

CHAPTER – 27
SPECIFIC HEAT CAPACITIES OF GASES

- $N = 1$ mole, $W = 20$ g/mol, $V = 50$ m/s
 K.E. of the vessel = Internal energy of the gas
 $= (1/2)mv^2 = (1/2) \times 20 \times 10^{-3} \times 50 \times 50 = 25$ J
 $25 = n \frac{3}{2} r(\Delta T) \Rightarrow 25 = 1 \times \frac{3}{2} \times 8.31 \times \Delta T \Rightarrow \Delta T = \frac{50}{3 \times 8.3} \approx 2$ k.
- $m = 5$ g, $\Delta t = 25 - 15 = 10^\circ\text{C}$
 $C_V = 0.172$ cal/g- $^\circ\text{C}$ = 4.2 J/Cal.
 $dQ = du + dw$
 Now, $V = 0$ (for a rigid body)
 So, $dw = 0$.
 So $dQ = du$.
 $Q = msdt = 5 \times 0.172 \times 10 = 8.6$ cal = $8.6 \times 4.2 = 36.12$ Joule.
- $\gamma = 1.4$, w or piston = 50 kg., A of piston = 100 cm²
 $P_o = 100$ kpa, $g = 10$ m/s², $x = 20$ cm.
 $dw = pdv = \left(\frac{mg}{A} + P_o \right) A dx = \left(\frac{50 \times 10}{100 \times 10^{-4}} + 10^5 \right) 100 \times 10^{-4} \times 20 \times 10^{-2} = 1.5 \times 10^5 \times 20 \times 10^{-4} = 300$ J.
 $nRdt = 300 \Rightarrow dT = \frac{300}{nR}$
 $dQ = nC_p dT = nC_p \times \frac{300}{nR} = \frac{n\gamma R 300}{(\gamma - 1)nR} = \frac{300 \times 1.4}{0.4} = 1050$ J.
- $C_V H_2 = 2.4$ Cal/g $^\circ\text{C}$, $C_p H_2 = 3.4$ Cal/g $^\circ\text{C}$
 $M = 2$ g/ Mol, $R = 8.3 \times 10^7$ erg/mol- $^\circ\text{C}$
 We know, $C_p - C_V = 1$ Cal/g $^\circ\text{C}$
 So, difference of molar specific heats
 $= C_p \times M - C_V \times M = 1 \times 2 = 2$ Cal/g $^\circ\text{C}$
 Now, $2 \times J = R \Rightarrow 2 \times J = 8.3 \times 10^7$ erg/mol- $^\circ\text{C} \Rightarrow J = 4.15 \times 10^7$ erg/cal.
- $\frac{C_p}{C_V} = 7.6$, $n = 1$ mole, $\Delta T = 50$ K
 (a) Keeping the pressure constant, $dQ = du + dw$,
 $\Delta T = 50$ K, $\gamma = 7/6$, $m = 1$ mole,
 $dQ = du + dw \Rightarrow nC_V dT = du + RdT \Rightarrow du = nC_p dT - RdT$
 $= 1 \times \frac{R\gamma}{\gamma - 1} \times dT - RdT = \frac{R \times \frac{7}{6}}{\frac{7}{6} - 1} dT - RdT$
 $= DT - RdT = 7RdT - RdT = 6RdT = 6 \times 8.3 \times 50 = 2490$ J.
 (b) Kipping Volume constant, $dv = nC_V dT$
 $= 1 \times \frac{R}{\gamma - 1} \times dt = \frac{1 \times 8.3}{\frac{7}{6} - 1} \times 50$
 $= 8.3 \times 50 \times 6 = 2490$ J
 (c) Adiabatically $dQ = 0$, $du = -dw$
 $= \left[\frac{n \times R}{\gamma - 1} (T_1 - T_2) \right] = \frac{1 \times 8.3}{\frac{7}{6} - 1} (T_2 - T_1) = 8.3 \times 50 \times 6 = 2490$ J

6. $m = 1.18 \text{ g}$, $V = 1 \times 10^3 \text{ cm}^3 = 1 \text{ L}$ $T = 300 \text{ k}$, $P = 10^5 \text{ Pa}$

$$PV = nRT \text{ or } n = \frac{PV}{RT} = 10^5 = \text{atm.}$$

$$N = \frac{PV}{RT} = \frac{1}{8.2 \times 10^{-2} \times 3 \times 10^2} = \frac{1}{8.2 \times 3} = \frac{1}{24.6}$$

$$\text{Now, } C_v = \frac{1}{n} \times \frac{Q}{dt} = 24.6 \times 2 = 49.2$$

$$C_p = R + C_v = 1.987 + 49.2 = 51.187$$

$$Q = nC_p dT = \frac{1}{24.6} \times 51.187 \times 1 = 2.08 \text{ Cal.}$$

7. $V_1 = 100 \text{ cm}^3$, $V_2 = 200 \text{ cm}^3$ $P = 2 \times 10^5 \text{ Pa}$, $\Delta Q = 50 \text{ J}$

$$(a) \Delta Q = du + dw \Rightarrow 50 = du + 20 \times 10^5 (200 - 100 \times 10^{-6}) \Rightarrow 50 = du + 20 \Rightarrow du = 30 \text{ J}$$

$$(b) 30 = n \times \frac{3}{2} \times 8.3 \times 300 \quad [U = \frac{3}{2} nRT \text{ for monoatomic}]$$

