v = 57.4 m/sec $\approx$58 m/sec

22. t = 1 min 57.56 sec = 11.56 sec, p= 400 W, s =200 m

p = $\frac{w}{t}$, Work w = pt = 460 × 117.56 J

Again, W = FS = $\frac{460 \times 117.56}{200}$ = 270.3 N $\approx$ 270 N

23. S = 100 m, t = 10.54 sec, m = 50 kg

The motion can be assumed to be uniform because the time taken for acceleration is minimum.

a) Speed v = S/t = 9.487 e/s

So, K.E. = ½ mv^2 = 2250 J

b) Weight = mg = 490 J

given R = mg /10 = 49 J

so, work done against resistance W_F = – RS = – 49 × 100 = – 4900 J

c) To maintain her uniform speed, she has to exert 4900 j of energy to over come friction

P = $\frac{W}{t}$ = 4900 / 10.54 = 465 W

24. h = 10 m

flow rate = (m/t) = 30 kg/min = 0.5 kg/sec

power P = $\frac{mgh}{t}$ = (0.5) × 9.8 × 10 = 49 W

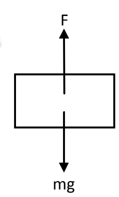
So, horse power (h.p) P/746 = 49/746 = 6.6 × 10^{-2}hp

25. m = 200g = 0.2kg, h = 150cm = 1.5m, v = 3m/sec, t = 1 sec

Total work done = ½ mv^2 + mgh = (1/2) × (0.2) ×9 + (0.2) × (9.8) × (1.5) = 3.84 J

h.p. used = $\frac{3.84}{746}$ = 5.14 × 10^{-3}

26. m = 200 kg, s = 12m, t = 1 min = 60 sec



So, work W = F cos θ = mgs cos0° [θ = 0°, for minimum work]

= 2000 × 10 × 12 = 240000 J

So, power p = $\frac{W}{t} = \frac{240000}{60}$ = 4000 watt

h.p = $\frac{4000}{746}$ = 5.3 hp.

27. The specification given by the company are

U = 0, m = 95 kg, P_m = 3.5 hp

V_m = 60 km/h = 50/3 m/sec t_m = 5 sec

So, the maximum acceleration that can be produced is given by,

a = $\frac{(50/3) - 0}{5} = \frac{10}{3}$

So, the driving force is given by

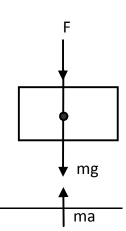
F = ma = 95 × $\frac{10}{3} = \frac{950}{3}$ N

So, the velocity that can be attained by maximum h.p. white supplying $\frac{950}{3}$ will be

v = $\frac{p}{F} \Rightarrow$ v = $\frac{3.5 \times 746 \times 5}{950}$ = 8.2 m/sec.

Because, the scooter can reach a maximum of 8.s m/sec while producing a force of 950/3 N, the specifications given are some what over claimed.

28. Given m = 30kg, v = 40 cm/sec = 0.4 m/sec s = 2m



From the free body diagram, the force given by the chain is,

F = (ma – mg) = m(a – g) [where a = acceleration of the block]

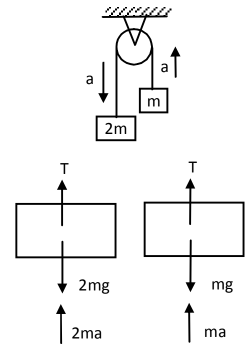
a = $\frac{(v2\ \ u2)}{2s} = \frac{0.16}{0.4}$ = 0.04 m/sec^2

So, work done $W = Fs \cos\theta = m(a - g)\, s \cos\theta$

$\Rightarrow W = 30\,(0.04 - 9.8) \times 2 \Rightarrow W = -585.5 \Rightarrow W = -586$ J.

So, $W = -586$ J

29. Given, T = 19 N



From the freebody diagrams,

$T - 2\,mg + 2\,ma = 0$ ...(i)

$T - mg - ma = 0$ ...(ii)

From, Equation (i) & (ii) $T = 4ma \Rightarrow a = \frac{T}{4m} \Rightarrow A = \frac{16}{4m} = \frac{4}{m}$ m/s^2.

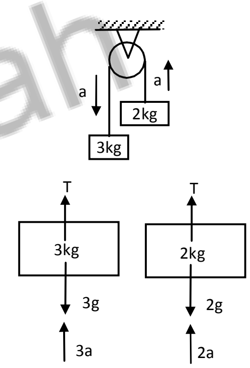
Now, $S = ut + \frac{1}{2} at^2$

$\Rightarrow S = \frac{1}{2} \times \frac{4}{m} \times 1 \Rightarrow S = \frac{2}{m}$ m [because u=0]

Net mass = 2m – m = m

Decrease in P.E. = mgh $\Rightarrow$ P.E. $= m \times g \times \frac{2}{m} \Rightarrow$ P.E. $= 9.8 \times 2 \Rightarrow$ P.E. = 19.6 J

