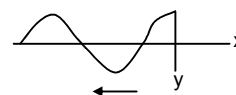


SOLUTIONS TO CONCEPTS CHAPTER 15

1. $v = 40 \text{ cm/sec}$

As velocity of a wave is constant location of maximum after 5 sec
 $= 40 \times 5 = 200 \text{ cm}$ along negative x-axis.



2. Given $y = Ae^{-[(x/a)+(t/T)]^2}$

a) $[A] = [M^0L^1T^0]$, $[T] = [M^0L^0T^1]$

$[a] = [M^0L^1T^0]$

b) Wave speed, $v = \lambda/T = a/T$ [Wave length $\lambda = a$]

c) If $y = f(t - x/v) \rightarrow$ wave is traveling in positive direction

and if $y = f(t + x/v) \rightarrow$ wave is traveling in negative direction

$$\text{So, } y = Ae^{-[(x/a)+(t/T)]^2} = Ae^{-\left(\frac{1}{T}\right)\left[\frac{x}{a/T}+t\right]^2}$$

$$= Ae^{-\left(\frac{1}{T}\right)\left[\frac{x}{v}+t\right]^2}$$

i.e. $y = f\{t + (x/v)\}$

d) Wave speed, $v = a/T$

\therefore Max. of pulse at $t = T$ is $(a/T) \times T = a$ (negative x-axis)

Max. of pulse at $t = 2T = (a/T) \times 2T = 2a$ (along negative x-axis)

So, the wave travels in negative x-direction.

3. At $t = 1 \text{ sec}$, $s_1 = vt = 10 \times 1 = 10 \text{ cm}$

$t = 2 \text{ sec}$, $s_2 = vt = 10 \times 2 = 20 \text{ cm}$

$t = 3 \text{ sec}$, $s_3 = vt = 10 \times 3 = 30 \text{ cm}$

4. The pulse is given by, $y = \left[\frac{a^3}{(x-vt)^2 + a^2}\right]$

$a = 5 \text{ mm} = 0.5 \text{ cm}$, $v = 20 \text{ cm/s}$

At $t = 0 \text{ s}$, $y = a^3 / (x^2 + a^2)$

The graph between y and x can be plotted by taking different values of x .

(left as exercise for the student)

similarly, at $t = 1 \text{ s}$, $y = a^3 / \{(x-v)^2 + a^2\}$

and at $t = 2 \text{ s}$, $y = a^3 / \{(x-2v)^2 + a^2\}$

5. At $x = 0$, $f(t) = a \sin(t/T)$

Wave speed = v

$\Rightarrow \lambda = \text{wavelength} = vT$ ($T = \text{Time period}$)

So, general equation of wave

$Y = A \sin \left[\left(\frac{t}{T} \right) - \left(\frac{x}{vT} \right) \right]$ [because $y = f\left(\frac{t}{T} - \frac{x}{\lambda}\right)$]

6. At $t = 0$, $g(x) = A \sin(x/a)$

a) $[M^0L^1T^0] = [L]$

$a = [M^0L^1T^0] = [L]$

b) Wave speed = v

\therefore Time period, $T = a/v$ ($a = \text{wave length} = \lambda$)

\therefore General equation of wave

$y = A \sin \left\{ \left(\frac{x}{a} \right) - \left(\frac{t}{a/v} \right) \right\}$

$= A \sin \left\{ \left(\frac{x-vt}{a} \right) \right\}$

7. At $t = t_0$, $g(x, t_0) = A \sin(x/a) \dots(1)$

For a wave traveling in the positive x-direction, the general equation is given by

$$y = f\left(\frac{x}{a} - \frac{t}{T}\right)$$

Putting $t = -t_0$ and comparing with equation (1), we get

$\Rightarrow g(x, 0) = A \sin \left\{ \left(\frac{x}{a} \right) + \left(\frac{t_0}{T} \right) \right\}$

$\Rightarrow g(x, t) = A \sin \left\{ \left(\frac{x}{a} \right) + \left(\frac{t_0}{T} \right) - \left(\frac{t}{T} \right) \right\}$

As $T = a/v$ ($a =$ wave length, $v =$ speed of the wave)

$$\Rightarrow y = A \sin \left(\frac{x}{a} + \frac{t_0}{(a/v)} - \frac{t}{(a/v)} \right)$$

$$= A \sin \left(\frac{x + v(t_0 - t)}{a} \right)$$

$$\Rightarrow y = A \sin \left[\frac{x - v(t - t_0)}{a} \right]$$

8. The equation of the wave is given by

$$y = (0.1 \text{ mm}) \sin [(31.4 \text{ m}^{-1})x + (314 \text{ s}^{-1})t] \quad y = r \sin \{(2\pi x / \lambda)\} + \omega t$$

a) Negative x-direction

b) $k = 31.4 \text{ m}^{-1}$

$$\Rightarrow 2\lambda/\lambda = 31.4 \Rightarrow \lambda = 2\pi/31.4 = 0.2 \text{ m} = 20 \text{ cm}$$

Again, $\omega = 314 \text{ s}^{-1}$

$$\Rightarrow 2\pi f = 314 \Rightarrow f = 314 / 2\pi = 314 / (2 \times (3/14)) = 50 \text{ sec}^{-1}$$

$$\therefore \text{wave speed, } v = \lambda f = 20 \times 50 = 1000 \text{ cm/s}$$

c) Max. displacement = 0.10 mm

$$\text{Max. velocity} = a\omega = 0.1 \times 10^{-1} \times 314 = 3.14 \text{ cm/sec.}$$

