

SOLUTIONS TO CONCEPTS CHAPTER 14

1. $F = mg$

$$\text{Stress} = \frac{F}{A}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

$$Y = \frac{FL}{A\Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{YA}$$

2. $\rho = \text{stress} = mg/A$

$$e = \text{strain} = \rho/Y$$

$$\text{Compression } \Delta L = eL$$

3. $y = \frac{F}{A} \frac{L}{\Delta L} \Rightarrow \Delta L = \frac{FL}{AY}$

4. $L_{\text{steel}} = L_{\text{cu}}$ and $A_{\text{steel}} = A_{\text{cu}}$

a) $\frac{\text{Stress of cu}}{\text{Stress of st}} = \frac{F_{\text{cu}}}{A_{\text{cu}}} \frac{A_{\text{g}}}{F_{\text{g}}} = \frac{F_{\text{cu}}}{F_{\text{st}}} = 1$

b) $\text{Strain} = \frac{\Delta L_{\text{st}}}{L_{\text{cu}}} = \frac{F_{\text{st}} L_{\text{st}}}{A_{\text{st}} Y_{\text{st}}} \cdot \frac{A_{\text{cu}} Y_{\text{cu}}}{F_{\text{cu}} L_{\text{cu}}} \quad (\because L_{\text{cu}} = L_{\text{st}} ; A_{\text{cu}} = A_{\text{st}})$

5. $\left(\frac{\Delta L}{L}\right)_{\text{st}} = \frac{F}{AY_{\text{st}}}$

$$\left(\frac{\Delta L}{L}\right)_{\text{cu}} = \frac{F}{AY_{\text{cu}}}$$

$$\frac{\text{strain steel wire}}{\text{Strain on copper wire}} = \frac{F}{AY_{\text{st}}} \times \frac{AY_{\text{cu}}}{F} (\because A_{\text{cu}} = A_{\text{st}}) = \frac{Y_{\text{cu}}}{Y_{\text{st}}}$$

6. $\text{Stress in lower rod} = \frac{T_1}{A_1} \Rightarrow \frac{m_1 g + \omega g}{A_1} \Rightarrow w = 14 \text{ kg}$

$$\text{Stress in upper rod} = \frac{T_2}{A_u} \Rightarrow \frac{m_2 g + m_1 g + \omega g}{A_u} \Rightarrow w = .18 \text{ kg}$$

For same stress, the max load that can be put is 14 kg. If the load is increased the lower wire will break first.

$$\frac{T_1}{A_1} = \frac{m_1 g + \omega g}{A_1} = 8 \times 10^8 \Rightarrow w = 14 \text{ kg}$$

$$\frac{T_2}{A_u} \Rightarrow \frac{m_2 g + m_1 g + \omega g}{A_u} = 8 \times 10^8 \Rightarrow \omega_0 = 2 \text{ kg}$$

The maximum load that can be put is 2 kg. Upper wire will break first if load is increased.

7. $Y = \frac{F}{A} \frac{L}{\Delta L}$

8. $Y = \frac{F}{A} \frac{L}{\Delta L} \Rightarrow F = \frac{YA \Delta L}{L}$

9. $m_2 g - T = m_2 a \quad \dots(1)$

and $T - F = m_1 a \quad \dots(2)$

$$\Rightarrow a = \frac{m_2 g - F}{m_1 + m_2}$$

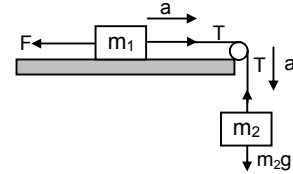
From equation (1) and (2), we get $\frac{m_2 g}{2(m_1 + m_2)}$

Again, $T = F + m_1 a$

$$\Rightarrow T = \frac{m_2 g}{2} + m_1 \frac{m_2 g}{2(m_1 + m_2)} \Rightarrow \frac{m_2^2 g + 2m_1 m_2 g}{2(m_1 + m_2)}$$

$$\text{Now } Y = \frac{FL}{A \Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{AY}$$

$$\Rightarrow \frac{\Delta L}{L} = \frac{(m_2^2 + 2m_1 m_2)g}{2(m_1 + m_2)AY} = \frac{m_2 g(m_2 + 2m_1)}{2AY(m_1 + m_2)}$$