$$\Rightarrow n = \frac{2}{3 \times 83} = \frac{2}{249} = 0.008$$

$$(c) du = nC_v dT \Rightarrow C_v = \frac{du}{n dT} = \frac{30}{0.008 \times 300} = 12.5$$

$$C_p = C_v + R = 12.5 + 8.3 = 20.3$$

$$(d) C_v = 12.5 \text{ (Proved above)}$$

8. $Q = \text{Amt of heat given}$

$$\text{Work done} = \frac{Q}{2}, \quad \Delta Q = W + \Delta U$$

$$\text{for monoatomic gas } \Rightarrow \Delta U = Q - \frac{Q}{2} = \frac{Q}{2}$$

$$V = n \frac{3}{2} RT = \frac{Q}{2} = nT \times \frac{3}{2} R = 3R \times nT$$

Again $Q = n C_p dT$ Where $C_p >$ Molar heat capacity at const. pressure.

$$3RnT = n dT C_p \Rightarrow C_p = 3R$$

9. $P = KV \Rightarrow \frac{nRT}{V} = KV \Rightarrow RT = KV^2 \Rightarrow R \Delta T = 2KV \Delta U \Rightarrow \frac{R \Delta T}{2KV} = dv$

$$dQ = du + dw \Rightarrow mcdT = C_v dT + pdv \Rightarrow msdT = C_v dT + \frac{PRdF}{2KV}$$

$$\Rightarrow ms = C_v + \frac{RKV}{2KV} \Rightarrow C_p + \frac{R}{2}$$

10. $\frac{C_p}{C_v} = \gamma$, $C_p - C_v = R$, $C_v = \frac{r}{\gamma - 1}$, $C_p = \frac{\gamma R}{\gamma - 1}$

$$PdV = \frac{1}{b+1} (Rdt)$$

$$\Rightarrow 0 = C_v dT + \frac{1}{b+1} (Rdt) \Rightarrow \frac{1}{b+1} = \frac{-C_v}{R}$$

$$\Rightarrow b+1 = \frac{-R}{C_v} = \frac{-(C_p - C_v)}{C_v} = -\gamma + 1 \Rightarrow b = -\gamma$$

11. Considering two gases, in Gas(1) we have,

γ , C_{p1} (Sp. Heat at const. 'P'), C_{v1} (Sp. Heat at const. 'V'), n_1 (No. of moles)

$$\frac{C_{p1}}{C_{v1}} = \gamma \text{ \& } C_{p1} - C_{v1} = R$$

$$\Rightarrow \gamma C_{V1} - C_{V1} = R \Rightarrow C_{V1} (\gamma - 1) = R$$

$$\Rightarrow C_{V1} = \frac{R}{\gamma - 1} \quad \& \quad C_{P1} = \frac{\gamma R}{\gamma - 1}$$

In Gas(2) we have, γ , C_{P2} (Sp. Heat at const. 'P'), C_{V2} (Sp. Heat at const. 'V'), n_2 (No. of moles)

$$\frac{C_{P2}}{C_{V2}} = \gamma \quad \& \quad C_{P2} - C_{V2} = R \Rightarrow \gamma C_{V2} - C_{V2} = R \Rightarrow C_{V2} (\gamma - 1) = R \Rightarrow C_{V2} = \frac{R}{\gamma - 1} \quad \& \quad C_{P2} = \frac{\gamma R}{\gamma - 1}$$

Given $n_1 : n_2 = 1 : 2$

$$dU_1 = nC_{V1} dT \quad \& \quad dU_2 = 2nC_{V2} dT = 3nC_{Vd}dT$$

$$\Rightarrow C_V = \frac{C_{V1} + 2C_{V2}}{3} = \frac{\frac{R}{\gamma-1} + \frac{2R}{\gamma-1}}{3} = \frac{3R}{3(\gamma-1)} = \frac{R}{\gamma-1} \quad \dots(1)$$

$$\& C_P = \gamma C_V = \frac{\gamma R}{\gamma-1} \quad \dots(2)$$

$$\text{So, } \frac{C_P}{C_V} = \gamma \text{ [from (1) \& (2)]}$$

12. $C_{P'} = 2.5 R$ $C_{P''} = 3.5 R$

$$C_{V'} = 1.5 R \quad C_{V''} = 2.5 R$$

$$n_1 = n_2 = 1 \text{ mol} \quad (n_1 + n_2)C_V dT = n_1 C_{V'} dT + n_2 C_{V''} dT$$

$$\Rightarrow C_V = \frac{n_1 C_{V'} + n_2 C_{V''}}{n_1 + n_2} = \frac{1.5R + 2.5R}{2} = 2R$$

$$C_P = C_V + R = 2R + R = 3R$$

$$\gamma = \frac{C_P}{C_V} = \frac{3R}{2R} = 1.5$$

13. $n = \frac{1}{2}$ mole, $R = \frac{25}{3}$ J/mol-k, $\gamma = \frac{5}{3}$

(a) Temp at A = T_a , $P_a V_a = nRT_a$

$$\Rightarrow T_a = \frac{P_a V_a}{nR} = \frac{5000 \times 10^{-6} \times 100 \times 10^3}{\frac{1}{2} \times \frac{25}{3}} = 120 \text{ k.}$$

Similarly temperatures at point b = 240 k at C it is 480 k and at D it is 240 k.