9. Wave speed, $v = 20 \text{ m/s}$

$$A = 0.20 \text{ cm}$$

$$\lambda = 2 \text{ cm}$$

a) Equation of wave along the x-axis

$$y = A \sin (kx - \omega t)$$

$$\therefore k = 2\pi/\lambda = 2\pi/2 = \pi \text{ cm}^{-1}$$

$$T = \lambda/v = 2/2000 = 1/1000 \text{ sec} = 10^{-3} \text{ sec}$$

$$\Rightarrow \omega = 2\pi/T = 2\pi \times 10^3 \text{ sec}^{-1}$$

So, the wave equation is,

$$\therefore y = (0.2 \text{ cm}) \sin [(\pi \text{ cm}^{-1})x - (2\pi \times 10^3 \text{ sec}^{-1})t]$$

b) At $x = 2 \text{ cm}$, and $t = 0$,

$$y = (0.2 \text{ cm}) \sin (\pi/2) = 0$$

$$\therefore v = r\omega \cos \pi x = 0.2 \times 2000 \pi \times \cos 2\pi = 400 \pi$$

$$= 400 \times (3.14) = 1256 \text{ cm/s}$$

$$= 400 \pi \text{ cm/s} = 4\pi \text{ m/s}$$

10. $Y = (1 \text{ mm}) \sin \pi \left[\frac{x}{2 \text{ cm}} - \frac{t}{0.01 \text{ sec}} \right]$

a) $T = 2 \times 0.01 = 0.02 \text{ sec} = 20 \text{ ms}$

$$\lambda = 2 \times 2 = 4 \text{ cm}$$

b) $v = dy/dt = d/dt [\sin 2\pi \{(x/4) - (t/0.02)\}] = -\cos 2\pi \{(x/4) - (t/0.02)\} \times 1/(0.02)$

$$\Rightarrow v = -50 \cos 2\pi \{(x/4) - (t/0.02)\}$$

$$\text{at } x = 1 \text{ and } t = 0.01 \text{ sec, } v = -50 \cos 2\pi [(1/4) - (1/2)] = 0$$

c) i) at $x = 3 \text{ cm}$, $t = 0.01 \text{ sec}$

$$v = -50 \cos 2\pi (3/4 - 1/2) = 0$$

ii) at $x = 5 \text{ cm}$, $t = 0.01 \text{ sec}$, $v = 0$ (putting the values)

iii) at $x = 7 \text{ cm}$, $t = 0.01 \text{ sec}$, $v = 0$

$$\text{at } x = 1 \text{ cm and } t = 0.011 \text{ sec}$$

$$v = -50 \cos 2\pi \{(1/4) - (0.011/0.02)\} = -50 \cos (3\pi/5) = -9.7 \text{ cm/sec}$$

(similarly the other two can be calculated)

11. Time period, $T = 4 \times 5 \text{ ms} = 20 \times 10^{-3} = 2 \times 10^{-2} \text{ s}$

$$\lambda = 2 \times 2 \text{ cm} = 4 \text{ cm}$$

$$\text{frequency, } f = 1/T = 1/(2 \times 10^{-2}) = 50 \text{ s}^{-1} = 50 \text{ Hz}$$

$$\text{Wave speed} = \lambda f = 4 \times 50 \text{ m/s} = 200 \text{ m/s} = 2 \text{ m/s}$$

12. Given that, $v = 200$ m/s
 a) Amplitude, $A = 1$ mm
 b) Wave length, $\lambda = 4$ cm
 c) wave number, $n = 2\pi/\lambda = (2 \times 3.14)/4 = 1.57 \text{ cm}^{-1}$ (wave number = k)
 d) frequency, $f = 1/T = (26/\lambda)/20 = 20/4 = 5$ Hz
 (where time period $T = \lambda/v$)

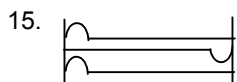
13. Wave speed = $v = 10$ m/sec
 Time period = $T = 20$ ms = $20 \times 10^{-3} = 2 \times 10^{-2}$ sec
 a) wave length, $\lambda = vT = 10 \times 2 \times 10^{-2} = 0.2$ m = 20 cm
 b) wave length, $\lambda = 20$ cm
 \therefore phase diffⁿ = $(2\pi/\lambda) x = (2\pi/20) \times 10 = \pi$ rad
 $y_1 = a \sin(\omega t - kx) \Rightarrow 1.5 = a \sin(\omega t - kx)$
 So, the displacement of the particle at a distance $x = 10$ cm.

$$[\phi = \frac{2\pi x}{\lambda} = \frac{2\pi \times 10}{20} = \pi] \text{ is given by}$$

$$y_2 = a \sin(\omega t - kx + \pi) \Rightarrow -a \sin(\omega t - kx) = -1.5 \text{ mm}$$

$$\therefore \text{displacement} = -1.5 \text{ mm}$$

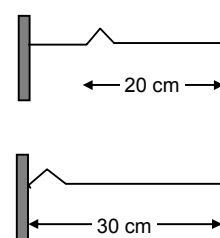
14. mass = 5 g, length $l = 64$ cm
 \therefore mass per unit length = $m = 5/64$ g/cm
 \therefore Tension, $T = 8\text{N} = 8 \times 10^5$ dyne
 $V = \sqrt{(T/m)} = \sqrt{(8 \times 10^5 \times 64)/5} = 3200$ cm/s = 32 m/s



- a) Velocity of the wave, $v = \sqrt{(T/m)} = \sqrt{(16 \times 10^5)/0.4} = 2000$ cm/sec
 \therefore Time taken to reach to the other end = $20/2000 = 0.01$ sec
 Time taken to see the pulse again in the original position = $0.01 \times 2 = 0.02$ sec
 b) At $t = 0.01$ s, there will be a 'thump' at the right end as it is reflected.
16. The crest reflects as a crest here, as the wire is traveling from denser to rarer medium.
 \Rightarrow phase change = 0

- a) To again original shape distance travelled by the wave $S = 20 + 20 = 40$ cm.
 Wave speed, $v = 20$ m/s \Rightarrow time = $s/v = 40/20 = 2$ sec
 b) The wave regains its shape, after traveling a periodic distance = $2 \times 30 = 60$ cm
 \therefore Time period = $60/20 = 3$ sec.

