

CHAPTER 28 HEAT TRANSFER

1. $t_1 = 90^\circ\text{C}$, $t_2 = 10^\circ\text{C}$

$$l = 1 \text{ cm} = 1 \times 10^{-3} \text{ m}$$

$$A = 10 \text{ cm} \times 10 \text{ cm} = 0.1 \times 0.1 \text{ m}^2 = 1 \times 10^{-2} \text{ m}^2$$

$$K = 0.80 \text{ w/m-}^\circ\text{C}$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{8 \times 10^{-1} \times 1 \times 10^{-2} \times 80}{1 \times 10^{-2}} = 64 \text{ J/s} = 64 \times 60 \text{ 3840 J.}$$

2. $t = 1 \text{ cm} = 0.01 \text{ m}$, $A = 0.8 \text{ m}^2$

$$\theta_1 = 300, \quad \theta_2 = 80$$

$$K = 0.025,$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{0.025 \times 0.8 \times (300 - 80)}{0.01} = 440 \text{ watt.}$$

3. $K = 0.04 \text{ J/m-}^\circ\text{C}$, $A = 1.6 \text{ m}^2$

$$t_1 = 97^\circ\text{F} = 36.1^\circ\text{C}$$

$$t_2 = 47^\circ\text{F} = 8.33^\circ\text{C}$$

$$l = 0.5 \text{ cm} = 0.005 \text{ m}$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{4 \times 10^{-2} \times 1.6 \times 27.78}{5 \times 10^{-3}} = 356 \text{ J/s}$$

4. $A = 25 \text{ cm}^2 = 25 \times 10^{-4} \text{ m}^2$

$$l = 1 \text{ mm} = 10^{-3} \text{ m}$$

$$K = 50 \text{ w/m-}^\circ\text{C}$$

$$\frac{Q}{t} = \text{Rate of conversion of water into steam}$$

$$= \frac{100 \times 10^{-3} \times 2.26 \times 10^6}{1 \text{ min}} = \frac{10^{-1} \times 2.26 \times 10^6}{60} = 0.376 \times 10^4$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} \Rightarrow 0.376 \times 10^4 = \frac{50 \times 25 \times 10^{-4} \times (\theta - 100)}{10^{-3}}$$

$$\Rightarrow \theta = \frac{10^{-3} \times 0.376 \times 10^4}{50 \times 25 \times 10^{-4}} = \frac{10^5 \times 0.376}{50 \times 25} = 30.1 \approx 30$$

5. $K = 46 \text{ w/m-s}^\circ\text{C}$

$$l = 1 \text{ m}$$

$$A = 0.04 \text{ cm}^2 = 4 \times 10^{-6} \text{ m}^2$$

$$L_{\text{fusion ice}} = 3.36 \times 10^5 \text{ J/Kg}$$

$$\frac{Q}{t} = \frac{46 \times 4 \times 10^{-6} \times 100}{1} = 5.4 \times 10^{-8} \text{ kg} \approx 5.4 \times 10^{-5} \text{ g.}$$

6. $A = 2400 \text{ cm}^2 = 2400 \times 10^{-4} \text{ m}^2$

$$l = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$$

$$K = 0.06 \text{ w/m-}^\circ\text{C}$$

$$\theta_1 = 20^\circ\text{C}$$

$$\theta_2 = 0^\circ\text{C}$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{0.06 \times 2400 \times 10^{-4} \times 20}{2 \times 10^{-3}} = 24 \times 6 \times 10^{-1} \times 10 = 24 \times 6 = 144 \text{ J/sec}$$

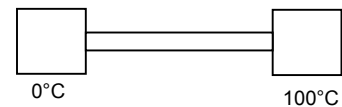
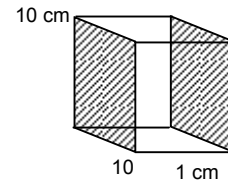
$$\text{Rate in which ice melts} = \frac{m}{t} = \frac{Q}{t \times L} = \frac{144}{3.4 \times 10^5} \text{ Kg/h} = \frac{144 \times 3600}{3.4 \times 10^5} \text{ Kg/s} = 1.52 \text{ kg/s.}$$

7. $l = 1 \text{ mm} = 10^{-3} \text{ m}$ $m = 10 \text{ kg}$

$$A = 200 \text{ cm}^2 = 2 \times 10^{-2} \text{ m}^2$$

$$L_{\text{vap}} = 2.27 \times 10^6 \text{ J/kg}$$

$$K = 0.80 \text{ J/m-s-}^\circ\text{C}$$



$$dQ = 2.27 \times 10^6 \times 10,$$

$$\frac{dQ}{dt} = \frac{2.27 \times 10^7}{10^5} = 2.27 \times 10^2 \text{ J/s}$$

Again we know

$$\frac{dQ}{dt} = \frac{0.80 \times 2 \times 10^{-2} \times (42 - T)}{1 \times 10^{-3}}$$

$$\text{So, } \frac{8 \times 2 \times 10^{-3} (42 - T)}{10^{-3}} = 2.27 \times 10^2$$

$$\Rightarrow 16 \times 42 - 16T = 227 \Rightarrow T = 27.8 \approx 28^\circ\text{C}$$

8. $K = 45 \text{ w/m}^\circ\text{C}$

$$\ell = 60 \text{ cm} = 60 \times 10^{-2} \text{ m}$$

$$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$$

Rate of heat flow,

$$= \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{45 \times 0.2 \times 10^{-4} \times 20}{60 \times 10^{-2}} = 30 \times 10^{-3} = 0.03 \text{ w}$$

9. $A = 10 \text{ cm}^2$, $h = 10 \text{ cm}$

$$\frac{\Delta Q}{\Delta t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{200 \times 10^{-3} \times 30}{1 \times 10^{-3}} = 6000$$

Since heat goes out from both surfaces. Hence net heat coming out.

$$= \frac{\Delta Q}{\Delta t} = 6000 \times 2 = 12000,$$

$$\frac{\Delta Q}{\Delta t} = MS \frac{\Delta \theta}{\Delta t}$$

$$\Rightarrow 6000 \times 2 = 10^{-3} \times 10^{-1} \times 1000 \times 4200 \times \frac{\Delta \theta}{\Delta t}$$

$$\Rightarrow \frac{\Delta \theta}{\Delta t} = \frac{72000}{420} = 28.57$$

So, in 1 Sec. 28.57°C is dropped

$$\text{Hence for drop of } 1^\circ\text{C } \frac{1}{28.57} \text{ sec.} = 0.035 \text{ sec. is required}$$

10. $\ell = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$

$$A = 0.2 \text{ cm}^2 = 0.2 \times 10^{-4} \text{ m}^2$$

$$\theta_1 = 80^\circ\text{C},$$

$$\theta_2 = 20^\circ\text{C},$$

$$K = 385$$

$$(a) \frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell} = \frac{385 \times 0.2 \times 10^{-4} (80 - 20)}{20 \times 10^{-2}} = 385 \times 6 \times 10^{-4} \times 10 = 2310 \times 10^{-3} = 2.31$$