(b) For ab process,

$$dQ = nC_P dT \quad [\text{since ab is isobaric}]$$

$$= \frac{1}{2} \times \frac{R\gamma}{\gamma-1} (T_b - T_a) = \frac{1}{2} \times \frac{\frac{25}{3} \times \frac{5}{3}}{\frac{5}{3}-1} \times (240 - 120) = \frac{1}{2} \times \frac{125}{9} \times \frac{3}{2} \times 120 = 1250 \text{ J}$$

For bc, $dQ = du + dw$ [$dq = 0$, Isochoric process]

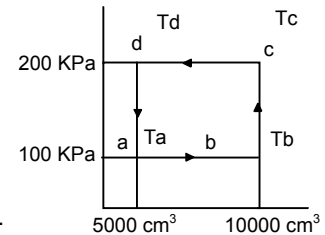
$$\Rightarrow dQ = du = nC_V dT = \frac{nR}{\gamma-1} (T_c - T_a) = \frac{1}{2} \times \frac{\frac{25}{3}}{\left(\frac{5}{3}-1\right)} (240) = \frac{1}{2} \times \frac{25}{3} \times \frac{3}{2} \times 240 = 1500 \text{ J}$$

(c) Heat liberated in cd = $-nC_P dT$

$$= \frac{-1}{2} \times \frac{nR}{\gamma-1} (T_d - T_c) = \frac{-1}{2} \times \frac{125}{3} \times \frac{3}{2} \times 240 = 2500 \text{ J}$$

Heat liberated in da = $-nC_V dT$

$$= \frac{-1}{2} \times \frac{R}{\gamma-1} (T_a - T_d) = \frac{-1}{2} \times \frac{25}{2} \times (120 - 240) = 750 \text{ J}$$

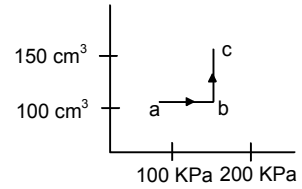


14. (a) For a, b 'V' is constant

$$\text{So, } \frac{P_1}{T_1} = \frac{P_2}{T_2} \Rightarrow \frac{100}{300} = \frac{200}{T_2} \Rightarrow T_2 = \frac{200 \times 300}{100} = 600 \text{ k}$$

For b,c 'P' is constant

$$\text{So, } \frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{100}{600} = \frac{150}{T_2} \Rightarrow T_2 = \frac{600 \times 150}{100} = 900 \text{ k}$$



(b) Work done = Area enclosed under the graph $50 \text{ cc} \times 200 \text{ kPa} = 50 \times 10^{-6} \times 200 \times 10^3 \text{ J} = 10 \text{ J}$

(c) 'Q' Supplied = $nC_v dT$

$$\text{Now, } n = \frac{PV}{RT} \text{ considering at pt. 'b'}$$

$$C_v = \frac{R}{\gamma - 1} dT = 300 \text{ a, b.}$$

$$Q_{bc} = \frac{PV}{RT} \times \frac{R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 100 \times 10^{-6}}{600 \times 0.67} \times 300 = 14.925 \quad (\because \gamma = 1.67)$$

$$Q \text{ supplied to be } nC_p dT \quad [\because C_p = \frac{\gamma R}{\gamma - 1}]$$

$$= \frac{PV}{RT} \times \frac{\gamma R}{\gamma - 1} dT = \frac{200 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 900} \times \frac{1.67 \times 8.3}{0.67} \times 300 = 24.925$$

(d) $Q = \Delta U + w$

$$\text{Now, } \Delta U = Q - w = \text{Heat supplied} - \text{Work done} = (24.925 + 14.925) - 1 = 29.850$$

15. In Joly's differential steam calorimeter

$$C_v = \frac{m_2 L}{m_1(\theta_2 - \theta_1)}$$

$$m_2 = \text{Mass of steam condensed} = 0.095 \text{ g, } L = 540 \text{ Cal/g} = 540 \times 4.2 \text{ J/g}$$

$$m_1 = \text{Mass of gas present} = 3 \text{ g, } \theta_1 = 20^\circ\text{C, } \theta_2 = 100^\circ\text{C}$$

$$\Rightarrow C_v = \frac{0.095 \times 540 \times 4.2}{3(100 - 20)} = 0.89 \approx 0.9 \text{ J/g-K}$$

16. $\gamma = 1.5$

Since it is an adiabatic process, So $PV^\gamma = \text{const.}$

$$(a) P_1 V_1^\gamma = P_2 V_2^\gamma \quad \text{Given } V_1 = 4 \text{ L, } V_2 = 3 \text{ L, } \quad \frac{P_2}{P_1} = ?$$

$$\Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = \left(\frac{4}{3}\right)^{1.5} = 1.5396 \approx 1.54$$

(b) $TV^{\gamma-1} = \text{Const.}$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{4}{3}\right)^{0.5} = 1.154$$

17. $P_1 = 2.5 \times 10^5 \text{ Pa, } V_1 = 100 \text{ cc, } T_1 = 300 \text{ k}$

$$(a) P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$\Rightarrow 2.5 \times 10^5 \times V^{1.5} = \left(\frac{V}{2}\right)^{1.5} \times P_2$$

$$\Rightarrow P_2 = 2^{1.5} \times 2.5 \times 10^5 = 7.07 \times 10^5 \approx 7.1 \times 10^5$$

$$(b) T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow 300 \times (100)^{1.5-1} = T_2 \times (50)^{1.5-1}$$

$$\Rightarrow T_2 = \frac{3000}{7.07} = 424.32 \text{ k} \approx 424 \text{ k}$$

(c) Work done by the gas in the process

$$W = \frac{mR}{\gamma-1} [T_2 - T_1] = \frac{P_1 V_1}{T(\gamma-1)} [T_2 - T_1]$$

$$= \frac{2.5 \times 10^5 \times 100 \times 10^{-6}}{300(1.5-1)} [424 - 300] = \frac{2.5 \times 10}{300 \times 0.5} \times 124 = 20.72 \approx 21 \text{ J}$$