- c) Frequency, $n = (1/3 \text{ sec}^{-1})$
 $n = (1/2l) \sqrt{(T/m)}$ $m =$ mass per unit length = 0.5 g/cm
 $\Rightarrow 1/3 = 1/(2 \times 30) \sqrt{(T/0.5)}$
 $\Rightarrow T = 400 \times 0.5 = 200$ dyne = 2×10^{-3} Newton.



17. Let $v_1 =$ velocity in the 1st string

$$\Rightarrow v_1 = \sqrt{(T/m_1)}$$

Because $m_1 =$ mass per unit length = $(\rho_1 a_1 l_1 / l_1) = \rho_1 a_1$ where $a_1 =$ Area of cross section

$$\Rightarrow v_1 = \sqrt{(T/\rho_1 a_1)} \dots (1)$$

Let $v_2 =$ velocity in the second string

$$\Rightarrow v_2 = \sqrt{(T/m_2)}$$

$$\Rightarrow v_2 = \sqrt{(T/\rho_2 a_2)} \dots (2)$$

Given that, $v_1 = 2v_2$

$$\Rightarrow \sqrt{(T/\rho_1 a_1)} = 2 \sqrt{(T/\rho_2 a_2)} \Rightarrow (T/a_1 \rho_1) = 4(T/a_2 \rho_2)$$

$$\Rightarrow \rho_1/\rho_2 = 1/4 \Rightarrow \rho_1 : \rho_2 = 1 : 4 \quad (\text{because } a_1 = a_2)$$

18. $m = \text{mass per unit length} = 1.2 \times 10^{-4} \text{ kg/mt}$

$$Y = (0.02\text{m}) \sin [(1.0 \text{ m}^{-1})x + (30 \text{ s}^{-1})t]$$

Here, $k = 1 \text{ m}^{-1} = 2\pi/\lambda$

$$\omega = 30 \text{ s}^{-1} = 2\pi f$$

\therefore velocity of the wave in the stretched string

$$v = \lambda f = \omega/k = 30/1 = 30 \text{ m/s}$$

$$\Rightarrow v = \sqrt{T/m} \Rightarrow 30\sqrt{(T/1.2) \times 10^{-4} \text{ N}}$$

$$\Rightarrow T = 10.8 \times 10^{-2} \text{ N} \Rightarrow T = 1.08 \times 10^{-1} \text{ Newton.}$$

19. Amplitude, $A = 1 \text{ cm}$, Tension $T = 90 \text{ N}$

Frequency, $f = 200/2 = 100 \text{ Hz}$

Mass per unit length, $m = 0.1 \text{ kg/mt}$

a) $\Rightarrow V = \sqrt{T/m} = 30 \text{ m/s}$

$$\lambda = V/f = 30/100 = 0.3 \text{ m} = 30 \text{ cm}$$

b) The wave equation $y = (1 \text{ cm}) \cos 2\pi (t/0.01 \text{ s}) - (x/30 \text{ cm})$
[because at $x = 0$, displacement is maximum]

c) $y = 1 \cos 2\pi(x/30 - t/0.01)$

$$\Rightarrow v = dy/dt = (1/0.01)2\pi \sin 2\pi \{(x/30) - (t/0.01)\}$$

$$a = dv/dt = -\{4\pi^2 / (0.01)^2\} \cos 2\pi \{(x/30) - (t/0.01)\}$$

When, $x = 50 \text{ cm}$, $t = 10 \text{ ms} = 10 \times 10^{-3} \text{ s}$

$$x = (2\pi / 0.01) \sin 2\pi \{(5/3) - (0.01/0.01)\}$$

$$= (p/0.01) \sin (2\pi \times 2 / 3) = (1/0.01) \sin (4\pi/3) = -200 \pi \sin (\pi/3) = -200 \pi x (\sqrt{3}/2)$$

$$= 544 \text{ cm/s} = 5.4 \text{ m/s}$$

Similarly

$$a = \{4\pi^2 / (0.01)^2\} \cos 2\pi \{(5/3) - 1\}$$

$$= 4\pi^2 \times 10^4 \times 1/2 \Rightarrow 2 \times 10^5 \text{ cm/s}^2 \Rightarrow 2 \text{ km/s}^2$$

20. $l = 40 \text{ cm}$, mass = 10 g

\therefore mass per unit length, $m = 10 / 40 = 1/4 \text{ (g/cm)}$

spring constant $K = 160 \text{ N/m}$

deflection = $x = 1 \text{ cm} = 0.01 \text{ m}$

$$\Rightarrow T = kx = 160 \times 0.01 = 1.6 \text{ N} = 16 \times 10^4 \text{ dyne}$$

$$\text{Again } v = \sqrt{(T/m)} = \sqrt{(16 \times 10^4)/(1/4)} = 8 \times 10^2 \text{ cm/s} = 800 \text{ cm/s}$$

\therefore Time taken by the pulse to reach the spring

$$t = 40/800 = 1/20 = 0/05 \text{ sec.}$$

21. $m_1 = m_2 = 3.2 \text{ kg}$

mass per unit length of AB = $10 \text{ g/mt} = 0.01 \text{ kg.mt}$

mass per unit length of CD = $8 \text{ g/mt} = 0.008 \text{ kg/mt}$

for the string CD, $T = 3.2 \times g$

$$\Rightarrow v = \sqrt{(T/m)} = \sqrt{(3.2 \times 10)/0.008} = \sqrt{(32 \times 10^3)/8} = 2 \times 10\sqrt{10} = 20 \times 3.14 = 63 \text{ m/s}$$

for the string AB, $T = 2 \times 3.2 \text{ g} = 6.4 \times g = 64 \text{ N}$

$$\Rightarrow v = \sqrt{(T/m)} = \sqrt{(64/0.01)} = \sqrt{6400} = 80 \text{ m/s}$$