10. At equilibrium $\Rightarrow T = mg$

When it moves to an angle θ , and released, the tension the T' at lowest point is

$$\Rightarrow T' = mg + \frac{mv^2}{r}$$

The change in tension is due to centrifugal force $\Delta T = \frac{mv^2}{r} \dots(1)$

\Rightarrow Again, by work energy principle,

$$\Rightarrow \frac{1}{2}mv^2 - 0 = mgr(1 - \cos\theta)$$

$$\Rightarrow v^2 = 2gr(1 - \cos\theta) \dots(2)$$

$$\text{So, } \Delta T = \frac{m[2gr(1 - \cos\theta)]}{r} = 2mg(1 - \cos\theta)$$

$$\Rightarrow F = \Delta T$$

$$\Rightarrow F = \frac{YA \Delta L}{L} = 2mg - 2mg \cos\theta \Rightarrow 2mg \cos\theta = 2mg - \frac{YA \Delta L}{L}$$

$$= \cos\theta = 1 - \frac{YA \Delta L}{L(2mg)}$$

11. From figure $\cos\theta = \frac{x}{\sqrt{x^2 + l^2}} = \frac{x}{l} \left[1 + \frac{x^2}{l^2} \right]^{-1/2}$
 $= x/l \dots(1)$

Increase in length $\Delta L = (AC + CB) - AB$

$$\text{Here, } AC = (l^2 + x^2)^{1/2}$$

$$\text{So, } \Delta L = 2(l^2 + x^2)^{1/2} - 100 \dots(2)$$

$$Y = \frac{F l}{A \Delta l} \dots(3)$$

From equation (1), (2) and (3) and the freebody diagram,

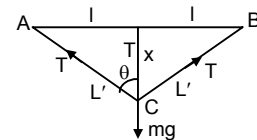
$$2l \cos\theta = mg.$$

12. $Y = \frac{FL}{A \Delta L} \Rightarrow \frac{\Delta L}{L} = \frac{F}{Ay}$

$$\sigma = \frac{\Delta D/D}{\Delta L/L} \Rightarrow \frac{\Delta D}{D} = \frac{\Delta L}{L}$$

$$\text{Again, } \frac{\Delta A}{A} = \frac{2\Delta r}{r}$$

$$\Rightarrow \Delta A = \frac{2\Delta r}{r}$$



$$13. B = \frac{Pv}{\Delta v} \Rightarrow P = B \left(\frac{\Delta v}{v} \right)$$

$$14. \rho_0 = \frac{m}{V_0} = \frac{m}{V_d}$$

$$\text{so, } \frac{\rho_d}{\rho_0} = \frac{V_0}{V_d} \quad \dots(1)$$

$$\text{vol.strain} = \frac{V_0 - V_d}{V_0}$$

$$B = \frac{\rho_0 gh}{(V_0 - V_d)/V_0} \Rightarrow 1 - \frac{V_d}{V_0} = \frac{\rho_0 gh}{B}$$

$$\Rightarrow \frac{vD}{v_0} = \left(1 - \frac{\rho_0 gh}{B} \right) \quad \dots(2)$$

Putting value of (2) in equation (1), we get

$$\frac{\rho_d}{\rho_0} = \frac{1}{1 - \rho_0 gh/B} \Rightarrow \rho_d = \frac{1}{(1 - \rho_0 gh/B)} \times \rho_0$$

$$15. \eta = \frac{F}{A\theta}$$

Lateral displacement = $l\theta$.