30. Given, $m_1 = 3$ kg, $m_2 = 2$kg, t = during 4th second



From the freebody diagram

$T - 3g + 3a = 0$ ..(i)

$T - 2g - 2a = 0$ ..(ii)

Equation (i) & (ii), we get $3g - 3a = 2g + 2a \Rightarrow a = \frac{g}{5}$ m/sec^2

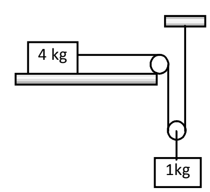
Distance travelled in 4th sec is given by

$$S_{4th} = \frac{a}{2}(2n-1) = \frac{\left(\frac{g}{5}\right)}{s}(2\times 4-1) = \frac{7g}{10} = \frac{7\times 9.8}{10} \text{ m}$$

Net mass 'm' = $m_1 - m_2 = 3 - 2 = 1$kg

So, decrease in P.E. = mgh $= 1 \times 9.8 \times \frac{7}{10} \times 9.8 = 67.2 = 67$ J

31. $m_1 = 4$kg, $m_2 = 1$kg, $V_2 = 0.3$m/sec $V_1 = 2 \times (0.3) = 0.6$ m/sec



($v_1 = 2x_2$ m this system)

h = 1m = height descent by 1kg block

s = 2 × 1 = 2m distance travelled by 4kg block

u = 0

Applying change in K.E. = work done (for the system)

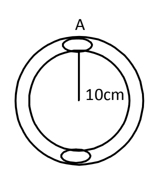
$[(1/2)m_1v_1^2 + (1/2)\, m_2v_m^2] - 0 = (-\mu R)S + m_2g$ [R = 4g = 40 N]

$\Rightarrow \frac{1}{2} \times 4 \times (0.36) \times \frac{1}{2} \times 1 \times (0.09) = -\mu \times 40 \times 2 + 1 \times 40 \times 1$

$\Rightarrow 0.72 + 0.045 = -80\mu + 10$

$\Rightarrow \mu = \frac{9.235}{80} = 0.12$

32. Given, m = 100g = 0.1kg, v = 5m/sec, r = 10cm



Work done by the block = total energy at A – total energy at B

$(1/2\, mv^2 + mgh) - 0$

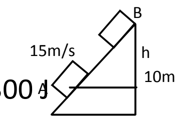
$\Rightarrow W = \frac{1}{2} mv^2 + mgh - 0 = \frac{1}{2} \times (0.1) \times 25 + (0.1) \times 10 \times (0.2)$ [h = 2r = 0.2m]

$\Rightarrow$ W = 1.25 – 0.2 $\Rightarrow$ W = 1.45 J

So, the work done by the tube on the body is

W_t = –1.45 J

33. m = 1400kg, v = 54km/h = 15m/sec, h = 10m



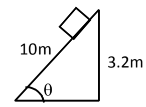
Work done = (total K.E.) – total P.E.

= 0 + ½ mv^2 – mgh = ½ × 1400 × $(15)^2$ – 1400 × 9.8 × 10 = 157500 – 137200 = 20300 J

So, work done against friction, W_t = 20300 J

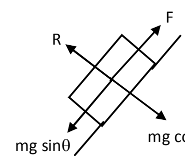
34. m = 200g = 0.2kg, s = 10m, h = 3.2m, g = 10 m/sec^2

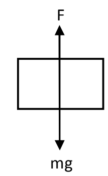
a) Work done W = mgh = 0.2 × 10 × 3.2 = 6.4 J



b) Work done to slide the block up the incline

w = (mg sin θ) = (0.2) × 10 × $\frac{3.2}{10}$ × 10 = 6.4 J





c) Let, the velocity be v when falls on the ground vertically,

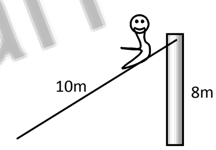
½ mv^2 – 0 = 6.4J $\Rightarrow$ v = 8 m/s

d) Let V be the velocity when reaches the ground by liding

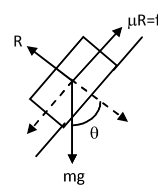
½ mV^2 – 0 = 6.4 J $\Rightarrow$ V = 8m/sec

35. ℓ = 10m, h = 8m, mg = 200N

f = 200 × $\frac{3}{10}$ = 60N



a) Work done by the ladder on the boy is zero when the boy is going up because the work is done by the boy himself.



b) Work done against frictional force, W = μRS = f ℓ = (–60) × 10 = – 600 J

c) Work done by the forces inside the boy is

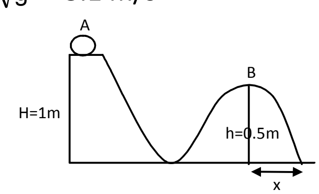
W_b = (mg sinθ) × 10 = 200 × $\frac{8}{10}$ × 10 = 1600 J

36. H = 1m, h = 0.5m

Applying law of conservation of Energy for point A & B

mgH = ½ mv^2 + mgh $\Rightarrow$ g = (1/2) v^2 + 0.5g $\Rightarrow$ v^2 2(g – 0.59) = g $\Rightarrow$ v = $\sqrt{g}$ = 3.1 m/s

After point B the body exhibits projectile motion for which



θ = 0°, v = – 0.5

So, –0.5 = (u sinθ) t - (1/2) gt^2 $\Rightarrow$ 0.5 = 4.9 t^2 $\Rightarrow$ t = 0.31 sec.