(b) Let the temp of the 11 cm point be θ

$$\frac{\Delta \theta}{\Delta l} = \frac{Q}{tKA}$$

$$\Rightarrow \frac{\Delta \theta}{\Delta l} = \frac{2.31}{385 \times 0.2 \times 10^{-4}}$$

$$\Rightarrow \frac{\theta - 20}{11 \times 10^{-2}} = \frac{2.31}{385 \times 0.2 \times 10^{-4}}$$

$$\Rightarrow \theta - 20 = \frac{2.31 \times 10^4}{385 \times 0.2} \times 11 \times 10^{-2} = 33$$

$$\Rightarrow \theta = 33 + 20 = 53$$

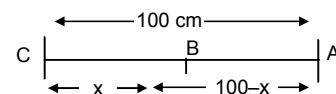
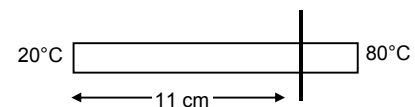
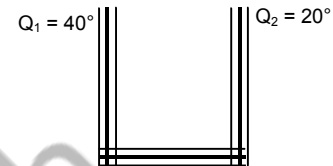
11. Let the point to be touched be 'B'

No heat will flow when, the temp at that point is also 25°C

$$\text{i.e. } Q_{AB} = Q_{BC}$$

$$\text{So, } \frac{KA(100 - 25)}{100 - x} = \frac{KA(25 - 0)}{x}$$

$$\Rightarrow 75x = 2500 - 25x \Rightarrow 100x = 2500 \Rightarrow x = 25 \text{ cm from the end with } 0^\circ\text{C}$$

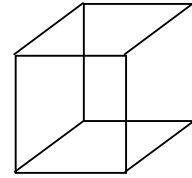


12. $V = 216 \text{ cm}^3$
 $a = 6 \text{ cm}$, Surface area = $6 a^2 = 6 \times 36 \text{ m}^2$
 $t = 0.1 \text{ cm}$ $\frac{Q}{t} = 100 \text{ W}$,

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{\ell}$$

$$\Rightarrow 100 = \frac{K \times 6 \times 36 \times 10^{-4} \times 5}{0.1 \times 10^{-2}}$$

$$\Rightarrow K = \frac{100}{6 \times 36 \times 5 \times 10^{-1}} = 0.9259 \text{ W/m}^\circ\text{C} \approx 0.92 \text{ W/m}^\circ\text{C}$$



13. Given $\theta_1 = 1^\circ\text{C}$, $\theta_2 = 0^\circ\text{C}$
 $K = 0.50 \text{ w/m}^\circ\text{C}$, $d = 2 \text{ mm} = 2 \times 10^{-3} \text{ m}$
 $A = 5 \times 10^{-2} \text{ m}^2$, $v = 10 \text{ cm/s} = 0.1 \text{ m/s}$
Power = Force \times Velocity = $Mg \times v$

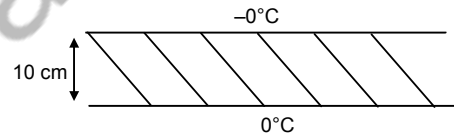
$$\text{Again Power} = \frac{dQ}{dt} = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\text{So, } Mg v = \frac{KA(\theta_1 - \theta_2)}{d}$$

$$\Rightarrow M = \frac{KA(\theta_1 - \theta_2)}{d v g} = \frac{5 \times 10^{-1} \times 5 \times 10^{-2} \times 1}{2 \times 10^{-3} \times 10^{-1} \times 10} = 12.5 \text{ kg.}$$

14. $K = 1.7 \text{ W/m}^\circ\text{C}$ $f_w = 1000 \text{ Kg/m}^3$
 $L_{\text{ice}} = 3.36 \times 10^5 \text{ J/kg}$ $T = 10 \text{ cm} = 10 \times 10^{-2} \text{ m}$

$$\begin{aligned} \text{(a) } \frac{Q}{t} &= \frac{KA(\theta_1 - \theta_2)}{\ell} \Rightarrow \frac{\ell}{t} = \frac{KA(\theta_1 - \theta_2)}{Q} = \frac{KA(\theta_1 - \theta_2)}{mL} \\ &= \frac{KA(\theta_1 - \theta_2)}{A t f_w L} = \frac{1.7 \times [0 - (-10)]}{10 \times 10^{-2} \times 1000 \times 3.36 \times 10^5} \\ &= \frac{17}{3.36} \times 10^{-7} = 5.059 \times 10^{-7} \approx 5 \times 10^{-7} \text{ m/sec} \end{aligned}$$

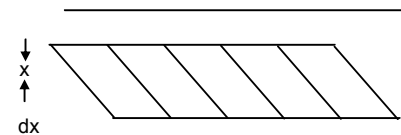


(b) let us assume that x length of ice has become formed to form a small strip of ice of length dx , dt time is required.

$$\frac{dQ}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{dmL}{dt} = \frac{KA(\Delta\theta)}{x} \Rightarrow \frac{A dx f_w L}{dt} = \frac{KA(\Delta\theta)}{x}$$

$$\Rightarrow \frac{dx f_w L}{dt} = \frac{K(\Delta\theta)}{x} \Rightarrow dt = \frac{x dx f_w L}{K(\Delta\theta)}$$

$$\Rightarrow \int_0^t dt = \frac{f_w L}{K(\Delta\theta)} \int_0^t x dx \Rightarrow t = \frac{f_w L}{K(\Delta\theta)} \left[\frac{x^2}{2} \right]_0^t = \frac{f_w L}{K \Delta\theta} \frac{L^2}{2}$$



Putting values

$$\Rightarrow t = \frac{1000 \times 3.36 \times 10^5 \times (10 \times 10^{-2})^2}{1.7 \times 10 \times 2} = \frac{3.36}{2 \times 17} \times 10^6 \text{ sec.} = \frac{3.36 \times 10^6}{2 \times 17 \times 3600} \text{ hrs} = 27.45 \text{ hrs} \approx 27.5 \text{ hrs.}$$

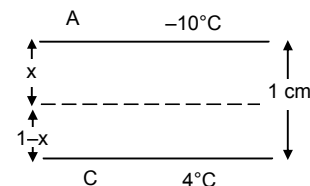
15. let 'B' be the maximum level upto which ice is formed. Hence the heat conducted at that point from both the levels is the same.

Let $AB = x$

$$\text{i.e. } \frac{Q}{t}_{\text{ice}} = \frac{Q}{t}_{\text{water}} \Rightarrow \frac{K_{\text{ice}} \times A \times 10}{x} = \frac{K_{\text{water}} \times A \times 4}{(1-x)}$$

$$\Rightarrow \frac{1.7 \times 10}{x} = \frac{5 \times 10^{-1} \times 4}{1-x} \Rightarrow \frac{17}{x} = \frac{2}{1-x}$$

$$\Rightarrow 17 - 17x = 2x \Rightarrow 19x = 17 \Rightarrow x = \frac{17}{19} = 0.894 \approx 89 \text{ cm}$$

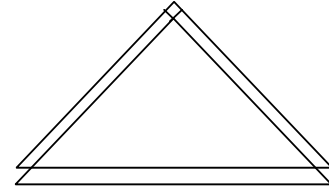


16. $K_{AB} = 50 \text{ J/m-s-}^\circ\text{C}$ $\theta_A = 40^\circ\text{C}$
 $K_{BC} = 200 \text{ J/m-s-}^\circ\text{C}$ $\theta_B = 80^\circ\text{C}$
 $K_{AC} = 400 \text{ J/m-s-}^\circ\text{C}$ $\theta_C = 80^\circ\text{C}$
Length = 20 cm = $20 \times 10^{-2} \text{ m}$
 $A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$

$$(a) \frac{Q_{AB}}{t} = \frac{K_{AB} \times A(\theta_B - \theta_A)}{l} = \frac{50 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 1 \text{ W.}$$

$$(b) \frac{Q_{AC}}{t} = \frac{K_{AC} \times A(\theta_C - \theta_A)}{l} = \frac{400 \times 1 \times 10^{-4} \times 40}{20 \times 10^{-2}} = 800 \times 10^{-2} = 8$$

$$(c) \frac{Q_{BC}}{t} = \frac{K_{BC} \times A(\theta_B - \theta_C)}{l} = \frac{200 \times 1 \times 10^{-4} \times 0}{20 \times 10^{-2}} = 0$$