22. Total length of string $2 + 0.25 = 2.25 \text{ mt}$

$$\text{Mass per unit length } m = \frac{4.5 \times 10^{-3}}{2.25} = 2 \times 10^{-3} \text{ kg/m}$$

$$T = 2g = 20 \text{ N}$$

$$\text{Wave speed, } v = \sqrt{(T/m)} = \sqrt{20/(2 \times 10^{-3})} = \sqrt{10^4} = 10^2 \text{ m/s} = 100 \text{ m/s}$$

Time taken to reach the pully, $t = (s/v) = 2/100 = 0.02 \text{ sec.}$

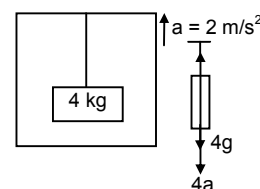
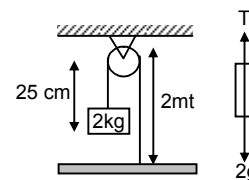
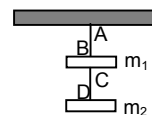
23. $m = 19.2 \times 10^{-3} \text{ kg/m}$

from the freebody diagram,

$$T - 4g - 4a = 0$$

$$\Rightarrow T = 4(a + g) = 48 \text{ N}$$

$$\text{wave speed, } v = \sqrt{(T/m)} = 50 \text{ m/s}$$



24. Let M = mass of the heavy ball
(m = mass per unit length)

Wave speed, $v_1 = \sqrt{T/m} = \sqrt{(Mg/m)}$ (because $T = Mg$)

$$\Rightarrow 60 = \sqrt{(Mg/m)} \Rightarrow Mg/m = 60^2 \quad \dots(1)$$

From the freebody diagram (2),

$$v_2 = \sqrt{T'/m}$$

$$\Rightarrow v_2 = \frac{[(Ma)^2 + (Mg)^2]^{1/4}}{m^{1/2}} \quad (\text{because } T' = \sqrt{(Ma)^2 + (Mg)^2})$$

$$\Rightarrow 62 = \frac{[(Ma)^2 + (Mg)^2]^{1/4}}{m^{1/2}}$$

$$\Rightarrow \frac{\sqrt{(Ma)^2 + (Mg)^2}}{m} = 62^2 \quad \dots(2)$$

$$\text{Eq(1) + Eq(2)} \Rightarrow (Mg/m) \times [m / \sqrt{(Ma)^2 + (Mg)^2}] = 3600 / 3844$$

$$\Rightarrow g / \sqrt{(a^2 + g^2)} = 0.936 \Rightarrow g^2 / (a^2 + g^2) = 0.876$$

$$\Rightarrow (a^2 + 100) 0.876 = 100$$

$$\Rightarrow a^2 \times 0.876 = 100 - 87.6 = 12.4$$

$$\Rightarrow a^2 = 12.4 / 0.876 = 14.15 \Rightarrow a = 3.76 \text{ m/s}^2$$

\therefore Acceⁿ of the car = 3.7 m/s^2

25. m = mass per unit length of the string
 R = Radius of the loop

ω = angular velocity, V = linear velocity of the string

Consider one half of the string as shown in figure.

The half loop experiences centrifugal force at every point, away from centre, which is balanced by tension $2T$.

Consider an element of angular part $d\theta$ at angle θ . Consider another element symmetric to this centrifugal force experienced by the element = $(mRd\theta)\omega^2 R$.

(...Length of element = $Rd\theta$, mass = $mRd\theta$)

Resolving into rectangular components net force on the two symmetric elements,

$DF = 2mR^2 d\theta\omega^2 \sin \theta$ [horizontal components cancels each other]

$$\text{So, total } F = \int_0^{\pi/2} 2mR^2\omega^2 \sin \theta d\theta = 2mR^2\omega^2 [-\cos \theta] \Rightarrow 2mR^2\omega^2$$

$$\text{Again, } 2T = 2mR^2\omega^2 \Rightarrow T = mR^2\omega^2$$

Velocity of transverse vibration $V = \sqrt{T/m} = \omega R = V$

So, the speed of the disturbance will be V .

26. a) m \rightarrow mass per unit of length of string

consider an element at distance ' x ' from lower end.

Here w_t acting down ward = $(mx)g =$ Tension in the string of upper part

Velocity of transverse vibration = $v = \sqrt{T/m} = \sqrt{(mgx/m)} = \sqrt{(gx)}$

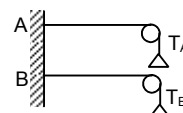
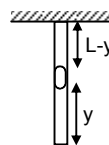
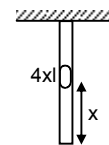
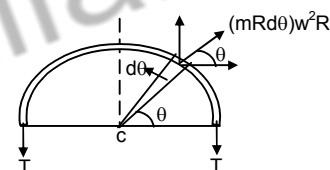
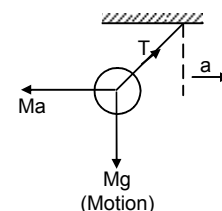
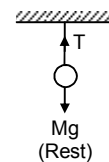
- b) For small displacement dx , $dt = dx / \sqrt{(gx)}$

$$\text{Total time } T = \int_0^L dx / \sqrt{gx} = \sqrt{(4L/g)}$$

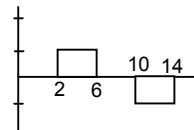
- c) Suppose after time ' t ' from start the pulse meet the particle at distance y from lower end.

$$t = \int_0^y dx / \sqrt{gx} = \sqrt{(4y/g)}$$

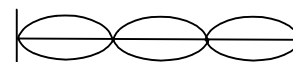
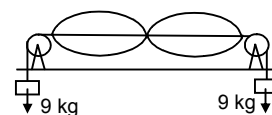
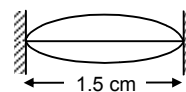
\therefore Distance travelled by the particle in this time is $(L - y)$