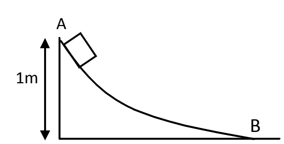
So, x = (4 cos θ) t = 3.1 × 3.1 = 1m.

So, the particle will hit the ground at a horizontal distance in from B.

37. mg = 10N, μ = 0.2, H = 1m, u = v = 0

change in P.E. = work done.

Increase in K.E.



$\Rightarrow$ w = mgh = 10 × 1 = 10 J

Again, on the horizontal surface the fictional force

F = μR = μmg = 0.2 × 10 = 2 N

So, the K.E. is used to overcome friction

$\Rightarrow$ S = $\frac{W}{F}$ = $\frac{10J}{2N}$ = 5m

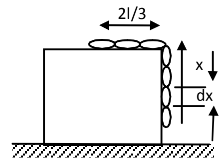
38. Let ‘dx’ be the length of an element at a distance × from the table

mass of ‘dx’ length = (m/ℓ) dx

Work done to put dx part back on the table

W = (m/ℓ) dx g(x)

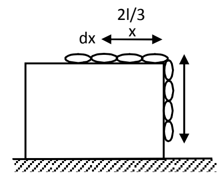
So, total work done to put ℓ/3 part back on the table



$$W = \int_0^{1/3} (m/\ell) gx\, dx \Rightarrow w = (m/\ell)\, g \left[\frac{x^2}{2}\right]_0^{\ell/3} = \frac{mg\ell^2}{18\ell} = \frac{mg\ell}{18}$$

39. Let, x length of chain is on the table at a particular instant.

So, work done by frictional force on a small element ‘dx’



$$dW_f = \mu Rx = \mu\left(\frac{M}{L}dx\right)gx \qquad \left[\text{where } dx = \frac{M}{L}dx\right]$$

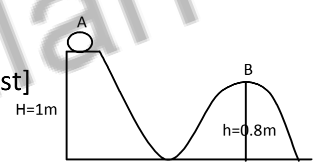
Total work don by friction,

$$W_f = \int_{2L/3}^{0} \mu \frac{M}{L} gx\ dx$$

$$\therefore\ W_f = \mu \frac{m}{L} g \left[\frac{x^2}{2}\right]_{2L/3}^{0} = \mu \frac{M}{L}\left[\frac{4L^2}{18}\right] = 2\mu Mg\ L/9$$

40. Given, m = 1kg, H = 1m, h = 0.8m

Here, work done by friction = change in P.E. [as the body comes to rest]



$\Rightarrow W_f = mgh - mgH$

$= mg\,(h - H)$

$= 1 \times 10\,(0.8 - 1) = -2J$

41. m = 5kg, x = 10cm = 0.1m, v = 2m/sec,

h =? G = 10m/sec^2

S0, $k = \frac{mg}{x} = \frac{50}{0.1} = 500$ N/m

Total energy just after the blow $E = ½\, mv^2 + ½\, kx^2$ ...(i)

Total energy a a height h = $½\, k\,(h - x)^2 + mgh$ ...(ii)

$½\, mv^2 + ½\, kx^2 = ½\, k\,(h - x)^2 + mgh$

On, solving we can get,

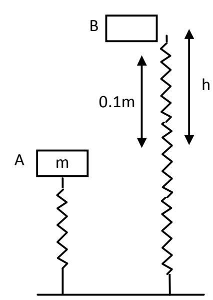
H = 0.2 m = 20 cm

42. m = 250 g = 0.250 kg,

k = 100 N/m, m = 10 cm = 0.1m

g = 10 m/sec^2

Applying law of conservation of energy



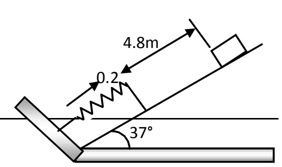
$$½\, kx^2 = mgh \Rightarrow h = \frac{1}{2}\left(\frac{kx^2}{mg}\right) = \frac{100 \times (0.1)^2}{2 \times 0.25 \times 10} = 0.2 \text{ m} = 20 \text{ cm}$$

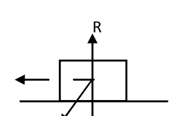
43. m = 2kg, s_1 = 4.8m, x = 20cm = 0.2m, s_2 = 1m,

sin 37° = 0.60 = 3/5, θ = 37°, cos 37° = .79 = 0.8 = 4/5

g = 10m/sec^2

Applying work – Energy principle for downward motion of the body





$0 - 0 = mg \sin 37° \times 5 - \mu R \times 5 - ½\ kx^2$

$\Rightarrow 20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½\ k\ (0.2)^2 = 0$