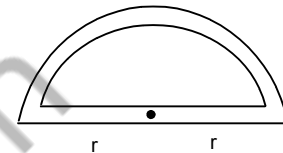
17. We know $Q = \frac{KA(\theta_1 - \theta_2)}{d}$

$$Q_1 = \frac{KA(\theta_1 - \theta_2)}{d_1},$$

$$Q_2 = \frac{KA(\theta_1 - \theta_2)}{d_2}$$

$$\frac{Q_1}{Q_2} = \frac{\frac{KA(\theta_1 - \theta_1)}{\pi r}}{\frac{KA(\theta_1 - \theta_1)}{2r}} = \frac{2r}{\pi r} = \frac{2}{\pi}$$

$$[d_1 = \pi r, \quad d_2 = 2r]$$



18. The rate of heat flow per sec.

$$= \frac{dQ_A}{dt} = KA \frac{d\theta}{dt}$$

The rate of heat flow per sec.

$$= \frac{dQ_B}{dt} = KA \frac{d\theta_B}{dt}$$

This part of heat is absorbed by the red.

$$\frac{Q}{t} = \frac{ms\Delta\theta}{dt} \quad \text{where } \frac{d\theta}{dt} = \text{Rate of net temp. variation}$$

$$\Rightarrow \frac{msd\theta}{dt} = KA \frac{d\theta_A}{dt} - KA \frac{d\theta_B}{dt} \Rightarrow ms \frac{d\theta}{dt} = KA \left(\frac{d\theta_A}{dt} - \frac{d\theta_B}{dt} \right)$$

$$\Rightarrow 0.4 \times \frac{d\theta}{dt} = 200 \times 1 \times 10^{-4} (5 - 2.5) \text{ }^\circ\text{C/cm}$$

$$\Rightarrow 0.4 \times \frac{d\theta}{dt} = 200 \times 10^{-4} \times 2.5$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{200 \times 2.5 \times 10^{-4}}{0.4 \times 10^{-2}} \text{ }^\circ\text{C/m} = 1250 \times 10^{-2} = 12.5 \text{ }^\circ\text{C/m}$$

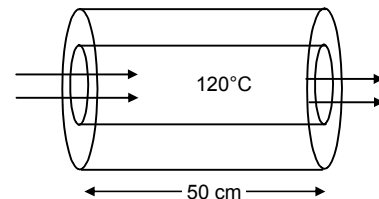
19. Given

$$K_{\text{rubber}} = 0.15 \text{ J/m-s-}^\circ\text{C} \quad T_2 - T_1 = 90^\circ\text{C}$$

We know for radial conduction in a Cylinder

$$\frac{Q}{t} = \frac{2\pi Kl(T_2 - T_1)}{\ln(R_2/R_1)}$$

$$= \frac{2 \times 3.14 \times 15 \times 10^{-2} \times 50 \times 10^{-1} \times 90}{\ln(1.2/1)} = 232.5 \approx 233 \text{ J/s.}$$



20. $\frac{dQ}{dt}$ = Rate of flow of heat

Let us consider a strip at a distance r from the center of thickness dr .

$$\frac{dQ}{dt} = \frac{K \times 2\pi r d \times d\theta}{dr} \quad [d\theta = \text{Temperature diff across the thickness } dr]$$

$$\Rightarrow C = \frac{K \times 2\pi r d \times d\theta}{dr} \quad \left[c = \frac{d\theta}{dr} \right]$$

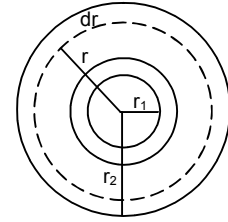
$$\Rightarrow C \frac{dr}{r} = K2\pi d d\theta$$

Integrating

$$\Rightarrow C \int_{r_1}^{r_2} \frac{dr}{r} = K2\pi d \int_{\theta_1}^{\theta_2} d\theta \quad \Rightarrow C [\log r]_{r_1}^{r_2} = K2\pi d (\theta_2 - \theta_1)$$

$$\Rightarrow C(\log r_2 - \log r_1) = K2\pi d (\theta_2 - \theta_1) \Rightarrow C \log \left(\frac{r_2}{r_1} \right) = K2\pi d (\theta_2 - \theta_1)$$

$$\Rightarrow C = \frac{K2\pi d(\theta_2 - \theta_1)}{\log(r_2/r_1)}$$



21. $T_1 > T_2$

$$A = \pi(R_2^2 - R_1^2)$$

$$\text{So, } Q = \frac{KA(T_2 - T_1)}{l} = \frac{KA(R_2^2 - R_1^2)(T_2 - T_1)}{l}$$

Considering a concentric cylindrical shell of radius 'r' and thickness 'dr'. The radial heat flow through the shell

$$H = \frac{dQ}{dt} = -KA \frac{d\theta}{dt} \quad [(-)\text{ve because as } r \text{ increases } \theta \text{ decreases}]$$

$$A = 2\pi r l \quad H = -2\pi r l K \frac{d\theta}{dt}$$

$$\text{or } \int_{R_1}^{R_2} \frac{dr}{r} = -\frac{2\pi l K}{H} \int_{T_1}^{T_2} d\theta$$

Integrating and simplifying we get

$$H = \frac{dQ}{dt} = \frac{2\pi K L (T_2 - T_1)}{\text{Loge}(R_2/R_1)} = \frac{2\pi K L (T_2 - T_1)}{\ln(R_2/R_1)}$$

22. Here the thermal conductivities are in series,

$$\frac{K_1 A (\theta_1 - \theta_2)}{l_1} \times \frac{K_2 A (\theta_1 - \theta_2)}{l_2} = \frac{KA(\theta_1 - \theta_2)}{l_1 + l_2}$$

$$\Rightarrow \frac{K_1 \times K_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} = \frac{K}{l_1 + l_2}$$

$$\Rightarrow \frac{K_1 K_2}{K_1 l_2 + K_2 l_1} = \frac{K}{l_1 + l_2} \Rightarrow K = \frac{(K_1 K_2)(l_1 + l_2)}{K_1 l_2 + K_2 l_1}$$

23. $K_{Cu} = 390 \text{ w/m}^\circ\text{C}$ $K_{St} = 46 \text{ w/m}^\circ\text{C}$

Now, Since they are in series connection,

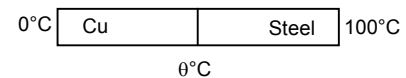
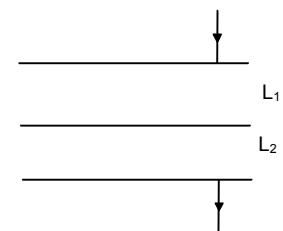
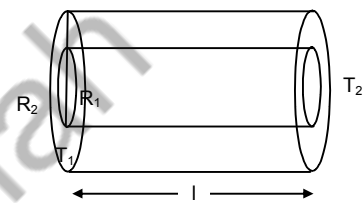
So, the heat passed through the crosssections in the same.