18. $\gamma = 1.4$, $T_1 = 20^\circ\text{C} = 293 \text{ k}$, $P_1 = 2 \text{ atm}$, $p_2 = 1 \text{ atm}$

We know for adiabatic process,

$$P_1^{1-\gamma} \times T_1^\gamma = P_2^{1-\gamma} \times T_2^\gamma \text{ or } (2)^{1-1.4} \times (293)^{1.4} = (1)^{1-1.4} \times T_2^{1.4}$$

$$\Rightarrow (2)^{0.4} \times (293)^{1.4} = T_2^{1.4} \Rightarrow 2153.78 = T_2^{1.4} \Rightarrow T_2 = (2153.78)^{1/1.4} = 240.3 \text{ K}$$

19. $P_1 = 100 \text{ KPa} = 10^5 \text{ Pa}$, $V_1 = 400 \text{ cm}^3 = 400 \times 10^{-6} \text{ m}^3$, $T_1 = 300 \text{ k}$,

$$\gamma = \frac{C_P}{C_V} = 1.5$$

(a) Suddenly compressed to $V_2 = 100 \text{ cm}^3$

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow 10^5 (400)^{1.5} = P_2 \times (100)^{1.5}$$

$$\Rightarrow P_2 = 10^5 \times (4)^{1.5} = 800 \text{ KPa}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow 300 \times (400)^{1.5-1} = T_2 \times (100)^{1.5-1} \Rightarrow T_2 = \frac{300 \times 20}{10} = 600 \text{ K}$$

(b) Even if the container is slowly compressed the walls are adiabatic so heat transferred is 0.

Thus the values remain, $P_2 = 800 \text{ KPa}$, $T_2 = 600 \text{ K}$.

20. Given $\frac{C_P}{C_V} = \gamma$ P_0 (Initial Pressure), V_0 (Initial Volume)

(a) (i) Isothermal compression, $P_1 V_1 = P_2 V_2$ or $P_0 V_0 = \frac{P_2 V_0}{2} \Rightarrow P_2 = 2P_0$

(ii) Adiabatic Compression $P_1 V_1^\gamma = P_2 V_2^\gamma$ or $2P_0 \left(\frac{V_0}{2}\right)^\gamma = P_1 \left(\frac{V_0}{4}\right)^\gamma$

$$\Rightarrow P' = \frac{V_0^\gamma}{2^\gamma} \times 2P_0 \times \frac{4^\gamma}{V_0^\gamma} = 2^\gamma \times 2 P_0 \Rightarrow P_0 2^{\gamma+1}$$

(b) (i) Adiabatic compression $P_1 V_1^\gamma = P_2 V_2^\gamma$ or $P_0 V_0^\gamma = P' \left(\frac{V_0}{2}\right)^\gamma \Rightarrow P' = P_0 2^\gamma$

(ii) Isothermal compression $P_1 V_1 = P_2 V_2$ or $2^\gamma P_0 \times \frac{V_0}{2} = P_2 \times \frac{V_0}{4} \Rightarrow P_2 = P_0 2^{\gamma+1}$

21. Initial pressure = P_0

Initial Volume = V_0

$$\gamma = \frac{C_P}{C_V}$$

(a) Isothermally to pressure $\frac{P_0}{2}$

$$P_0 V_0 = \frac{P_0}{2} V_1 \Rightarrow V_1 = 2 V_0$$

Adiabatically to pressure = $\frac{P_0}{4}$

$$\frac{P_0}{2} (V_1)^\gamma = \frac{P_0}{4} (V_2)^\gamma \Rightarrow \frac{P_0}{2} (2V_0)^\gamma = \frac{P_0}{4} (V_2)^\gamma$$

$$\Rightarrow 2^{\gamma+1} V_0^\gamma = V_2^\gamma \Rightarrow V_2 = 2^{(\gamma+1)/\gamma} V_0$$

$$\therefore \text{Final Volume} = 2^{(\gamma+1)/\gamma} V_0$$

(b) Adiabatically to pressure $\frac{P_0}{2}$ to P_0

$$P_0 \times (2^{\gamma+1} V_0^\gamma) = \frac{P_0}{2} \times (V')^\gamma$$

Isothermal to pressure $\frac{P_0}{4}$

$$\frac{P_0}{2} \times 2^{1/\gamma} V_0 = \frac{P_0}{4} \times V'' \Rightarrow V'' = 2^{(\gamma+1)/\gamma} V_0$$

$$\therefore \text{Final Volume} = 2^{(\gamma+1)/\gamma} V_0$$

22. $PV = nRT$

Given $P = 150 \text{ KPa} = 150 \times 10^3 \text{ Pa}$, $V = 150 \text{ cm}^3 = 150 \times 10^{-6} \text{ m}^3$, $T = 300 \text{ K}$

(a) $n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 150 \times 10^{-6}}{8.3 \times 300} = 9.036 \times 10^{-3} = 0.009 \text{ moles.}$