- $\therefore S = ut + \frac{1}{2}gt^2$
 $\Rightarrow L - y = \frac{1}{2}g \times \left\{ \sqrt{\frac{4y}{g}} \right\}^2 \quad \{u = 0\}$
 $\Rightarrow L - y = 2y \Rightarrow 3y = L$
 $\Rightarrow y = L/3$. So, the particles meet at distance $L/3$ from lower end.
27. $m_A = 1.2 \times 10^{-2} \text{ kg/m}$, $T_A = 4.8 \text{ N}$
 $\Rightarrow V_A = \sqrt{T/m} = 20 \text{ m/s}$
 $m_B = 1.2 \times 10^{-2} \text{ kg/m}$, $T_B = 7.5 \text{ N}$
 $\Rightarrow V_B = \sqrt{T/m} = 25 \text{ m/s}$
 $t = 0$ in string A
 $t_1 = 0 + 20 \text{ ms} = 20 \times 10^{-3} = 0.02 \text{ sec}$
 In 0.02 sec A has travelled $20 \times 0.02 = 0.4 \text{ m}$
 Relative speed between A and B = $25 - 20 = 5 \text{ m/s}$
 Time taken for B to overtake A = $s/v = 0.4/5 = 0.08 \text{ sec}$
28. $r = 0.5 \text{ mm} = 0.5 \times 10^{-3} \text{ m}$
 $f = 100 \text{ Hz}$, $T = 100 \text{ N}$
 $v = 100 \text{ m/s}$
 $v = \sqrt{T/m} \Rightarrow v^2 = (T/m) \Rightarrow m = (T/v^2) = 0.01 \text{ kg/m}$
 $P_{\text{ave}} = 2\pi^2 m v r^2 f^2$
 $= 2(3.14)^2(0.01) \times 100 \times (0.5 \times 10^{-3})^2 \times (100)^2 \Rightarrow 49 \times 10^{-3} \text{ watt} = 49 \text{ mW}$
29. $A = 1 \text{ mm} = 10^{-3} \text{ m}$, $m = 6 \text{ g/m} = 6 \times 10^{-3} \text{ kg/m}$
 $T = 60 \text{ N}$, $f = 200 \text{ Hz}$
 $\therefore V = \sqrt{T/m} = 100 \text{ m/s}$
 a) $P_{\text{average}} = 2\pi^2 m v A^2 f^2 = 0.47 \text{ W}$
 b) Length of the string is 2 m. So, $t = 2/100 = 0.02 \text{ sec}$.
 Energy = $2\pi^2 m v f^2 A^2 t = 9.46 \text{ mJ}$.
30. $f = 440 \text{ Hz}$, $m = 0.01 \text{ kg/m}$, $T = 49 \text{ N}$, $r = 0.5 \times 10^{-3} \text{ m}$
 a) $v = \sqrt{T/m} = 70 \text{ m/s}$
 b) $v = \lambda f \Rightarrow \lambda = v/f = 16 \text{ cm}$
 c) $P_{\text{average}} = 2\pi^2 m v r^2 f^2 = 0.67 \text{ W}$.
31. Phase difference $\phi = \pi/2$
 f and λ are same. So, ω is same.
 $y_1 = r \sin \omega t$, $y_2 = r \sin(\omega t + \pi/2)$
 From the principle of superposition
 $y = y_1 + y_2 \rightarrow r \sin \omega t + r \sin(\omega t + \pi/2)$
 $= r[\sin \omega t + \sin(\omega t + \pi/2)]$
 $= r[2\sin\{(\omega t + \omega t + \pi/2)/2\} \cos\{(\omega t - \omega t - \pi/2)/2\}]$
 $\Rightarrow y = 2r \sin(\omega t + \pi/4) \cos(-\pi/4)$
 Resultant amplitude = $\sqrt{2} r = 4\sqrt{2} \text{ mm}$ (because $r = 4 \text{ mm}$)
32. The distance travelled by the pulses are shown below.
 $t = 4 \text{ ms} = 4 \times 10^{-3} \text{ s}$ $s = vt = 50 \times 10 \times 4 \times 10^{-3} = 2 \text{ mm}$
 $t = 8 \text{ ms} = 8 \times 10^{-3} \text{ s}$ $s = vt = 50 \times 10 \times 8 \times 10^{-3} = 4 \text{ mm}$
 $t = 6 \text{ ms} = 6 \times 10^{-3} \text{ s}$ $s = 3 \text{ mm}$
 $t = 12 \text{ ms} = 12 \times 10^{-3} \text{ s}$ $s = 50 \times 10 \times 12 \times 10^{-3} = 6 \text{ mm}$
 The shape of the string at different times are shown in the figure.
33. $f = 100 \text{ Hz}$, $\lambda = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$
 \therefore wave speed, $v = f\lambda = 2 \text{ m/s}$
 a) in 0.015 sec 1st wave has travelled
 $x = 0.015 \times 2 = 0.03 \text{ m} = \text{path diff}^n$
 \therefore corresponding phase difference, $\phi = 2\pi x/\lambda = \{2\pi / (2 \times 10^{-2})\} \times 0.03 = 3\pi$.
 b) Path difference $x = 4 \text{ cm} = 0.04 \text{ m}$