$\Rightarrow 60 - 80\mu - 0.02k = 0 \Rightarrow 80\mu + 0.02k = 60$ ...(i)

Similarly, for the upward motion of the body the equation is

$0 - 0 = (-mg \sin 37°) \times 1 - \mu\ R \times 1 + ½\ k\ (0.2)^2$

$\Rightarrow -20 \times (0.60) \times 1 - \mu \times 20 \times (0.80) \times 1 + ½\ k\ (0.2)^2 = 0$

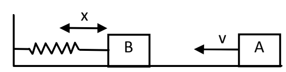
$\Rightarrow -12 - 16\mu + 0.02\ K = 0$ ..(ii)

Adding equation (i) & equation (ii), we get $96\ \mu = 48$

$\Rightarrow \mu = 0.5$

Now putting the value of μ in equation (i) K = 1000N/m

44. Let the velocity of the body at A be v



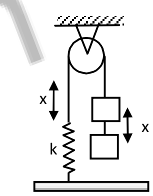
So, the velocity of the body at B is v/2

Energy at point A = Energy at point B

So, $½\ mv_A^2 = ½\ mv_B^2 + ½\ kx^{2+}$

$\Rightarrow ½\ kx^2 = ½\ mv_A^2 - ½\ mv_B^2 \Rightarrow kx^2 = m\ (v_A^{2+} - v_B^2) \Rightarrow kx^2 = m\left(v^2 - \frac{v^2}{4}\right) \Rightarrow k = \frac{3mv^2}{3x^2}$

45. Mass of the body = m

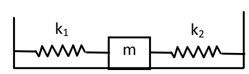


Let the elongation be x

So, $½\ kx^2 = mgx$

$\Rightarrow x = 2mg / k$

46. The body is displaced x towards right



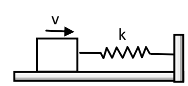
Let the velocity of the body be v at its mean position

Applying law of conservation of energy

$½\ mv^2 = ½\ k_1x^2 + ½\ k_2x^2 \Rightarrow mv^2 = x^2\ (k_1 + k_2) \Rightarrow v^2 = \frac{x^2(k_1 + k_2)}{m}$

$\Rightarrow v = x\sqrt{\frac{k_1 + k_2}{m}}$

47. Let the compression be x



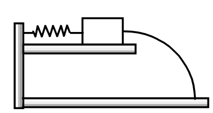
According to law of conservation of energy

$½\ mv^2 = ½\ kx^2 \Rightarrow x^2 = mv^2 / k \Rightarrow x = v\sqrt{(m/k)}$

b) No. It will be in the opposite direction and magnitude will be less due to loss in spring.

48. m = 100g = 0.1kg, x = 5cm = 0.05m, k = 100N/m

when the body leaves the spring, let the velocity be v



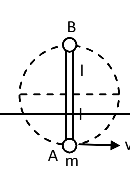
$½\ mv^2 = ½\ kx^2 \Rightarrow v = x\sqrt{k/m} = 0.05 \times \sqrt{\frac{100}{0.1}} = 1.58\text{m/sec}$

For the projectile motion, $\theta = 0°$, Y = –2

Now, $y = (u \sin \theta)t - ½\ gt^2$

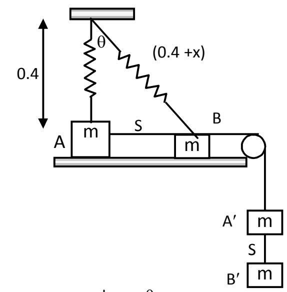
$\Rightarrow -2 = (-1/2) \times 9.8 \times t^2 \Rightarrow t = 0.63$ sec.

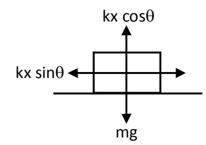
So, $x = (u \cos \theta)\ t \Rightarrow 1.58 \times 0.63 = 1\text{m}$



49. Let the velocity of the body at A is 'V' for minimum velocity given at A velocity of the body at point B is zero.
Applying law of conservation of energy at A & B
$\frac{1}{2}mv^2 = mg(2\ell) \Rightarrow v = \sqrt{(4g\ell)} = 2\sqrt{g\ell}$

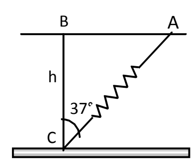
50. m = 320g = 0.32kg
k = 40N/m
h = 40cm = 0.4m
$g = 10\ m/s^2$
From the free body diagram,
$kx\cos\theta = mg$
(when the block breaks off R = 0)
$\Rightarrow \cos\theta = mg/kx$
So, $\frac{0.4}{0.4+x} = \frac{3.2}{40\times x} \Rightarrow 16x = 3.2x + 1.28 \Rightarrow x = 0.1$ m
S0, $s = AB = \sqrt{(h+x)^2 - h^2} = \sqrt{(0.5)^2 - (0.4)^2} = 0.3$ m



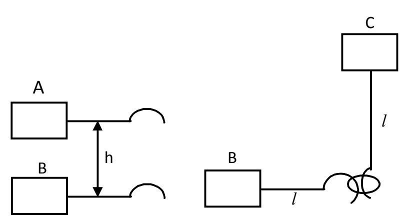


Let the velocity of the body at B be v
Charge in K.E. = work done (for the system)
$(1/2\ mv^2 + \frac{1}{2}mv^2) = -1/2\ kx^2 + mgs$
$\Rightarrow (0.32) \times v^2 = -(1/2) \times 40 \times (0.1)^2 + 0.32 \times 10 \times (0.3) \Rightarrow v = 1.5$ m/s.