(b) $\frac{C_P}{C_V} = \gamma \Rightarrow \frac{\gamma R}{(\gamma-1)C_V} = \gamma \quad \left[\because C_P = \frac{\gamma R}{\gamma-1} \right]$

$\Rightarrow C_V = \frac{R}{\gamma-1} = \frac{8.3}{1.5-1} = \frac{8.3}{0.5} = 2R = 16.6 \text{ J/mole}$

(c) Given $P_1 = 150 \text{ KPa} = 150 \times 10^3 \text{ Pa}$, $P_2 = ?$

$V_1 = 150 \text{ cm}^3 = 150 \times 10^{-6} \text{ m}^3$, $\gamma = 1.5$

$V_2 = 50 \text{ cm}^3 = 50 \times 10^{-6} \text{ m}^3$, $T_1 = 300 \text{ K}$, $T_2 = ?$

Since the process is adiabatic Hence $-P_1 V_1^\gamma = P_2 V_2^\gamma$

$\Rightarrow 150 \times 10^3 (150 \times 10^{-6})^\gamma = P_2 \times (50 \times 10^{-6})^\gamma$

$\Rightarrow P_2 = 150 \times 10^3 \times \left(\frac{150 \times 10^{-6}}{50 \times 10^{-6}} \right)^{1.5} = 150000 \times 3^{1.5} = 779.422 \times 10^3 \text{ Pa} \approx 780 \text{ KPa}$

(d) $\Delta Q = W + \Delta U$ or $W = -\Delta U$ [$\because \Delta U = 0$, in adiabatic]

$= -nC_V dT = -0.009 \times 16.6 \times (520 - 300) = -0.009 \times 16.6 \times 220 = -32.8 \text{ J} \approx -33 \text{ J}$

(e) $\Delta U = nC_V dT = 0.009 \times 16.6 \times 220 \approx 33 \text{ J}$

23. $V_A = V_B = V_C$

For A, the process is isothermal

$P_A V_A = P_A' V_A' \Rightarrow P_A' = P_A \frac{V_A}{V_A'} = P_A \times \frac{1}{2}$

For B, the process is adiabatic,

$P_A (V_B)^\gamma = P_A' (V_B)^\gamma = P_B' = P_B \left(\frac{V_B}{V_B'} \right)^\gamma = P_B \times \left(\frac{1}{2} \right)^{1.5} = \frac{P_B}{2^{1.5}}$

For, C, the process is isobaric

$\frac{V_C}{T_C} = \frac{V_C'}{T_C'} \Rightarrow \frac{V_C}{T_C} = \frac{2V_C'}{T_C'} \Rightarrow T_C' = \frac{2}{T_C}$

Final pressures are equal.

$= \frac{P_A}{2} = \frac{P_B}{2^{1.5}} = P_C \Rightarrow P_A : P_B : P_C = 2 : 2^{1.5} : 1 = 2 : 2\sqrt{2} : 1$

24. $P_1 = \text{Initial Pressure}$ $V_1 = \text{Initial Volume}$ $P_2 = \text{Final Pressure}$ $V_2 = \text{Final Volume}$

Given, $V_2 = 2V_1$, Isothermal workdone $= nRT_1 \ln \left(\frac{V_2}{V_1} \right)$

$$\text{Adiabatic workdone} = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}$$

Given that workdone in both cases is same.

$$\text{Hence } nRT_1 \ln \left(\frac{V_2}{V_1} \right) = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1} \Rightarrow (\gamma - 1) \ln \left(\frac{V_2}{V_1} \right) = \frac{P_1 V_1 - P_2 V_2}{nRT_1}$$

$$\Rightarrow (\gamma - 1) \ln \left(\frac{V_2}{V_1} \right) = \frac{nRT_1 - nRT_2}{nRT_1} \Rightarrow (\gamma - 1) \ln 2 = \frac{T_1 - T_2}{T_1} \quad \dots(i) \quad [\because V_2 = 2V_1]$$

We know $TV^{\gamma-1} = \text{const.}$ in adiabatic Process.

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}, \text{ or } T_1 (V_2)^{\gamma-1} = T_2 \times (2)^{\gamma-1} \times (V_1)^{\gamma-1}$$

$$\text{Or, } T_1 = 2^{\gamma-1} \times T_2 \text{ or } T_2 = T_1^{1-\gamma} \quad \dots(ii)$$

From (i) & (ii)

$$(\gamma - 1) \ln 2 = \frac{T_1 - T_1 \times 2^{1-\gamma}}{T_1} \Rightarrow (\gamma - 1) \ln 2 = 1 - 2^{1-\gamma}$$

25. $\gamma = 1.5, \quad T = 300 \text{ k}, \quad V = 1 \text{ L} = \frac{1}{2} \text{ m}^3$

(a) The process is adiabatic as it is sudden,

$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_1 (V_0)^\gamma = P_2 \left(\frac{V_0}{2} \right)^\gamma \Rightarrow P_2 = P_1 \left(\frac{1}{1/2} \right)^{1.5} = P_1 (2)^{1.5} \Rightarrow \frac{P_2}{P_1} = 2^{1.5} = 2\sqrt{2}$$