- $\Rightarrow \phi = (2\pi/\lambda)x = \{(2\pi/2 \times 10^{-2}) \times 0.04\} = 4\pi$.
 c) The waves have same frequency, same wavelength and same amplitude.
 Let, $y_1 = r \sin wt$, $y_2 = r \sin (wt + \phi)$
 $\Rightarrow y = y_1 + y_2 = r[\sin wt + (wt + \phi)]$
 $= 2r \sin (wt + \phi/2) \cos (\phi/2)$
 \therefore resultant amplitude $= 2r \cos \phi/2$
 So, when $\phi = 3\pi$, $r = 2 \times 10^{-3}$ m
 $R_{\text{res}} = 2 \times (2 \times 10^{-3}) \cos (3\pi/2) = 0$
 Again, when $\phi = 4\pi$, $R_{\text{res}} = 2 \times (2 \times 10^{-3}) \cos (4\pi/2) = 4$ mm.
34. $l = 1$ m, $V = 60$ m/s
 \therefore fundamental frequency, $f_0 = V/2l = 30 \text{ sec}^{-1} = 30$ Hz.
35. $l = 2$ m, $f_0 = 100$ Hz, $T = 160$ N
 $f_0 = 1/2l\sqrt{(T/m)}$
 $\Rightarrow m = 1$ g/m. So, the linear mass density is 1 g/m.
36. $m = (4/80)$ g/cm $= 0.005$ kg/m
 $T = 50$ N, $l = 80$ cm $= 0.8$ m
 $v = \sqrt{(T/m)} = 100$ m/s
 fundamental frequency $f_0 = 1/2l\sqrt{(T/m)} = 62.5$ Hz
 First harmonic $= 62.5$ Hz
 $f_4 =$ frequency of fourth harmonic $= 4f_0 = F_3 = 250$ Hz
 $V = f_4 \lambda_4 \Rightarrow \lambda_4 = (v/f_4) = 40$ cm.
37. $l = 90$ cm $= 0.9$ m
 $m = (6/90)$ g/cm $= (6/900)$ kg/m
 $f = 261.63$ Hz
 $f = 1/2l\sqrt{(T/m)} \Rightarrow T = 1478.52$ N $= 1480$ N.
38. First harmonic be f_0 , second harmonic be f_1
 $\therefore f_1 = 2f_0$
 $\Rightarrow f_0 = f_1/2$
 $f_1 = 256$ Hz
 \therefore 1st harmonic or fundamental frequency
 $f_0 = f_1/2 = 256 / 2 = 128$ Hz
 $\lambda/2 = 1.5$ m $\Rightarrow \lambda = 3$ m (when fundamental wave is produced)
 \Rightarrow Wave speed $= V = f_0 \lambda = 384$ m/s.
39. $l = 1.5$ m, mass $= 12$ g
 $\Rightarrow m = 12/1.5$ g/m $= 8 \times 10^{-3}$ kg/m
 $T = 9 \times g = 90$ N
 $\lambda = 1.5$ m, $f_1 = 2/2l\sqrt{T/m}$
 [for, second harmonic two loops are produced]
 $f_1 = 2f_0 \Rightarrow 70$ Hz.
40. A string of mass 40 g is attached to the tuning fork
 $m = (40 \times 10^{-3})$ kg/m
 The fork vibrates with $f = 128$ Hz
 $\lambda = 0.5$ m
 $v = f\lambda = 128 \times 0.5 = 64$ m/s
 $v = \sqrt{T/m} \Rightarrow T = v^2 m = 163.84$ N $\Rightarrow 164$ N.
41. This wire makes a resonant frequency of 240 Hz and 320 Hz.
 The fundamental frequency of the wire must be divisible by both 240 Hz and 320 Hz.
 a) So, the maximum value of fundamental frequency is 80 Hz.
 b) Wave speed, $v = 40$ m/s
 $\Rightarrow 80 = (1/2l) \times 40 \Rightarrow 0.25$ m.



42. Let there be 'n' loops in the 1st case
 \Rightarrow length of the wire, $l = (n\lambda_1)/2$ [$\lambda_1 = 2 \times 2 = 4$ cm]
 So there are (n + 1) loops with the 2nd case
 \Rightarrow length of the wire, $l = \{(n+1)\lambda_2/2$ [$\lambda = 2 \times 1.6 = 3.2$ cm]

$$\Rightarrow n\lambda_1/2 = \frac{(n+1)\lambda_2}{2}$$

$$\Rightarrow n \times 4 = (n + 1) (3.2) \Rightarrow n = 4$$

$$\therefore \text{length of the string, } l = (n\lambda_1)/2 = 8 \text{ cm.}$$

43. Frequency of the tuning fork, $f = 660$ Hz
 Wave speed, $v = 220$ m/s $\Rightarrow \lambda = v/f = 1/3$ m
 No. of loops = 3

a) So, $f = (3/2l)v \Rightarrow l = 50$ cm

- b) The equation of resultant stationary wave is given by

$$y = 2A \cos(2\pi x/ql) \sin(2\pi vt/\lambda)$$

$$\Rightarrow y = (0.5 \text{ cm}) \cos(0.06 \pi \text{ cm}^{-1}) \sin(1320 \pi \text{ s}^{-1}t)$$

44. $l_1 = 30$ cm = 0.3 m

$$f_1 = 196 \text{ Hz, } f_2 = 220 \text{ Hz}$$

We know $f \propto (1/l)$ (as V is constant for a medium)

$$\Rightarrow \frac{f_1}{f_2} = \frac{l_2}{l_1} \Rightarrow l_2 = 26.7 \text{ cm}$$

Again $f_3 = 247$ Hz

$$\Rightarrow \frac{f_3}{f_1} = \frac{l_1}{l_3} \Rightarrow \frac{0.3}{l_3}$$

$$\Rightarrow l_3 = 0.224 \text{ m} = 22.4 \text{ cm and } l_3 = 20 \text{ cm}$$

45. Fundamental frequency $f_1 = 200$ Hz

Let l_4 Hz be nth harmonic

$$\Rightarrow F_2/F_1 = 14000/200$$

$$\Rightarrow NF_1/F_1 = 70 \Rightarrow N = 70$$

\therefore The highest harmonic audible is 70th harmonic.