So, $Q_1 = Q_2$

$$\text{Or } \frac{K_{Cu} \times A \times (\theta - 0)}{l} = \frac{K_{St} \times A \times (100 - \theta)}{l}$$

$$\Rightarrow 390(\theta - 0) = 46 \times 100 - 46 \theta \Rightarrow 436 \theta = 4600$$

$$\Rightarrow \theta = \frac{4600}{436} = 10.55 \approx 10.6^\circ\text{C}$$



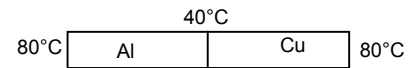
24. As the Aluminum rod and Copper rod joined are in parallel

$$\frac{Q}{t} = \left(\frac{Q}{t}\right)_{Al} + \left(\frac{Q}{t}\right)_{Cu}$$

$$\Rightarrow \frac{KA(\theta_1 - \theta_2)}{l} = \frac{K_1 A(\theta_1 - \theta_2)}{l} + \frac{K_2 A(\theta_1 - \theta_2)}{l}$$

$$\Rightarrow K = K_1 + K_2 = (390 + 200) = 590$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{590 \times 1 \times 10^{-4} \times (60 - 20)}{1} = 590 \times 10^{-4} \times 40 = 2.36 \text{ Watt}$$



25. $K_{Al} = 200 \text{ w/m}^\circ\text{C}$ $K_{Cu} = 400 \text{ w/m}^\circ\text{C}$

$$A = 0.2 \text{ cm}^2 = 2 \times 10^{-5} \text{ m}^2$$

$$l = 20 \text{ cm} = 2 \times 10^{-1} \text{ m}$$

Heat drawn per second

$$= Q_{Al} + Q_{Cu} = \frac{K_{Al} \times A(80 - 40)}{l} + \frac{K_{Cu} \times A(80 - 40)}{l} = \frac{2 \times 10^{-5} \times 40}{2 \times 10^{-1}} [200 + 400] = 2.4 \text{ J}$$

$$\text{Heat drawn per min} = 2.4 \times 60 = 144 \text{ J}$$

26. $(Q/t)_{AB} = (Q/t)_{BE \text{ bent}} + (Q/t)_{BE}$

$$(Q/t)_{BE \text{ bent}} = \frac{KA(\theta_1 - \theta_2)}{70} \quad (Q/t)_{BE} = \frac{KA(\theta_1 - \theta_2)}{60}$$

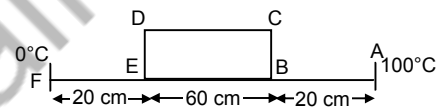
$$\frac{(Q/t)_{BE \text{ bent}}}{(Q/t)_{BE}} = \frac{60}{70} = \frac{6}{7}$$

$$(Q/t)_{BE \text{ bent}} + (Q/t)_{BE} = 130$$

$$\Rightarrow (Q/t)_{BE \text{ bent}} + (Q/t)_{BE} \frac{7}{6} = 130$$

$$\Rightarrow \left(\frac{7}{6} + 1\right) (Q/t)_{BE \text{ bent}} = 130$$

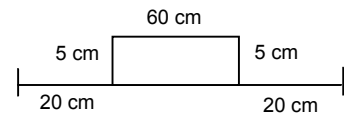
$$\Rightarrow (Q/t)_{BE \text{ bent}} = \frac{130 \times 6}{13} = 60$$



27. $\frac{Q}{t} \text{ bent} = \frac{780 \times A \times 100}{70}$

$$\frac{Q}{t} \text{ str} = \frac{390 \times A \times 100}{60}$$

$$\frac{(Q/t) \text{ bent}}{(Q/t) \text{ str}} = \frac{780 \times A \times 100}{70} \times \frac{60}{390 \times A \times 100} = \frac{12}{7}$$



28. (a) $\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l} = \frac{1 \times 2 \times 1(40 - 32)}{2 \times 10^{-3}} = 8000 \text{ J/sec.}$

(b) Resistance of glass = $\frac{l}{ak_g} + \frac{l}{ak_g}$

Resistance of air = $\frac{l}{ak_a}$

Net resistance = $\frac{l}{ak_g} + \frac{l}{ak_g} + \frac{l}{ak_a}$

$$= \frac{l}{a} \left(\frac{2}{k_g} + \frac{1}{k_a} \right) = \frac{l}{a} \left(\frac{2k_a + k_g}{K_g k_a} \right)$$

$$= \frac{1 \times 10^{-3}}{2} \left(\frac{2 \times 0.025 + 1}{0.025} \right)$$

$$= \frac{1 \times 10^{-3} \times 1.05}{0.05}$$

$$\frac{Q}{t} = \frac{\theta_1 - \theta_2}{R} = \frac{8 \times 0.05}{1 \times 10^{-3} \times 1.05} = 380.9 \approx 381 \text{ W}$$



29. Now; Q/t remains same in both cases

$$\text{In Case I: } \frac{K_A \times A \times (100 - 70)}{\ell} = \frac{K_B \times A \times (70 - 0)}{\ell}$$

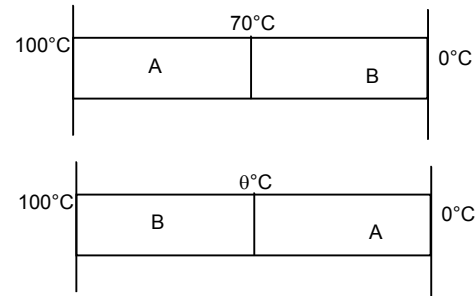
$$\Rightarrow 30 K_A = 70 K_B$$

$$\text{In Case II: } \frac{K_B \times A \times (100 - \theta)}{\ell} = \frac{K_A \times A \times (\theta - 0)}{\ell}$$

$$\Rightarrow 100K_B - K_B \theta = K_A \theta$$

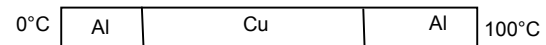
$$\Rightarrow 100K_B - K_B \theta = \frac{70}{30} K_B \theta$$

$$\Rightarrow 100 = \frac{7}{3} \theta + \theta \quad \Rightarrow \theta = \frac{300}{10} = 30^\circ\text{C}$$



30. $\theta_1 - \theta_2 = 100$

$$\frac{Q}{t} = \frac{\theta_1 - \theta_2}{R}$$



$$R = R_1 + R_2 + R_3 = \frac{\ell}{aK_{Al}} + \frac{\ell}{aK_{Cu}} + \frac{\ell}{aK_{Al}} = \frac{\ell}{a} \left(\frac{2}{200} + \frac{1}{400} \right) = \frac{\ell}{a} \left(\frac{4+1}{400} \right) = \frac{\ell}{a} \frac{1}{80}$$

$$\frac{Q}{t} = \frac{100}{(\ell/a)(1/80)} \Rightarrow 40 = 80 \times 100 \times \frac{a}{\ell}$$

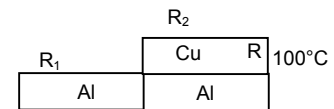
$$\Rightarrow \frac{a}{\ell} = \frac{1}{200}$$