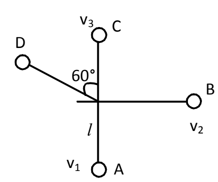
51. $\theta = 37°$; l = h = natural length
Let the velocity when the spring is vertical be 'v'.
Cos 37° = BC/AC = 0.8 = 4/5
Ac = (h + x) = 5h/4 (because BC = h)
So, x = (5h/4) – h = h/4
Applying work energy principle $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$
$\Rightarrow v = x\sqrt{(k/m)} = \frac{h}{4}\sqrt{\frac{k}{m}}$

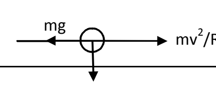


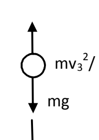
52. The minimum velocity required to cross the height point c = $\sqrt{2gl}$
Let the rod released from a height h.
Total energy at A = total energy at B
$mgh = 1/2\ mv^2$; $mgh = 1/2\ m\ (2gl)$
[Because v = required velocity at B such that the block makes a complete circle. [Refer Q – 49]
So, h = l.



53. a) Let the velocity at B be v_2
$1/2\ mv_1^2 = 1/2\ mv_2^2 + mgl$
$\Rightarrow 1/2\ m\ (10\ gl) = 1/2\ mv_2^2 + mgl$
$v_2^2 = 8\ gl$
So, the tension in the string at horizontal position







$T = \frac{mv^2}{R} = \frac{m8gl}{l} = 8\ mg$

b) Let the velocity at C be V_3

$1/2\ mv_1^2 = 1/2\ mv_3^2 + mg\ (2l)$

$\Rightarrow 1/2\ m\ (log\ l) = 1/2\ mv_3^2 + 2mgl$

$\Rightarrow v_3^2 = 6\ mgl$

So, the tension in the string is given by

$T_c = \frac{mv^2}{l} - mg = \frac{6\ glm}{l}\ mg = 5\ mg$

c) Let the velocity at point D be v_4

Again, $1/2\ mv_1^2 = 1/2\ mv_4^2 + mgh$

$1/2 \times m \times (10\ gl) = 1.2\ mv_4^2 + mgl\ (1 + \cos 60°)$

$\Rightarrow v_4^2 = 7\ gl$

So, the tension in the string is

$T_D = (mv^2/l) - mg \cos 60°$

$= m(7\ gl)/l - l - 0.5\ mg \Rightarrow 7\ mg - 0.5\ mg = 6.5\ mg.$

54. From the figure, $\cos \theta = AC/AB$

$\Rightarrow AC = AB \cos \theta \Rightarrow (0.5) \times (0.8) = 0.4.$

So, CD = (0.5) – (0.4) = (0.1) m

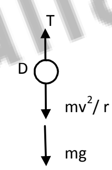
Energy at D = energy at B

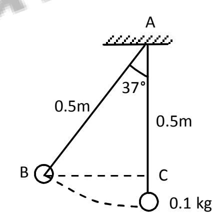
$1/2\ mv^2 = mg\ (CD)$

$v^2 = 2 \times 10 \times (0.1) = 2$

So, the tension is given by,

$T = \frac{mv^2}{r} + mg = (0.1)\left(\frac{2}{0.5} + 10\right) = 1.4\ N.$





55. Given, N = mg

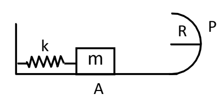
As shown in the figure, $mv^2 / R = mg$

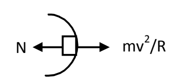
$\Rightarrow v^2 = gR$ ...(1)

Total energy at point A = energy at P

$1/2\ kx^2 = \frac{mgR + 2mgR}{2}$ [because $v^2 = gR$]

$\Rightarrow x^2 = 3mgR/k \Rightarrow x = \sqrt{(3mgR)/k}$.