46. The resonant frequencies of a string are

$$f_1 = 90 \text{ Hz, } f_2 = 150 \text{ Hz, } f_3 = 120 \text{ Hz}$$

- a) The highest possible fundamental frequency of the string is $f = 30$ Hz

[because f_1, f_2 and f_3 are integral multiple of 30 Hz]

- b) The frequencies are $f_1 = 3f, f_2 = 5f, f_3 = 7f$

So, f_1, f_2 and f_3 are 3rd harmonic, 5th harmonic and 7th harmonic respectively.

- c) The frequencies in the string are $f, 2f, 3f, 4f, 5f, \dots$

So, $3f = 2^{\text{nd}}$ overtone and 3rd harmonic

$5f = 4^{\text{th}}$ overtone and 5th harmonic

$7f = 6^{\text{th}}$ overtone and 7th harmonic

- d) length of the string is $l = 80$ cm

$$\Rightarrow f_1 = (3/2l)v \quad (v = \text{velocity of the wave})$$

$$\Rightarrow 90 = \{3/(2 \times 80)\} \times K$$

$$\Rightarrow K = (90 \times 2 \times 80) / 3 = 4800 \text{ cm/s} = 48 \text{ m/s.}$$

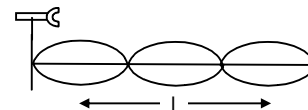
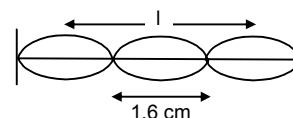
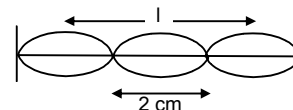
47. Frequency $f = \frac{1}{lD} \sqrt{\frac{T}{\pi\rho}} \Rightarrow f_1 = \frac{1}{l_1 D_1} \sqrt{\frac{T_1}{\pi\rho_1}} \Rightarrow f_2 = \frac{1}{l_2 D_2} \sqrt{\frac{T_2}{\pi\rho_2}}$

Given that, $T_1/T_2 = 2, r_1 / r_2 = 3 = D_1/D_2$

$$\frac{\rho_1}{\rho_2} = \frac{1}{2}$$

$$\text{So, } \frac{f_1}{f_2} = \frac{l_2 D_2}{l_1 D_1} \sqrt{\frac{T_1}{T_2}} \sqrt{\frac{\pi\rho_2}{\pi\rho_1}} \quad (l_1 = l_2 = \text{length of string})$$

$$\Rightarrow f_1 : f_2 = 2 : 3$$



48. Length of the rod = $L = 40 \text{ cm} = 0.4 \text{ m}$
 Mass of the rod $m = 1.2 \text{ kg}$
 Let the 4.8 kg mass be placed at a distance
 'x' from the left end.

Given that, $f_i = 2f_r$

$$\therefore \frac{1}{2l} \sqrt{T_i} = \frac{2}{2l} \sqrt{T_r}$$

$$\Rightarrow \sqrt{\frac{T_i}{T_r}} = 2 \Rightarrow \frac{T_i}{T_r} = 4 \quad \dots(1)$$

From the freebody diagram,

$$T_i + T_r = 60 \text{ N}$$

$$\Rightarrow 4T_r + T_r = 60 \text{ N}$$

$$\therefore T_r = 12 \text{ N and } T_i = 48 \text{ N}$$

Now taking moment about point A,

$$T_r \times (0.4) = 48x + 12(0.2) \Rightarrow x = 5 \text{ cm}$$

So, the mass should be placed at a distance 5 cm from the left end.

49. $\rho_s = 7.8 \text{ g/cm}^3$, $\rho_A = 2.6 \text{ g/cm}^3$

$$m_s = \rho_s A_s = 7.8 \times 10^{-2} \text{ g/cm} \quad (m = \text{mass per unit length})$$

$$m_A = \rho_A A_A = 2.6 \times 10^{-2} \times 3 \text{ g/cm} = 7.8 \times 10^{-3} \text{ kg/m}$$

A node is always placed in the joint. Since aluminium and steel rod has same mass per unit length, velocity of wave in both of them is same.

$$\Rightarrow v = \sqrt{T/m} \Rightarrow 500/7 \text{ m/s}$$

For minimum frequency there would be maximum wavelength for maximum wavelength minimum no of loops are to be produced.

$$\therefore \text{maximum distance of a loop} = 20 \text{ cm}$$

$$\Rightarrow \text{wavelength} = \lambda = 2 \times 20 = 40 \text{ cm} = 0.4 \text{ m}$$

$$\therefore f = v/\lambda = 180 \text{ Hz.}$$

50. Fundamental frequency

$$V = 1/2l \sqrt{T/m} \Rightarrow \sqrt{T/m} = v2l \quad [\sqrt{T/m} = \text{velocity of wave}]$$

a) wavelength, $\lambda = \text{velocity} / \text{frequency} = v2l / v = 2l$

and wave number = $K = 2\pi/\lambda = 2\pi/2l = \pi/l$

b) Therefore, equation of the stationary wave is

$$y = A \cos(2\pi x/\lambda) \sin(2\pi Vt/L)$$

$$= A \cos(2\pi x/2l) \sin(2\pi Vt/2L)$$

$$v = V/2L \quad [\text{because } v = (v/2l)]$$

51. $V = 200 \text{ m/s}$, $2A = 0.5 \text{ m}$

a) The string is vibrating in its 1st overtone

$$\Rightarrow \lambda = 1 = 2 \text{ m}$$

$$\Rightarrow f = v/\lambda = 100 \text{ Hz}$$

b) The stationary wave equation is given by

$$y = 2A \cos \frac{2\pi x}{\lambda} \sin \frac{2\pi Vt}{\lambda}$$

$$= (0.5 \text{ cm}) \cos [(\pi \text{ m}^{-1})x] \sin [(200 \pi \text{ s}^{-1})t]$$