(b) $P_1 = 100 \text{ KPa} = 10^5 \text{ Pa}$ $W = \frac{nR}{\gamma-1} [T_1 - T_2]$

$$T_1 V_1^{\gamma-1} = P_2 V_2^{\gamma-1} \Rightarrow 300 \times (1)^{1.5-1} = T_2 (0.5)^{1.5-1} \Rightarrow 300 \times 1 = T_2 \sqrt{0.5}$$

$$T_2 = 300 \times \sqrt{\frac{1}{0.5}} = 300\sqrt{2} \text{ K}$$

$$P_1 V_1 = nRT_1 \Rightarrow n = \frac{P_1 V_1}{RT_1} = \frac{10^5 \times 10^{-3}}{R \times 300} = \frac{1}{3R} \quad (V \text{ in m}^3)$$

$$w = \frac{nR}{\gamma-1} [T_1 - T_2] = \frac{1R}{3R(1.5-1)} [300 - 300\sqrt{2}] = \frac{300}{3 \times 0.5} (1 - \sqrt{2}) = -82.8 \text{ J} \approx -82 \text{ J.}$$

(c) Internal Energy,

$$dQ = 0, \quad \Rightarrow du = -dw = -(-82.8) \text{ J} = 82.8 \text{ J} \approx 82 \text{ J.}$$

(d) Final Temp = $300\sqrt{2} = 300 \times 1.414 \times 100 = 424.2 \text{ k} \approx 424 \text{ k.}$

(e) The pressure is kept constant. \therefore The process is isobaric.

$$\text{Work done} = nRdT = \frac{1}{3R} \times R \times (300 - 300\sqrt{2}) \quad \text{Final Temp} = 300 \text{ K}$$

$$= -\frac{1}{3} \times 300 (0.414) = -41.4 \text{ J. Initial Temp} = 300\sqrt{2}$$

(f) Initial volume $\Rightarrow \frac{V_1}{T_1} = \frac{V_1'}{T_1'} = V_1' = \frac{V_1}{T_1} \times T_1' = \frac{1}{2 \times 300 \times \sqrt{2}} \times 300 = \frac{1}{2\sqrt{2}} \text{ L.}$

Final volume = 1L

$$\text{Work done in isothermal} = nRT \ln \frac{V_2}{V_1}$$

$$= \frac{1}{3R} \times R \times 300 \ln \left(\frac{1}{1/2\sqrt{2}} \right) = 100 \times \ln(2\sqrt{2}) = 100 \times 1.039 \approx 103$$

(g) Net work done = $W_A + W_B + W_C = -82 - 41.4 + 103 = -20.4 \text{ J.}$

26. Given $\gamma = 1.5$

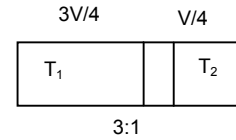
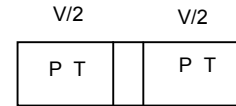
We know from adiabatic process $TV^{\gamma-1} = \text{Const.}$

So, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$... (eq)

As, it is an adiabatic process and all the other conditions are same. Hence the above equation can be applied.

So, $T_1 \times \left(\frac{3V}{4}\right)^{1.5-1} = T_2 \times \left(\frac{V}{4}\right)^{1.5-1} \Rightarrow T_1 \times \left(\frac{3V}{4}\right)^{0.5} = T_2 \times \left(\frac{V}{4}\right)^{0.5}$

$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V}{4}\right)^{0.5} \times \left(\frac{4}{3V}\right)^{0.5} = \frac{1}{\sqrt{3}}$ So, $T_1 : T_2 = 1 : \sqrt{3}$



27. $V = 200 \text{ cm}^3$, $C = 12.5 \text{ J/mol-k}$, $T = 300 \text{ k}$, $P = 75 \text{ cm}$

(a) No. of moles of gas in each vessel,

$\frac{PV}{RT} = \frac{75 \times 13.6 \times 980 \times 200}{8.3 \times 10^7 \times 300} = 0.008$

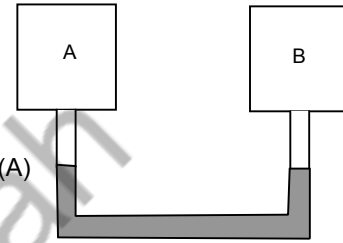
(b) Heat is supplied to the gas but $dv = 0$

$dQ = du \Rightarrow 5 = nC_V dT \Rightarrow 5 = 0.008 \times 12.5 \times dT \Rightarrow dT = \frac{5}{0.008 \times 12.5}$ for (A)

For (B) $dT = \frac{10}{0.008 \times 12.5}$ $\therefore \frac{P}{T} = \frac{P_A}{T_A}$ [For container A]

$\Rightarrow \frac{75}{300} = \frac{P_A \times 0.008 \times 12.5}{5} \Rightarrow P_A = \frac{75 \times 5}{300 \times 0.008 \times 12.5} = 12.5 \text{ cm of Hg.}$

$\therefore \frac{P}{T} = \frac{P_B}{T_B}$ [For Container B] $\Rightarrow \frac{75}{300} = \frac{P_B \times 0.008 \times 12.5}{10} \Rightarrow P_B = 2 P_A = 25 \text{ cm of Hg.}$



Mercury moves by a distance $P_B - P_A = 25 - 12.5 = 12.5 \text{ Cm.}$

28. $m_{\text{He}} = 0.1 \text{ g}$, $\gamma = 1.67$, $\mu = 4 \text{ g/mol}$, $m_{\text{H}_2} = ?$

$\mu = 28/\text{mol}$ $\gamma_2 = 1.4$

Since it is an adiabatic surrounding

He $dQ = nC_V dT = \frac{0.1}{4} \times \frac{R}{\gamma-1} \times dT = \frac{0.1}{4} \times \frac{R}{(1.67-1)} \times dT$... (i)

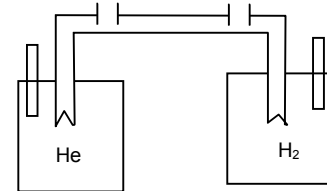
$\text{H}_2 = nC_V dT = \frac{m}{2} \times \frac{R}{\gamma-1} \times dT = \frac{m}{2} \times \frac{R}{1.4-1} \times dT$ [Where m is the reqd.]