56. $V = \sqrt{3gl}$

$1/2\ mv^2 - 1/2\ mu^2 = -mgh$

$v^2 = u^2 - 2g(l + l\cos\theta)$

$\Rightarrow v^2 = 3gl - 2gl\ (1 + \cos\theta)$ ...(1)

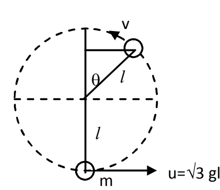
Again,

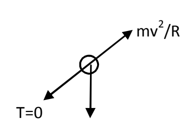
$mv^2/l = mg \cos\theta$

$v^2 = lg \cos\theta$

From equation (1) and (2), we get

$3gl - 2gl - 2gl \cos\theta = gl \cos\theta$



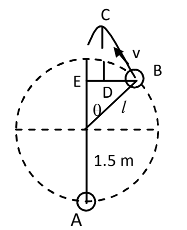


$3 \cos \theta = 1 \Rightarrow \cos \theta = 1/3$

$\theta = \cos^{-1} (1/3)$

So, angle rotated before the string becomes slack

$= 180° - \cos^{-1} (1/3) = \cos^{-1} (-1/3)$



57. $l = 1.5$ m; $u = \sqrt{57}$ m/sec.

a) $mg \cos \theta = mv^2 / l$

$v^2 = lg \cos \theta$ …(1)

change in K.E. = work done

$1/2\ mv^2 - 1/2\ mu^2 = mgh$

$\Rightarrow v^2 - 57 = -2 \times 1.5\ g\ (1 + \cos \theta)$…(2)

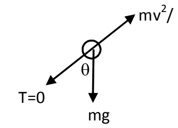
$\Rightarrow v^2 = 57 - 3g(1 + \cos \theta)$

Putting the value of v from equation (1)

$15 \cos \theta = 57 - 3g\ (1 + \cos \theta) \Rightarrow 15 \cos \theta = 57 - 30 - 30 \cos \theta$

$\Rightarrow 45 \cos \theta = 27 \Rightarrow \cos \theta = 3/5.$

$\Rightarrow \theta = \cos^{-1} (3/5) = 53°$



b) $v = \sqrt{57 - 3g(1 + \cos\theta)}$ from equation (2)

$= \sqrt{9} = 3$ m/sec.

c) As the string becomes slack at point B, the particle will start making projectile motion.

$H = OE + DC = 1.5 \cos \theta + \dfrac{u^2 \sin^2 \theta}{2g}$

$= (1.5) \times (3/5) + \dfrac{9 \times (0.8)^2}{2 \times 10} = 1.2$ m.

58.

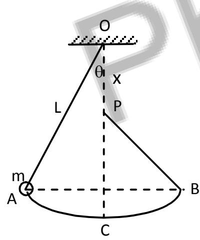


Fig-1

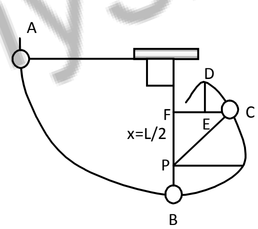


Fig-2

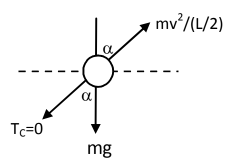


Fig-3

a) When the bob has an initial height less than the peg and then released from rest (figure 1), let body travels from A to B.

Since, Total energy at A = Total energy at B

$\therefore (K.E)_A = (PE)_A = (KE)_B + (PE)_B$

$\Rightarrow$ $(PE)_A = (PE)_B$ [because, $(KE)_A = (KE)_B = 0$]

So, the maximum height reached by the bob is equal to initial height.

b) When the pendulum is released with $\theta = 90°$ and $x = L/2$, (figure 2) the path of the particle is shown in the figure 2.

At point C, the string will become slack and so the particle will start making projectile motion. (Refer Q.No. 56)

$(1/2)mv_c^2 - 0 = mg\ (L/2)\ (1 - \cos \alpha)$

because, distance between A nd C in the vertical direction is L/2 (1 – cos α)

$\Rightarrow v_c^2 = gL(1 - \cos\theta)$ ..(1)

Again, form the freebody diagram (fig – 3)

$\frac{mv_c^2}{L/2} = mg\cos\alpha$ {because $T_c = 0$}

So, $V_C^2 = \frac{gL}{2}\cos\alpha$ ..(2)

From Eqn.(1) and equn (2),

$gL(1 - \cos\alpha) = \frac{gL}{2}\cos\alpha$

$\Rightarrow 1 - \cos\alpha = 1/2\cos\alpha$

$\Rightarrow 3/2\cos\alpha = 1 \Rightarrow \cos\alpha = 2/3$ ..(3)

To find highest position C, before the string becomes slack

$BF = \frac{L}{2} + \frac{L}{2}\cos\theta = \frac{L}{2} + \frac{L}{2}\times\frac{2}{3} = L\left(\frac{1}{2} + \frac{1}{3}\right)$

So, BF = (5L/6)

c) If the particle has to complete a vertical circle, at the point C.