For (b)

$$R = R_1 + R_2 = R_1 + \frac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = R_{Al} + \frac{R_{Cu}R_{Al}}{R_{Cu} + R_{Al}} = \frac{\frac{\ell}{AK_{Al}} + \frac{\ell}{AK_{Cu}} + \frac{\ell}{AK_{Al}}}{\frac{\ell}{A_{Cu}} + \frac{\ell}{A_{Al}}}$$

$$= \frac{\frac{\ell}{AK_{Al}} + \frac{\ell}{A} + \frac{\ell}{K_{Al} + K_{Cu}}}{\frac{\ell}{A} \left(\frac{1}{200} + \frac{1}{200 + 400} \right)} = \frac{\ell}{A} \times \frac{4}{600}$$

$$\frac{Q}{t} = \frac{\theta_1 - \theta_2}{R} = \frac{100}{(\ell/A)(4/600)} = \frac{100 \times 600 A}{4 \ell} = \frac{100 \times 600}{4} \times \frac{1}{200} = 75$$



For (c)

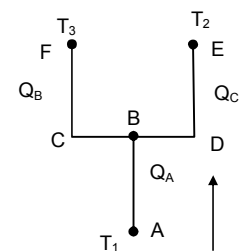
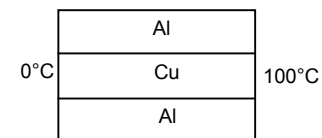
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{\frac{\ell}{aK_{Al}}} + \frac{1}{\frac{\ell}{aK_{Cu}}} + \frac{1}{\frac{\ell}{aK_{Al}}}$$

$$= \frac{a}{\ell} (K_{Al} + K_{Cu} + K_{Al}) = \frac{a}{\ell} (2 \times 200 + 400) = \frac{a}{\ell} (800)$$

$$\Rightarrow R = \frac{\ell}{a} \times \frac{1}{800}$$

$$\Rightarrow \frac{Q}{t} = \frac{\theta_1 - \theta_2}{R} = \frac{100 \times 800 \times a}{\ell}$$

$$= \frac{100 \times 800}{200} = 400 \text{ W}$$



31. Let the temp. at B be T

$$\frac{Q_A}{t} = \frac{Q_B}{t} + \frac{Q_C}{t}$$

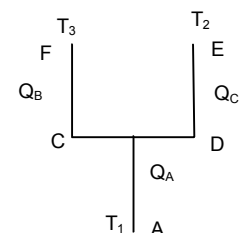
$$\Rightarrow \frac{KA(T_1 - T)}{\ell} = \frac{KA(T - T_3)}{\ell + (\ell/2)} + \frac{KA(T - T_2)}{\ell + (\ell/2)}$$

$$\Rightarrow \frac{T_1 - T}{\ell} = \frac{T - T_3}{3\ell/2} + \frac{T - T_2}{3\ell/2}$$

$$\Rightarrow 3T_1 - 3T = 4T - 2(T_2 + T_3)$$

$$\Rightarrow -7T = -3T_1 - 2(T_2 + T_3)$$

$$\Rightarrow T = \frac{3T_1 + 2(T_2 + T_3)}{7}$$



32. The temp at the both ends of bar F is same

Rate of Heat flow to right = Rate of heat flow through left

$$\Rightarrow (Q/t)_A + (Q/t)_C = (Q/t)_B + (Q/t)_D$$

$$\Rightarrow \frac{K_A(T_1 - T)A}{l} + \frac{K_C(T_1 - T)A}{l} = \frac{K_B(T - T_2)A}{l} + \frac{K_D(T - T_2)A}{l}$$

$$\Rightarrow 2K_0(T_1 - T) = 2 \times 2K_0(T - T_2)$$

$$\Rightarrow T_1 - T = 2T - 2T_2$$

$$\Rightarrow T = \frac{T_1 + 2T_2}{3}$$

- 33.
- $\tan \phi = \frac{r_2 - r_1}{L} = \frac{(y - r_1)}{x}$

$$\Rightarrow xr_2 - xr_1 = yL - r_1L$$

Differentiating wr to 'x'

$$\Rightarrow r_2 - r_1 = \frac{Ldy}{dx} - 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{r_2 - r_1}{L} \Rightarrow dx = \frac{dyL}{(r_2 - r_1)} \quad \dots(1)$$

$$\text{Now } \frac{Q}{T} = \frac{K\pi y^2 d\theta}{dx} \Rightarrow \frac{\theta dx}{T} = K\pi y^2 d\theta$$

$$\Rightarrow \frac{\theta L dy}{r_2 r_1} = K\pi y^2 d\theta \quad \text{from(1)}$$

$$\Rightarrow d\theta = \frac{QLdy}{(r_2 - r_1)K\pi y^2}$$

Integrating both side

$$\Rightarrow \int_{\theta_1}^{\theta_2} d\theta = \frac{QL}{(r_2 - r_1)K\pi} \int_{r_1}^{r_2} \frac{dy}{y}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{-1}{y} \right]_{r_1}^{r_2}$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{1}{r_1} - \frac{1}{r_2} \right]$$

$$\Rightarrow (\theta_2 - \theta_1) = \frac{QL}{(r_2 - r_1)K\pi} \times \left[\frac{r_2 - r_1}{r_1 + r_2} \right]$$

$$\Rightarrow Q = \frac{K\pi r_1 r_2 (\theta_2 - \theta_1)}{L}$$

- 34.
- $\frac{d\theta}{dt} = \frac{60}{10 \times 60} = 0.1^\circ\text{C/sec}$

$$\frac{dQ}{dt} = \frac{KA}{d} (\theta_1 - \theta_2)$$

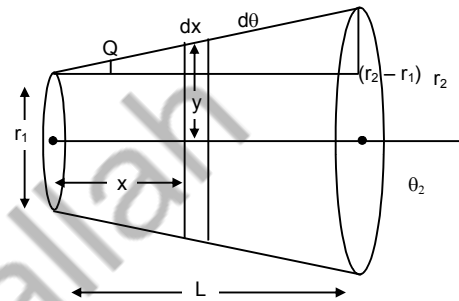
$$= \frac{KA \times 0.1}{d} + \frac{KA \times 0.2}{d} + \dots + \frac{KA \times 60}{d}$$

$$= \frac{KA}{d} (0.1 + 0.2 + \dots + 60) = \frac{KA}{d} \times \frac{600}{2} \times (2 \times 0.1 + 599 \times 0.1)$$

$$[\therefore a + 2a + \dots + na = n/2\{2a + (n-1)a\}]$$

$$= \frac{200 \times 1 \times 10^{-4}}{20 \times 10^{-2}} \times 300 \times (0.2 + 59.9) = \frac{200 \times 10^{-2} \times 300 \times 60.1}{20}$$

$$= 3 \times 10 \times 60.1 = 1803 \text{ w} \approx 1800 \text{ w}$$



35. $a = r_1 = 5 \text{ cm} = 0.05 \text{ m}$

$b = r_2 = 20 \text{ cm} = 0.2 \text{ m}$

$\theta_1 = T_1 = 50^\circ\text{C}$

$\theta_2 = T_2 = 10^\circ\text{C}$

Now, considering a small strip of thickness 'dr' at a distance 'r'.