Mass of H_2

Since equal amount of heat is given to both and ΔT is same in both.

Equating (i) & (ii) we get

$\frac{0.1}{4} \times \frac{R}{0.67} \times dT = \frac{m}{2} \times \frac{R}{0.4} \times dT \Rightarrow m = \frac{0.1}{2} \times \frac{0.4}{0.67} = 0.0298 \approx 0.03 \text{ g}$



29. Initial pressure = P_0 , Initial Temperature = T_0

Initial Volume = V_0

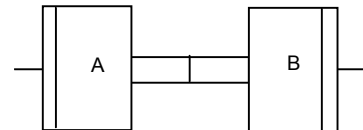
$\frac{C_P}{C_V} = \gamma$

(a) For the diathermic vessel the temperature inside remains constant

$P_1 V_1 = P_2 V_2 \Rightarrow P_0 V_0 = P_2 \times 2V_0 \Rightarrow P_2 = \frac{P_0}{2}$, Temperature = T_0

For adiabatic vessel the temperature does not remain constant. The process is adiabatic

$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow T_0 V_0^{\gamma-1} = T_2 \times (2V_0)^{\gamma-1} \Rightarrow T_2 = T_0 \left(\frac{V_0}{2V_0}\right)^{\gamma-1} = T_0 \times \left(\frac{1}{2}\right)^{\gamma-1} = \frac{T_0}{2^{\gamma-1}}$



$$P_1 V_1^\gamma = P_2 V_2^\gamma \Rightarrow P_0 V_0^\gamma = p_1 (2V_0)^\gamma \Rightarrow P_1 = P_0 \left(\frac{V_0}{2V_0} \right)^\gamma = \frac{P_0}{2^\gamma}$$

(b) When the values are opened, the temperature remains T_0 through out

$$P_1 = \frac{n_1 RT_0}{4V_0}, P_2 = \frac{n_2 RT_0}{4V_0} \quad [\text{Total value after the expt} = 2V_0 + 2V_0 = 4V_0]$$

$$P = P_1 + P_2 = \frac{(n_1 + n_2)RT_0}{4V_0} = \frac{2nRT_0}{4V_0} = \frac{nRT_0}{2V} = \frac{P_0}{2}$$

30. For an adiabatic process, $PV^\gamma = \text{Const.}$

There will be a common pressure 'P' when the equilibrium is reached

$$\text{Hence } P_1 \left(\frac{V_0}{2} \right)^\gamma = P(V')^\gamma$$

$V_0/2$	$V_0/2$
$P_1 T_1$	$P_2 T_2$

$$\text{For left } P = P_1 \left(\frac{V_0}{2} \right)^\gamma (V')^\gamma \quad \dots(1)$$

$$\text{For Right } P = P_2 \left(\frac{V_0}{2} \right)^\gamma (V_0 - V')^\gamma \quad \dots(2)$$

V'	$V_0 - V'$
1	

Equating 'P' for both left & right

$$= \frac{P_1}{(V')^\gamma} = \frac{P_2}{(V_0 - V')^\gamma} \quad \text{or} \quad \frac{V_0 - V'}{V'} = \left(\frac{P_2}{P_1} \right)^{1/\gamma}$$

$$\Rightarrow \frac{V_0}{V'} - 1 = \frac{P_2^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow \frac{V_0}{V'} = \frac{P_2^{1/\gamma} + P_1^{1/\gamma}}{P_1^{1/\gamma}} \Rightarrow V' = \frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \quad \text{For left} \dots\dots(3)$$

$$\text{Similarly } V_0 - V' = \frac{V_0 P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \quad \text{For right} \dots\dots(4)$$

(b) Since the whole process takes place in adiabatic surroundings. The separator is adiabatic. Hence heat given to the gas in the left part = Zero.

$$\text{(c) From (1) Final pressure } P = \frac{P_1 \left(\frac{V_0}{2} \right)^\gamma}{(V')^\gamma}$$

$$\text{Again from (3) } V' = \frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \quad \text{or} \quad P = \frac{P_1 \left(\frac{V_0}{2} \right)^\gamma}{\left(\frac{V_0 P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}} \right)^\gamma} = \frac{P_1 (V_0)^\gamma}{2^\gamma} \times \frac{(P_1^{1/\gamma} + P_2^{1/\gamma})^\gamma}{(V_0)^\gamma P_1} = \left(\frac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2} \right)^\gamma$$

31. $A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$, $M = 0.03 \text{ g} = 0.03 \times 10^{-3} \text{ kg}$,

$P = 1 \text{ atm} = 10^5 \text{ pascal}$, $L = 40 \text{ cm} = 0.4 \text{ m}$.