$\frac{mv_c^2}{(L-x)} = mg$

$\Rightarrow v_c^2 = g(L - x)$ ..(1)

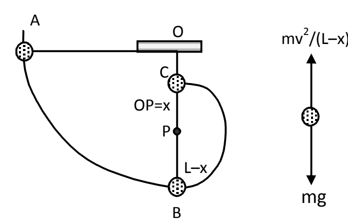
Again, applying energy principle between A and C,

$1/2\, mv_c^2 - 0 = mg(OC)$

$\Rightarrow 1/2\, v_c^2 = mg[L - 2(L - x)] = mg(2x - L)$

$\Rightarrow v_c^2 = 2g(2x - L)$ ..(2)

From equn. (1) and equn (2)



$g(L - x) = 2g(2x - L)$

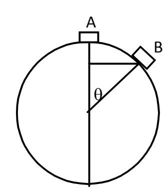
$\Rightarrow L - x = 4x - 2L$

$\Rightarrow 5x = 3L$

$\therefore \frac{x}{L} = \frac{3}{5} = 0.6$

So, the rates (x/L) should be 0.6

59. Let the velocity be v when the body leaves the surface.



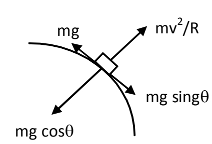
From the freebody diagram,

$\frac{mv^2}{R} = mg\cos\theta$ [Because normal reaction]

$v^2 = Rg\cos\theta$ ..(1)

Again, form work-energy principle,

Change in K.E. = work done



$\Rightarrow 1/2\, mv^2 - 0 = mg(R - R\cos\theta)$

$\Rightarrow v^2 = 2gR(1 - \cos\theta)$ ..(2)

From (1) and (2)

$Rg\cos\theta = 2gR(1 - \cos\theta)$

3gR cos θ = 2 gR

Cos θ = 2/3

$\theta = \cos^{-1}(2/3)$

60. a) When the particle is released from rest (fig–1), the centrifugal force is zero.

N force is zero = mg cos θ

$= mg \cos 30° = \frac{\sqrt{3}mg}{2}$

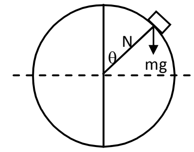


Fig-1

b) When the particle leaves contact with the surface (fig–2), N = 0.

So, $\frac{mv^2}{R}$ mg cos θ

$\Rightarrow v^2 = Rg \cos\theta$ ..(1)

Again, $\frac{1}{2}mv^2 = mgR(\cos 30° - \cos\theta)$

$\Rightarrow v^2 = 2Rg\left(\frac{\sqrt{3}}{2} - \cos\theta\right)$ ..(2)

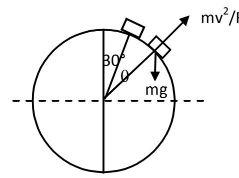


Fig-2

From equn. (1) and equn. (2)

$Rg \cos\theta = \sqrt{3}\,Rg - 2Rg\cos\theta$

$\Rightarrow 3\cos\theta = \sqrt{3}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

So, the distance travelled by the particle before leaving contact,

$\ell = R(\theta - \pi/6)$ [because 30° = π/6]

putting the value of θ, we get $\ell = 0.43R$

61. a) Radius =R

horizontal speed = v

From the free body diagram, (fig–1)

N = Normal force = $mg - \frac{mv^2}{R}$

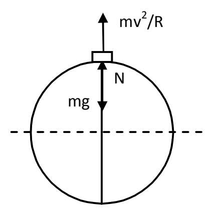


Fig-1

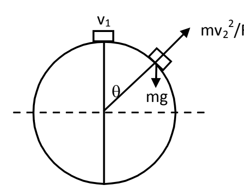


Fig-2

b) When the particle is given maximum velocity so that the centrifugal force balances the weight, the particle does not slip on the sphere.

$\frac{mv^2}{R} = mg \Rightarrow v = \sqrt{gR}$

c) If the body is given velocity v_1

$v_1 = \sqrt{gR}/2$

$v_1^2 - gR/4$

Let the velocity be v_2 when it leaves contact with the surface, (fig–2)

So, $\frac{mv^2}{R} = mg\cos\theta$

$\Rightarrow v_2^2 = Rg\cos\theta$ ..(1)

Again, $1/2\ mv_2^2 - 1/2\ mv_1^2 = mgR(1 - \cos\theta)$

$\Rightarrow v_2^2 = v_1^2 + 2gR(1 - \cos\theta)$ ..(2)

From equn. (1) and equn (2)

$Rg \cos\theta = (Rg/4) + 2gR(1 - \cos\theta)$

$\Rightarrow \cos\theta = (1/4) + 2 - 2\cos\theta$

$\Rightarrow 3\cos\theta = 9/4$

$\Rightarrow \cos\theta = 3/4$

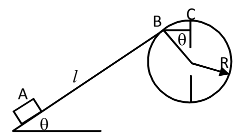
$\Rightarrow \theta = \cos^{-1}(3/4)$

62. a) Net force on the particle between A & B, $F = mg \sin\theta$

work done to reach B, $W = FS = mg \sin\theta\, \ell$

Again, work done to reach B to C = $mgh = mg R(1 - \cos\theta)$

So, Total workdone = $mg[\ell \sin\theta + R(1 - \cos\theta)]$



Now, change in K.E. = work done

$\Rightarrow 1/2\, mv_O^2 = mg[\ell \sin\theta + R(1 - \cos\theta)]$

$\Rightarrow v_o = \sqrt{2g(R(1-\cos\theta)+\ell\sin\theta)}$

b) When the block is projected at a speed $2v_o$.