$A = 4 \pi r^2$

$H = -4 \pi r^2 K \frac{d\theta}{dr}$ [(-)ve because with increase of r, θ decreases]

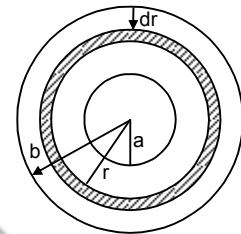
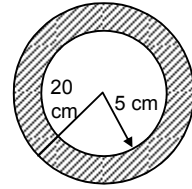
$= \int_a^b \frac{dr}{r^2} = \frac{-4\pi K}{H} \int_{\theta_1}^{\theta_2} d\theta$ On integration,

$H = \frac{dQ}{dt} = K \frac{4\pi ab(\theta_1 - \theta_2)}{(b-a)}$

Putting the values we get

$\frac{K \times 4 \times 3.14 \times 5 \times 20 \times 40 \times 10^{-3}}{15 \times 10^{-2}} = 100$

$\Rightarrow K = \frac{15}{4 \times 3.14 \times 4 \times 10^{-1}} = 2.985 \approx 3 \text{ w/m}^\circ\text{C}$



36. $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$

Rise in Temp. in $T_2 \Rightarrow \frac{KA(T_1 - T_2)}{Lms}$

Fall in Temp in $T_1 = \frac{KA(T_1 - T_2)}{Lms}$ Final Temp. $T_1 \Rightarrow T_1 - \frac{KA(T_1 - T_2)}{Lms}$

Final Temp. $T_2 = T_2 + \frac{KA(T_1 - T_2)}{Lms}$

Final $\frac{\Delta T}{dt} = T_1 - \frac{KA(T_1 - T_2)}{Lms} - T_2 - \frac{KA(T_1 - T_2)}{Lms}$

$= (T_1 - T_2) - \frac{2KA(T_1 - T_2)}{Lms} = \frac{dT}{dt} = -\frac{2KA(T_1 - T_2)}{Lms} \Rightarrow \int_{(T_1 - T_2)}^{(T_1 - T_2)/2} \frac{dt}{(T_1 - T_2)} = \frac{-2KA}{Lms} dt$

$\Rightarrow \ln \frac{(T_1 - T_2)/2}{(T_1 - T_2)} = \frac{-2KAt}{Lms} \Rightarrow \ln(1/2) = \frac{-2KAt}{Lms} \Rightarrow \ln 2 = \frac{2KAt}{Lms} \Rightarrow t = \ln 2 \frac{Lms}{2KA}$

37. $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$

Rise in Temp. in $T_2 \Rightarrow \frac{KA(T_1 - T_2)}{Lm_1s_1}$

Fall in Temp in $T_1 \Rightarrow \frac{KA(T_1 - T_2)}{Lm_2s_2}$ Final Temp. $T_1 = T_1 - \frac{KA(T_1 - T_2)}{Lm_1s_1}$

Final Temp. $T_2 = T_2 + \frac{KA(T_1 - T_2)}{Lm_1s_1}$

$\frac{\Delta T}{dt} = T_1 - \frac{KA(T_1 - T_2)}{Lm_1s_1} - T_2 - \frac{KA(T_1 - T_2)}{Lm_2s_2} = (T_1 - T_2) - \left[\frac{KA(T_1 - T_2)}{Lm_1s_1} + \frac{KA(T_1 - T_2)}{Lm_2s_2} \right]$

$\Rightarrow \frac{dT}{dt} = -\frac{KA(T_1 - T_2)}{L} \left(\frac{1}{m_1s_1} + \frac{1}{m_2s_2} \right) \Rightarrow \frac{dT}{(T_1 - T_2)} = -\frac{KA}{L} \left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2} \right) dt$

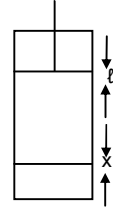
$\Rightarrow \ln \Delta t = -\frac{KA}{L} \left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2} \right) t + C$

At time $t = 0$, $T = T_0$, $\Delta T = \Delta T_0 \Rightarrow C = \ln \Delta T_0$

$\Rightarrow \ln \frac{\Delta T}{\Delta T_0} = -\frac{KA}{L} \left(\frac{m_2s_2 + m_1s_1}{m_1s_1m_2s_2} \right) t \Rightarrow \frac{\Delta T}{\Delta T_0} = e^{-\frac{KA}{L} \left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2} \right) t}$

$\Rightarrow \Delta T = \Delta T_0 e^{-\frac{KA}{L} \left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2} \right) t} = (T_2 - T_1) e^{-\frac{KA}{L} \left(\frac{m_1s_1 + m_2s_2}{m_1s_1m_2s_2} \right) t}$

$$\begin{aligned}
 38. \quad \frac{Q}{t} &= \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{nC_p dT}{dt} = \frac{KA(T_s - T_0)}{x} \\
 &\Rightarrow \frac{n(5/2)RdT}{dt} = \frac{KA(T_s - T_0)}{x} \Rightarrow \frac{dT}{dt} = \frac{-2LA}{5nRx}(T_s - T_0) \\
 &\Rightarrow \frac{dT}{(T_s - T_0)} = -\frac{2KA dt}{5nRx} \Rightarrow \ln(T_s - T_0)_{T_0}^T = -\frac{2KA dt}{5nRx} \\
 &\Rightarrow \ln \frac{T_s - T}{T_s - T_0} = -\frac{2KA dt}{5nRx} \Rightarrow T_s - T = (T_s - T_0)e^{-\frac{2KA dt}{5nRx}} \\
 &\Rightarrow T = T_s - (T_s - T_0)e^{-\frac{2KA dt}{5nRx}} = T_s + (T_0 - T_s)e^{-\frac{2KA dt}{5nRx}} \\
 &\Rightarrow \Delta T = T - T_0 = (T_s - T_0) + (T_0 - T_s)e^{-\frac{2KA dt}{5nRx}} = (T_s - T_0) \left(1 + e^{-\frac{2KA dt}{5nRx}} \right) \\
 &\Rightarrow \frac{P_a AL}{nR} = (T_s - T_0) \left(1 + e^{-\frac{2KA dt}{5nRx}} \right) \quad [p_a dv = nRdt \quad P_a A l = nRdt \quad dT = \frac{P_a AL}{nR}] \\
 &\Rightarrow L = \frac{nR}{P_a A} (T_s - T_0) \left(1 + e^{-\frac{2KA dt}{5nRx}} \right)
 \end{aligned}$$



$$\begin{aligned}
 39. \quad A &= 1.6 \text{ m}^2, \quad T = 37^\circ\text{C} = 310 \text{ K}, \quad \sigma = 6.0 \times 10^{-8} \text{ w/m}^2\text{-K}^4 \\
 \text{Energy radiated per second} \\
 &= A\sigma T^4 = 1.6 \times 6 \times 10^{-8} \times (310)^4 = 8865801 \times 10^{-4} = 886.58 \approx 887 \text{ J} \\
 40. \quad A &= 12 \text{ cm}^2 = 12 \times 10^{-4} \text{ m}^2 \quad T = 20^\circ\text{C} = 293 \text{ K} \\
 e &= 0.8 \quad \sigma = 6 \times 10^{-8} \text{ w/m}^2\text{-K}^4 \\
 \frac{Q}{t} &= Ae \sigma T^4 = 12 \times 10^{-4} \times 0.8 \times 6 \times 10^{-8} (293)^4 = 4.245 \times 10^{12} \times 10^{-13} = 0.4245 \approx 0.42
 \end{aligned}$$