52. The stationary wave equation is given by

$$y = (0.4 \text{ cm}) \sin [(0.314 \text{ cm}^{-1})x] \cos [(6.00 \pi \text{ s}^{-1})t]$$

a) $\omega = 600 \pi \Rightarrow 2\pi f = 600 \pi \Rightarrow f = 300 \text{ Hz}$

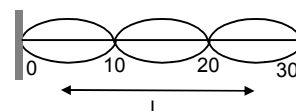
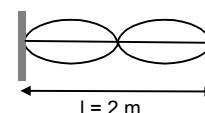
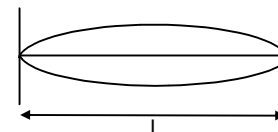
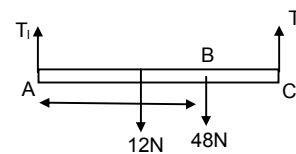
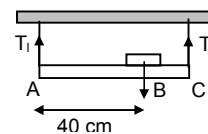
wavelength, $\lambda = 2\pi/0.314 = (2 \times 3.14) / 0.314 = 20 \text{ cm}$

b) Therefore, nodes are located at, $0, 10 \text{ cm}, 20 \text{ cm}, 30 \text{ cm}$

c) Length of the string = $3\lambda/2 = 3 \times 20/2 = 30 \text{ cm}$

d) $y = 0.4 \sin(0.314 x) \cos(600 \pi t) \Rightarrow 0.4 \sin\{(\pi/10)x\} \cos(600 \pi t)$

since, λ and v are the wavelength and velocity of the waves that interfere to give this vibration $\lambda = 20 \text{ cm}$



$$v = \omega/k = 6000 \text{ cm/sec} = 60 \text{ m/s}$$

53. The equation of the standing wave is given by
 $y = (0.4 \text{ cm}) \sin [(0.314 \text{ cm}^{-1})x] \cos [(6.00 \pi \text{ s}^{-1})t]$
 $\Rightarrow k = 0.314 = \pi/10$

$$\Rightarrow 2\pi/\lambda = \pi/10 \Rightarrow \lambda = 20 \text{ cm}$$

for smallest length of the string, as wavelength remains constant, the string should vibrate in fundamental frequency

$$\Rightarrow l = \lambda/2 = 20 \text{ cm} / 2 = 10 \text{ cm}$$

54. $L = 40 \text{ cm} = 0.4 \text{ m}$, mass = $3.2 \text{ kg} = 3.2 \times 10^{-3} \text{ kg}$
 \therefore mass per unit length, $m = (3.2)/(0.4) = 8 \times 10^{-3} \text{ kg/m}$
 change in length, $\Delta L = 40.05 - 40 = 0.05 \times 10^{-2} \text{ m}$
 strain = $\Delta L/L = 0.125 \times 10^{-2}$
 $f = 220 \text{ Hz}$

$$f = \frac{1}{2l} \sqrt{\frac{T}{m}} = \frac{1}{2 \times (0.4005)} \sqrt{\frac{T}{8 \times 10^{-3}}} \Rightarrow T = 248.19 \text{ N}$$

$$\text{Strain} = 248.19/1 \text{ mm}^2 = 248.19 \times 10^6$$

$$Y = \text{stress} / \text{strain} = 1.985 \times 10^{11} \text{ N/m}^2$$

55. Let, $\rho \rightarrow$ density of the block

Weight ρVg where $V =$ volume of block

The same tuning fork resonates with the string in the two cases

$$f_{10} = \frac{10}{2l} \sqrt{\frac{T - \rho_w Vg}{m}} = \frac{11}{2l} \sqrt{\frac{(\rho - \rho_w)Vg}{m}}$$

As the f of tuning fork is same,

$$f_{10} = f_{11} \Rightarrow \frac{10}{2l} \sqrt{\frac{\rho Vg}{m}} = \frac{11}{2l} \sqrt{\frac{(\rho - \rho_w)Vg}{m}}$$

$$\Rightarrow \frac{10}{11} = \sqrt{\frac{\rho - \rho_w}{\rho}} \Rightarrow \frac{\rho - 1}{\rho} = \frac{100}{121} \quad (\text{because, } \rho_w = 1 \text{ gm/cc})$$

$$\Rightarrow 100\rho = 121\rho - 121 \Rightarrow 5.8 \times 10^3 \text{ kg/m}^3$$

56. $l =$ length of rope = 2 m
 $M =$ mass = $80 \text{ gm} = 0.8 \text{ kg}$
 mass per unit length = $m = 0.08/2 = 0.04 \text{ kg/m}$
 Tension $T = 256 \text{ N}$

$$\text{Velocity, } V = \sqrt{T/m} = 80 \text{ m/s}$$

For fundamental frequency,

$$l = \lambda/4 \Rightarrow \lambda = 4l = 8 \text{ m}$$

$$\Rightarrow f = 80/8 = 10 \text{ Hz}$$

- a) Therefore, the frequency of 1st two overtones are

$$1^{\text{st}} \text{ overtone} = 3f = 30 \text{ Hz}$$

$$2^{\text{nd}} \text{ overtone} = 5f = 50 \text{ Hz}$$

- b) $\lambda_1 = 4l = 8 \text{ m}$

$$\lambda_1 = V/f_1 = 2.67 \text{ m}$$

$$\lambda_2 = V/f_2 = 1.6 \text{ m}$$

so, the wavelengths are 8 m , 2.67 m and 1.6 m respectively.

57. Initially because the end A is free, an antinode will be formed.

$$\text{So, } l = \lambda_1 / 4$$

Again, if the movable support is pushed to right by 10 m , so that the joint is placed on the pulley, a node will be formed there.

$$\text{So, } l = \lambda_2 / 2$$

Since, the tension remains same in both the cases, velocity remains same.

As the wavelength is reduced by half, the frequency will become twice as that of 120 Hz i.e. 240 Hz .