$$16. F = Tl$$

$$17. \text{a) } P = \frac{2T_{Hg}}{r} \quad \text{b) } P = \frac{4T_g}{r} \quad \text{c) } P = \frac{2T_g}{r}$$

$$18. \text{a) } F = P_0 A$$

$$\text{b) Pressure} = P_0 + (2T/r)$$

$$F = P'A = (P_0 + (2T/r))A$$

$$\text{c) } P = 2T/r$$

$$F = PA = \frac{2T}{r} A$$

$$19. \text{a) } h_A = \frac{2T \cos \theta}{r_A - \rho g} \quad \text{b) } h_B = \frac{2T \cos \theta}{r_B \rho g} \quad \text{c) } h_C = \frac{2T \cos \theta}{r_C \rho g}$$

$$20. h_{Hg} = \frac{2T_{Hg} \cos \theta_{Hg}}{r \rho_{Hg} g}$$

$$h_{\omega} = \frac{2T_{\omega} \cos \theta_{\omega}}{r \rho_{\omega} g} \quad \text{where, the symbols have their usual meanings.}$$

$$\frac{h_{\omega}}{h_{Hg}} = \frac{T_{\omega}}{T_{Hg}} \times \frac{\rho_{Hg}}{\rho_{\omega}} \times \frac{\cos \theta_{\omega}}{\cos \theta_{Hg}}$$

$$21. h = \frac{2T \cos \theta}{r \rho g}$$

$$22. P = \frac{2T}{r}$$

$$P = F/r$$

$$23. A = \pi r^2$$

$$24. \frac{4}{3} \pi R^3 = \frac{4}{3} \pi r^3 \times 8$$

$$\Rightarrow r = R/2 = 2$$

Increase in surface energy = $TA' - TA$

$$25. h = \frac{2T \cos \theta}{r\rho g}, h' = \frac{2T \cos \theta}{r\rho g}$$

$$\Rightarrow \cos \theta = \frac{h'r\rho g}{2T}$$

$$\text{So, } \theta = \cos^{-1}(1/2) = 60^\circ.$$

$$26. \text{ a) } h = \frac{2T \cos \theta}{r\rho g}$$

$$\text{ b) } T \times 2\pi r \cos \theta = \pi r^2 h \times \rho \times g$$

$$\therefore \cos \theta = \frac{hr\rho g}{2T}$$

$$27. T(2l) = [1 \times (10^{-3}) \times h]\rho g$$

$$28. \text{ Surface area} = 4\pi r^2$$

$$29. \text{ The length of small element} = r d\theta$$

$$dF = T \times r d\theta$$

considering symmetric elements,

$$dF_y = 2T r d\theta \cdot \sin \theta \quad [dF_x = 0]$$

$$\text{so, } F = 2Tr \int_0^{\pi/2} \sin \theta d\theta = 2Tr [\cos \theta]_0^{\pi/2} = T \times 2r$$

$$\text{Tension} \Rightarrow 2T_1 = T \times 2r \Rightarrow T_1 = Tr$$

$$30. \text{ a) Viscous force} = 6\pi\eta rv$$

$$\text{ b) Hydrostatic force} = B = \left(\frac{4}{3}\right)\pi r^3 \sigma g$$

$$\text{ c) } 6\pi\eta rv + \left(\frac{4}{3}\right)\pi r^3 \sigma g = mg$$

$$v = \frac{2r^2(\rho - \sigma)g}{9\eta} \Rightarrow \frac{2}{9}r^2 \frac{\left(\frac{m}{(4/3)\pi r^3} - \sigma\right)g}{n}$$

31. To find the terminal velocity of rain drops, the forces acting on the drop are,

i) The weight $(4/3)\pi r^3 \rho g$ downward.

ii) Force of buoyancy $(4/3)\pi r^3 \sigma g$ upward.

iii) Force of viscosity $6\pi\eta rv$ upward.

Because, σ of air is very small, the force of buoyancy may be neglected.

Thus,

$$6\pi\eta rv = \left(\frac{4}{3}\right)\pi r^2 \rho g \quad \text{or} \quad v = \frac{2r^2 \rho g}{9\eta}$$

$$32. v = \frac{R\eta}{\rho D} \Rightarrow R = \frac{v\rho D}{\eta}$$