Let the velocity at C will be V_c.

Applying energy principle,

$1/2\, mv_c^2 - 1/2\, m(2v_o)^2 = -mg[\ell \sin\theta + R(1 - \cos\theta)]$

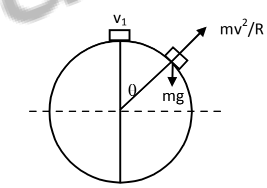
$\Rightarrow v_c^2 = 4v_o - 2g[\ell \sin\theta + R(1 - \cos\theta)]$

$4.2g[\ell \sin\theta + R(1 - \cos\theta)] - 2g[\ell \sin\theta + R(1 - \cos\theta)$

$= 6g[\ell \sin\theta + R(1 - \cos\theta)]$

So, force acting on the body,

$\Rightarrow N = \dfrac{mv_c^2}{R} = 6mg[(\ell/R)\sin\theta + 1 - \cos\theta]$



c) Let the block loose contact after making an angle θ

$\dfrac{mv^2}{R} = mg\cos\theta \Rightarrow v^2 = Rg\cos\theta$ ..(1)

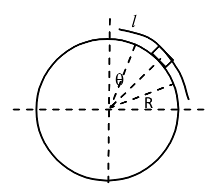
Again, $1/2\, mv^2 = mg(R - R\cos\theta) \Rightarrow v^2 = 2gR(1 - \cos\theta)$ ..(2)........(?)

From (1) and (2) $\cos\theta = 2/3 \Rightarrow \theta = \cos^{-1}(2/3)$

63. Let us consider a small element which makes angle 'dθ' at the centre.

$\therefore dm = (m/\ell)R\, d\theta$

a) Gravitational potential energy of 'dm' with respect to centre of the sphere



$= (dm)g\, R\cos\theta$

$= (mg/\ell)\, R\cos\theta\, d\theta$

So, Total G.P.E. = $\displaystyle\int_0^{\ell/r} \frac{mgR^2}{\ell}\cos\theta\, d\theta$ [$\alpha = (\ell/R)$](angle subtended by the chain at the centre)..........

$= \dfrac{mR^2 g}{\ell}[\sin\theta](\ell/R) = \dfrac{mRg}{\ell}\sin(\ell/R)$

b) When the chain is released from rest and slides down through an angle θ, the K.E. of the chain is given

K.E. = Change in potential energy.

$= \frac{mR^2g}{\ell} \sin(\ell/R) - m\int \frac{gR^2}{\ell} \cos\theta \, d\theta$..........?

$= \frac{mR^2g}{\ell} [\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

c) Since, K.E. = $1/2\, mv^2 = \frac{mR^2g}{\ell}[\sin(\ell/R) + \sin\theta - \sin\{\theta + (\ell/R)\}]$

Taking derivative of both sides with respect to 't'

$(1/2) \times 2v \times \frac{dv}{dt} = \frac{R^2g}{\ell}[\cos\theta \times \frac{d\theta}{dt} - \cos(\theta + \ell/R)\frac{d\theta}{dt}]$

$\therefore (R\frac{d\theta}{dt})\frac{dv}{dt} = \frac{R^2g}{\ell} \times \frac{d\theta}{dt}[\cos\theta - \cos(\theta + (\ell/R))]$

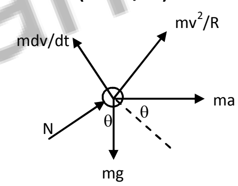
When the chain starts sliding down, $\theta = 0$.

So, $\frac{dv}{dt} = \frac{Rg}{\ell}[1 - \cos(\ell/R)]$

64. Let the sphere move towards left with an acceleration 'a

Let m = mass of the particle

The particle 'm' will also experience the inertia due to acceleration 'a' as it is on the sphere. It will also experience the tangential inertia force (m (dv/dt)) and centrifugal force (mv^2/R).

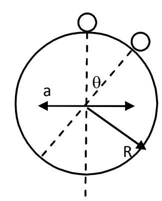


$m\frac{dv}{dt} = ma\cos\theta + mg\sin\theta \Rightarrow mv\frac{dv}{dt} = ma\cos\theta\left(R\frac{d\theta}{dt}\right) + mg\sin\theta\left(R\frac{d\theta}{dt}\right)$

Because, $v = R\frac{d\theta}{dt}$

$\Rightarrow v\,dv = aR\cos\theta\, d\theta + gR\sin\theta\, d\theta$

Integrating both sides we get,



$\frac{v^2}{2} = aR\sin\theta - gR\cos\theta + C$

Given that, at $\theta = 0$, $v = 0$, So, $C = gR$

So, $\frac{v^2}{2} = aR\sin\theta - gR\cos\theta + gR$

$\therefore v^2 = 2R(a\sin\theta + g - g\cos\theta) \Rightarrow v = [2R(a\sin\theta + g - g\cos\theta)]^{1/2}$

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