$L_1 = 80 \text{ cm} = 0.8 \text{ m}$, $P = 0.355 \text{ atm}$

The process is adiabatic

$$P(V)^\gamma = P(V')^\gamma \Rightarrow 1 \times (AL)^\gamma = 0.355 \times (A2L)^\gamma \Rightarrow 1 = 0.355 \cdot 2^\gamma \Rightarrow \frac{1}{0.355} = 2^\gamma$$

$$= \gamma \log 2 = \log \left(\frac{1}{0.355} \right) = 1.4941$$

$$V = \sqrt{\frac{\gamma P}{f}} = \sqrt{\frac{1.4941 \times 10^5}{\text{m/v}}} = \sqrt{\frac{1.4941 \times 10^5}{\left(\frac{0.03 \times 10^{-3}}{10^{-4} \times 1 \times 0.4} \right)^\gamma}} = \sqrt{\frac{1.441 \times 10^5 \times 4 \times 10^{-5}}{3 \times 10^{-5}}} = 446.33 \approx 447 \text{ m/s}$$

32. $V = 1280 \text{ m/s}$, $T = 0^\circ\text{C}$, $f_0\text{H}_2 = 0.089 \text{ kg/m}^3$, $R = 8.3 \text{ J/mol-k}$,
At STP, $P = 10^5 \text{ Pa}$, We know

$$V_{\text{sound}} = \sqrt{\frac{\gamma P}{f_0}} \Rightarrow 1280 = \sqrt{\frac{\gamma \times 10^5}{0.089}} \Rightarrow (1280)^2 = \frac{\gamma \times 10^5}{0.089} \Rightarrow \gamma = \frac{0.089 \times (1280)^2}{10^5} \approx 1.458$$

Again,

$$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.458 - 1} = 18.1 \text{ J/mol-k}$$

Again, $\frac{C_P}{C_V} = \gamma$ or $C_P = \gamma C_V = 1.458 \times 18.1 = 26.3 \text{ J/mol-k}$

33. $\mu = 4\text{g} = 4 \times 10^{-3} \text{ kg}$, $V = 22400 \text{ cm}^3 = 22400 \times 10^{-6} \text{ m}^3$
 $C_P = 5 \text{ cal/mol-ki} = 5 \times 4.2 \text{ J/mol-k} = 21 \text{ J/mol-k}$

$$C_P = \frac{\gamma R}{\gamma - 1} = \frac{\gamma \times 8.3}{\gamma - 1}$$

$$\Rightarrow 21(\gamma - 1) = \gamma(8.3) \Rightarrow 21\gamma - 21 = 8.3\gamma \Rightarrow \gamma = \frac{21}{12.7}$$

Since the condition is STP, $P = 1 \text{ atm} = 10^5 \text{ pa}$

$$V = \sqrt{\frac{\gamma f}{f}} = \sqrt{\frac{\frac{21}{12.7} \times 10^5}{4 \times 10^{-3}}} = \sqrt{\frac{21 \times 10^5 \times 22400 \times 10^{-6}}{12.7 \times 4 \times 10^{-3}}} = 962.28 \text{ m/s}$$

34. Given $f_0 = 1.7 \times 10^{-3} \text{ g/cm}^3 = 1.7 \text{ kg/m}^3$, $P = 1.5 \times 10^5 \text{ Pa}$, $R = 8.3 \text{ J/mol-k}$,
 $f = 3.0 \text{ KHz}$.

Node separation in a Kundt's tube = $\frac{\lambda}{2} = 6 \text{ cm}$, $\Rightarrow \lambda = 12 \text{ cm} = 12 \times 10^{-3} \text{ m}$

So, $V = f\lambda = 3 \times 10^3 \times 12 \times 10^{-2} = 360 \text{ m/s}$

We know, Speed of sound = $\sqrt{\frac{\gamma P}{f_0}} \Rightarrow (360)^2 = \frac{\gamma \times 1.5 \times 10^5}{1.7} \Rightarrow \gamma = \frac{(360)^2 \times 1.7}{1.5 \times 10^5} = 1.4688$

But $C_V = \frac{R}{\gamma - 1} = \frac{8.3}{1.4688 - 1} = 17.72 \text{ J/mol-k}$

Again $\frac{C_P}{C_V} = \gamma$ So, $C_P = \gamma C_V = 17.72 \times 1.468 = 26.01 \approx 26 \text{ J/mol-k}$

35. $f = 5 \times 10^3 \text{ Hz}$, $T = 300 \text{ Hz}$, $\frac{\lambda}{2} = 3.3 \text{ cm} \Rightarrow \lambda = 6.6 \times 10^{-2} \text{ m}$

$V = f\lambda = 5 \times 10^3 \times 6.6 \times 10^{-2} = (66 \times 5) \text{ m/s}$

$V = \frac{\lambda P}{f}$ [$P_V = nRT \Rightarrow P = \frac{m}{mV} \times Rt \Rightarrow PM = f_0 RT \Rightarrow \frac{P}{f_0} = \frac{RT}{m}$]

$$= \sqrt{\frac{\gamma RT}{m}} (66 \times 5) = \sqrt{\frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}}} \Rightarrow (66 \times 5)^2 = \frac{\gamma \times 8.3 \times 300}{32 \times 10^{-3}} \Rightarrow \gamma = \frac{(66 \times 5)^2 \times 32 \times 10^{-3}}{8.3 \times 300} = 1.3995$$

$C_V = \frac{R}{\gamma - 1} = \frac{8.3}{0.3995} = 20.7 \text{ J/mol-k}$,

$C_P = C_V + R = 20.77 + 8.3 = 29.07 \text{ J/mol-k}$.

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