$$\begin{aligned}
 41. \quad E &\rightarrow \text{Energy radiated per unit area per unit time} \\
 \text{Rate of heat flow} &\rightarrow \text{Energy radiated} \\
 \text{(a) Per time} &= E \times A \\
 \text{So, } E_{Al} &= \frac{e\sigma T^4 \times A}{e\sigma T^4 \times A} = \frac{4\pi r^2}{4\pi(2r)^2} = \frac{1}{4} \quad \therefore 1 : 4 \\
 \text{(b) Emissivity of both are same} \\
 &= \frac{m_1 S_1 dT_1}{m_2 S_2 dT_2} = 1 \\
 &\Rightarrow \frac{dT_1}{dT_2} = \frac{m_2 S_2}{m_1 S_1} = \frac{s_1 4\pi r_1^3 \times S_2}{s_2 4\pi r_2^3 \times S_1} = \frac{1 \times \pi \times 900}{3.4 \times 8\pi \times 390} = 1 : 2 : 9
 \end{aligned}$$

$$\begin{aligned}
 42. \quad \frac{Q}{t} &= Ae \sigma T^4 \\
 &\Rightarrow T^4 = \frac{\theta}{teA\sigma} \Rightarrow T^4 = \frac{100}{0.8 \times 2 \times 3.14 \times 4 \times 10^{-5} \times 1 \times 6 \times 10^{-8}} \\
 &\Rightarrow T = 1697.0 \approx 1700 \text{ K}
 \end{aligned}$$

$$\begin{aligned}
 43. \quad \text{(a) } A &= 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2, \quad T = 57^\circ\text{C} = 330 \text{ K} \\
 E &= A \sigma T^4 = 20 \times 10^{-4} \times 6 \times 10^{-8} \times (330)^4 \times 10^4 = 1.42 \text{ J}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) } \frac{E}{t} &= A\sigma e(T_1^4 - T_2^4), \quad A = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2 \\
 \sigma &= 6 \times 10^{-8} \quad T_1 = 473 \text{ K}, \quad T_2 = 330 \text{ K} \\
 &= 20 \times 10^{-4} \times 6 \times 10^{-8} \times 1[(473)^4 - (330)^4] \\
 &= 20 \times 6 \times [5.005 \times 10^{10} - 1.185 \times 10^{10}] \\
 &= 20 \times 6 \times 3.82 \times 10^{-2} = 4.58 \text{ w} \quad \text{from the ball.}
 \end{aligned}$$

44. $r = 1 \text{ cm} = 1 \times 10^{-3} \text{ m}$
 $A = 4\pi(10^{-2})^2 = 4\pi \times 10^{-4} \text{ m}^2$
 $E = 0.3, \quad \sigma = 6 \times 10^{-8}$
 $\frac{E}{t} = A\sigma e(T_1^4 - T_2^4)$
 $= 0.3 \times 6 \times 10^{-8} \times 4\pi \times 10^{-4} \times [(100)^4 - (300)^4]$
 $= 0.3 \times 6 \times 4\pi \times 10^{-12} \times [1 - 0.0081] \times 10^{12}$
 $= 0.3 \times 6 \times 4 \times 3.14 \times 9919 \times 10^{-4}$
 $= 4 \times 18 \times 3.14 \times 9919 \times 10^{-5} = 22.4 \approx 22 \text{ W}$
45. Since the Cube can be assumed as black body

$$e = 1$$

$$\sigma = 6 \times 10^{-8} \text{ w/m}^2\text{-k}^4$$

$$A = 6 \times 25 \times 10^{-4} \text{ m}^2$$

$$m = 1 \text{ kg}$$

$$s = 400 \text{ J/kg-}^\circ\text{K}$$

$$T_1 = 227^\circ\text{C} = 500 \text{ K}$$

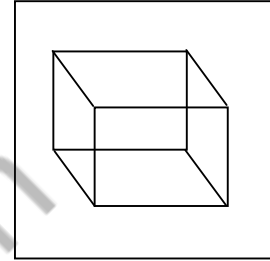
$$T_2 = 27^\circ\text{C} = 300 \text{ K}$$

$$\Rightarrow ms \frac{d\theta}{dt} = e\sigma A(T_1^4 - T_2^4)$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{e\sigma A(T_1^4 - T_2^4)}{ms}$$

$$= \frac{1 \times 6 \times 10^{-8} \times 6 \times 25 \times 10^{-4} \times [(500)^4 - (300)^4]}{1 \times 400}$$

$$= \frac{36 \times 25 \times 544}{400} \times 10^{-4} = 1224 \times 10^{-4} = 0.1224^\circ\text{C/s} \approx 0.12^\circ\text{C/s.}$$



46. $Q = e\sigma A(T_2^4 - T_1^4)$
 For any body, $210 = eA\sigma[(500)^4 - (300)^4]$
 For black body, $700 = 1 \times A\sigma[(500)^4 - (300)^4]$
 Dividing $\frac{210}{700} = \frac{e}{1} \Rightarrow e = 0.3$

47. $A_A = 20 \text{ cm}^2, \quad A_B = 80 \text{ cm}^2$
 $(mS)_A = 42 \text{ J/}^\circ\text{C}, \quad (mS)_B = 82 \text{ J/}^\circ\text{C},$
 $T_A = 100^\circ\text{C}, \quad T_B = 20^\circ\text{C}$

K_B is low thus it is a poor conductor and K_A is high.

Thus A will absorb no heat and conduct all

$$\left(\frac{E}{t}\right)_A = \sigma A_A [(373)^4 - (293)^4] \Rightarrow (mS)_A \left(\frac{d\theta}{dt}\right)_A = \sigma A_A [(373)^4 - (293)^4]$$

$$\Rightarrow \left(\frac{d\theta}{dt}\right)_A = \frac{\sigma A_A [(373)^4 - (293)^4]}{(mS)_A} = \frac{6 \times 10^{-8} [(373)^4 - (293)^4]}{42} = 0.03^\circ\text{C/S}$$

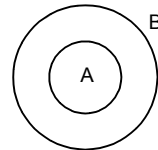
$$\text{Similarly } \left(\frac{d\theta}{dt}\right)_B = 0.043^\circ\text{C/S}$$

48. $\frac{Q}{t} = eAe(T_2^4 - T_1^4)$

$$\Rightarrow \frac{Q}{At} = 1 \times 6 \times 10^{-8} [(300)^4 - (290)^4] = 6 \times 10^{-8} (81 \times 10^8 - 70.7 \times 10^8) = 6 \times 10.3$$

$$\frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{l}$$

$$\Rightarrow \frac{Q}{tA} = \frac{K(\theta_1 - \theta_2)}{l} = \frac{K \times 17}{0.5} = 6 \times 10.3 = \frac{K \times 17}{0.5} \Rightarrow K = \frac{6 \times 10.3 \times 0.5}{17} = 1.8$$



49. $\sigma = 6 \times 10^{-8} \text{ w/m}^2\text{-k}^4$

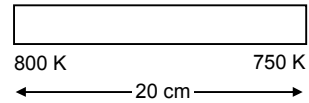
$L = 20 \text{ cm} = 0.2 \text{ m}, \quad K = ?$

300 K

$$\Rightarrow E = \frac{KA(\theta_1 - \theta_2)}{d} = A\sigma(T_1^4 - T_2^4)$$

$$\Rightarrow K = \frac{s(T_1 - T_2) \times d}{\theta_1 - \theta_2} = \frac{6 \times 10^{-8} \times (750^4 - 300^4) \times 2 \times 10^{-1}}{50}$$

$$\Rightarrow K = 73.993 \approx 74.$$



50. $v = 100 \text{ cc}$

$\Delta\theta = 5^\circ\text{C}$

$t = 5 \text{ min}$

For water

$$\frac{mS\Delta\theta}{dt} = \frac{KA}{l} \Delta\theta$$

$$\Rightarrow \frac{100 \times 10^{-3} \times 1000 \times 4200}{5} = \frac{KA}{l}$$

For Kerosene

$$\frac{ms}{at} = \frac{KA}{l}$$

$$\Rightarrow \frac{100 \times 10^{-3} \times 800 \times 2100}{t} = \frac{KA}{l}$$

$$\Rightarrow \frac{100 \times 10^{-3} \times 800 \times 2100}{t} = \frac{100 \times 10^{-3} \times 1000 \times 4200}{5}$$

$$\Rightarrow T = \frac{5 \times 800 \times 2100}{1000 \times 4200} = 2 \text{ min}$$

51. 50°C 45°C 40°C

Let the surrounding temperature be ' T ' $^\circ\text{C}$

$$\text{Avg. } t = \frac{50 + 45}{2} = 47.5$$

Avg. temp. diff. from surrounding

$$T = 47.5 - T$$

$$\text{Rate of fall of temp} = \frac{50 - 45}{5} = 1^\circ\text{C/mm}$$

From Newton's Law

$$1^\circ\text{C/mm} = bA \times t$$

$$\Rightarrow bA = \frac{1}{t} = \frac{1}{47.5 - T} \quad \dots(1)$$

In second case,

$$\text{Avg. temp} = \frac{40 + 45}{2} = 42.5$$

Avg. temp. diff. from surrounding

$$t' = 42.5 - t$$

$$\text{Rate of fall of temp} = \frac{45 - 40}{8} = \frac{5}{8}^\circ\text{C/mm}$$

From Newton's Law

$$\frac{5}{8} = bAt'$$

$$\Rightarrow \frac{5}{8} = \frac{1}{(47.5 - T)} \times (42.5 - T)$$

By C & D [Componendo & Dividendo method]

We find, $T = 34.1^\circ\text{C}$

52. Let the water eq. of calorimeter = m

$$\frac{(m + 50 \times 10^{-3}) \times 4200 \times 5}{10} = \text{Rate of heat flow}$$

$$\frac{(m + 100 \times 10^{-3}) \times 4200 \times 5}{18} = \text{Rate of flow}$$

$$\Rightarrow \frac{(m + 50 \times 10^{-3}) \times 4200 \times 5}{10} = \frac{(m + 100 \times 10^{-3}) \times 4200 \times 5}{18}$$

$$\Rightarrow (m + 50 \times 10^{-3})18 = 10m + 1000 \times 10^{-3}$$

$$\Rightarrow 18m + 18 \times 50 \times 10^{-3} = 10m + 1000 \times 10^{-3}$$

$$\Rightarrow 8m = 100 \times 10^{-3} \text{ kg}$$

$$\Rightarrow m = 12.5 \times 10^{-3} \text{ kg} = 12.5 \text{ g}$$

53. In steady state condition as no heat is absorbed, the rate of loss of heat by conduction is equal to that of the supplied.

i.e. $H = P$

$m = 1 \text{ Kg}$, Power of Heater = 20 W, Room Temp. = 20°C

(a) $H = \frac{d\theta}{dt} = P = 20 \text{ watt}$

(b) by Newton's law of cooling

$$\frac{-d\theta}{dt} = K(\theta - \theta_0)$$

$$-20 = K(50 - 20) \Rightarrow K = 2/3$$

$$\text{Again, } \frac{-d\theta}{dt} = K(\theta - \theta_0) = \frac{2}{3} \times (30 - 20) = \frac{20}{3} \text{ w}$$

(c) $\left(\frac{dQ}{dt}\right)_{20} = 0, \quad \left(\frac{dQ}{dt}\right)_{30} = \frac{20}{3}, \quad \left(\frac{dQ}{dt}\right)_{\text{avg}} = \frac{10}{3}$

$$T = 5 \text{ min} = 300'$$

$$\text{Heat liberated} = \frac{10}{3} \times 300 = 1000 \text{ J}$$

$$\text{Net Heat absorbed} = \text{Heat supplied} - \text{Heat Radiated} = 6000 - 1000 = 5000 \text{ J}$$

Now, $m\Delta\theta' = 5000$

$$\Rightarrow S = \frac{5000}{m\Delta\theta} = \frac{5000}{1 \times 10} = 500 \text{ J Kg}^{-1}\text{C}^{-1}$$

54. Given:

$$\text{Heat capacity} = m \times s = 80 \text{ J/}^\circ\text{C}$$

$$\left(\frac{d\theta}{dt}\right)_{\text{increase}} = 2 \text{ }^\circ\text{C/s}$$

$$\left(\frac{d\theta}{dt}\right)_{\text{decrease}} = 0.2 \text{ }^\circ\text{C/s}$$

(a) Power of heater = $mS\left(\frac{d\theta}{dt}\right)_{\text{increasing}} = 80 \times 2 = 160 \text{ W}$

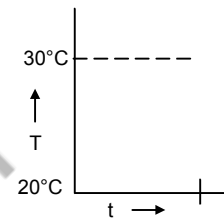
(b) Power radiated = $mS\left(\frac{d\theta}{dt}\right)_{\text{decreasing}} = 80 \times 0.2 = 16 \text{ W}$

(c) Now $mS\left(\frac{d\theta}{dt}\right)_{\text{decreasing}} = K(T - T_0)$

$$\Rightarrow 16 = K(30 - 20) \quad \Rightarrow K = \frac{16}{10} = 1.6$$

$$\text{Now, } \frac{d\theta}{dt} = K(T - T_0) = 1.6 \times (30 - 25) = 1.6 \times 5 = 8 \text{ W}$$

(d) $P.t = H \Rightarrow 8 \times t$



$$55. \frac{d\theta}{dt} = -K(T - T_0)$$

Temp. at $t = 0$ is θ_1

(a) Max. Heat that the body can loose = $\Delta Q_m = ms(\theta_1 - \theta_0)$

(\therefore as, $\Delta t = \theta_1 - \theta_0$)

(b) if the body loses 90% of the max heat the decrease in its temp. will be

$$\frac{\Delta Q_m \times 9}{10ms} = \frac{(\theta_1 - \theta_0) \times 9}{10}$$

If it takes time t_1 , for this process, the temp. at t_1

$$= \theta_1 - (\theta_1 - \theta_0) \frac{9}{10} = \frac{10\theta_1 - 9\theta_1 - 9\theta_0}{10} = \frac{\theta_1 - 9\theta_0}{10} \times 1$$

$$\text{Now, } \frac{d\theta}{dt} = -K(\theta - \theta_1)$$

Let $\theta = \theta_1$ at $t = 0$; & θ be temp. at time t

$$\int_{\theta}^{\theta_1} \frac{d\theta}{\theta - \theta_0} = -K \int_0^t dt$$

$$\text{or, } \ln \frac{\theta - \theta_0}{\theta_1 - \theta_0} = -Kt$$

$$\text{or, } \theta - \theta_0 = (\theta_1 - \theta_0) e^{-kt} \quad \dots(2)$$

Putting value in the Eq (1) and Eq (2)

$$\frac{\theta_1 - 9\theta_0}{10} - \theta_0 = (\theta_1 - \theta_0) e^{-kt}$$

$$\Rightarrow t_1 = \frac{\ln 10}{k}$$

♣ ♣ ♣ ♣ ♣