Ch 8- Electromagnetic Waves

Lect-01

Today's Goal

Displacement Current

Maxwell's Equation

MAXWELL'S DISPLACEMENT CURRENT

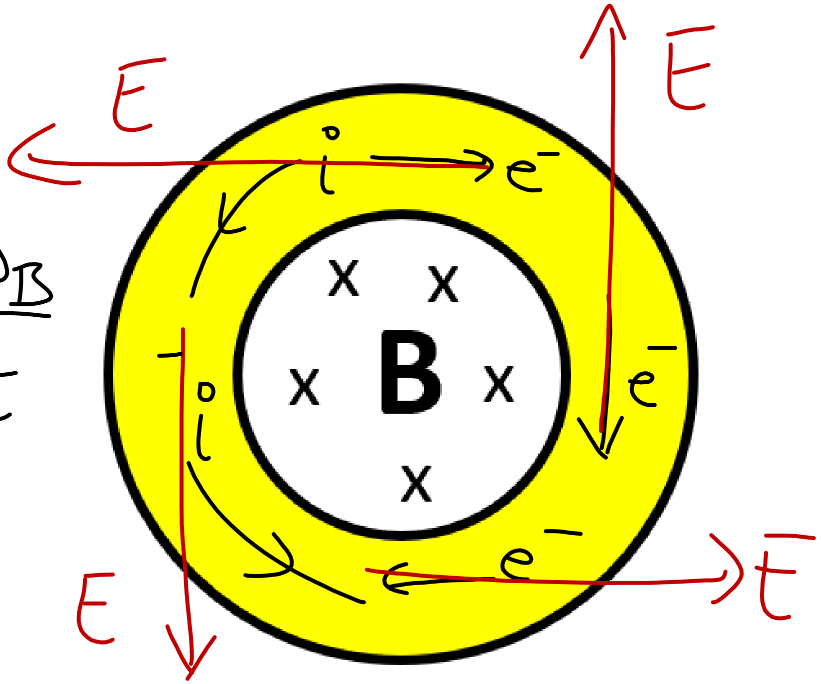
1. In 1831, Michael Faraday gave Faraday's Laws of electromagnetic induction , according to which a change in magnetic flux produces an induced emf

Changing Magnetic Field Produces an Electric Field

$$\oint \vec{E}\cdot\vec{dl} = -\frac{d\phi_B}{dt}$$

$$e = -\frac{d\phi_B}{dt}$$

Faraday's Law

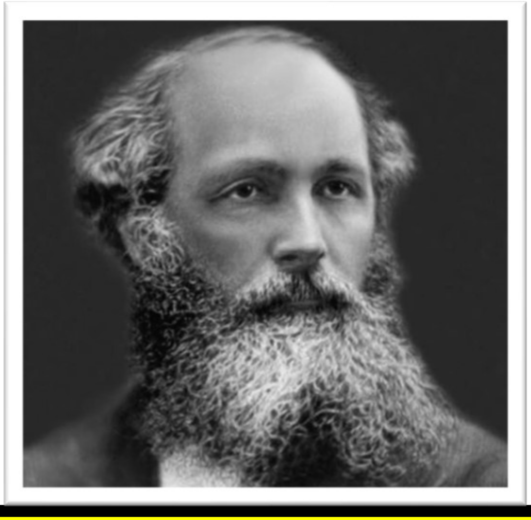


B increasing

2. So, a "changing Magnetic field produces an Electric Field" $\boxed{1831}$ → Faraday's Law

3. Now the Question arises " Can changing Electric Field produce a Magnetic Field ??"

4.



YES BACHOOO!!

James Clerk Maxwell

Maxwell's Experiment to Prove his point

Charging of Capacitor $\quad i = \frac{dq}{dt}$

X $\quad$ Ampere Circuital Law

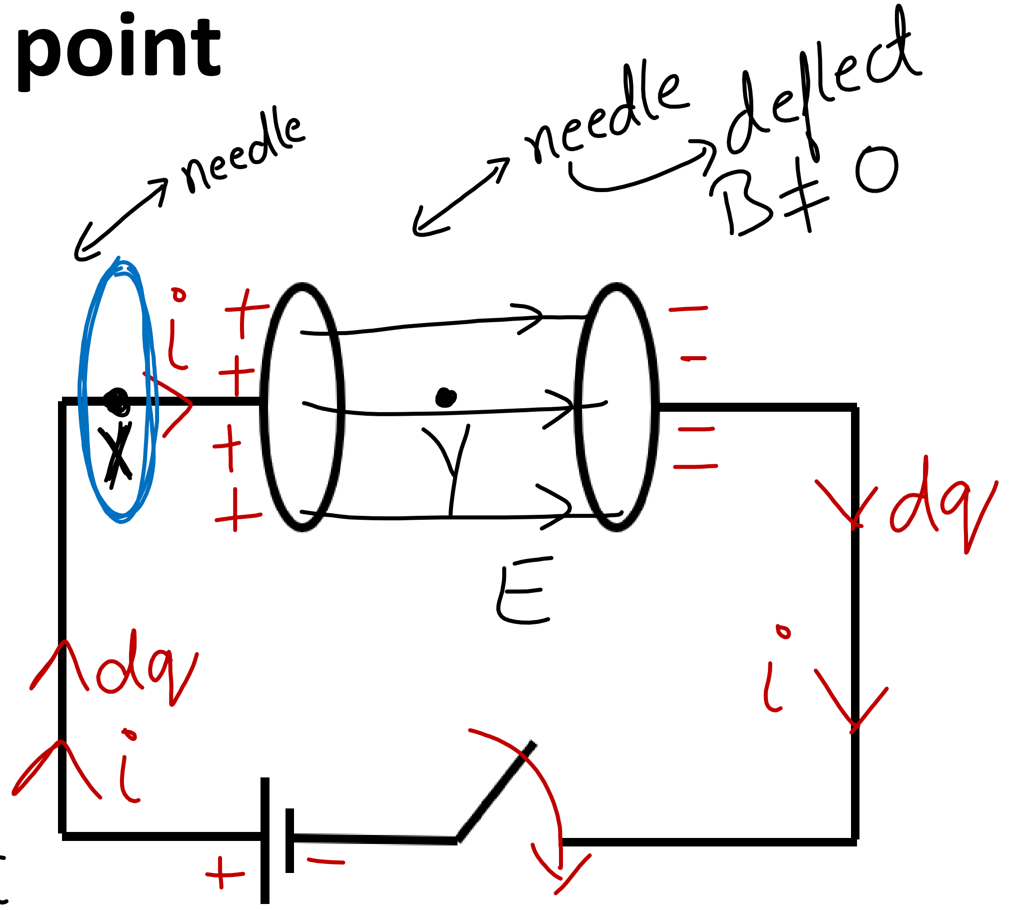
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 i$$ ✓

Y $\quad$ (Vacuum/free space)

$$\oint \vec{B}\cdot\vec{dl} = \mu_0(0)$$

$$B = 0$$

Inconsistent



Consider a Parallel Plate Capacitor being Charged by a battery

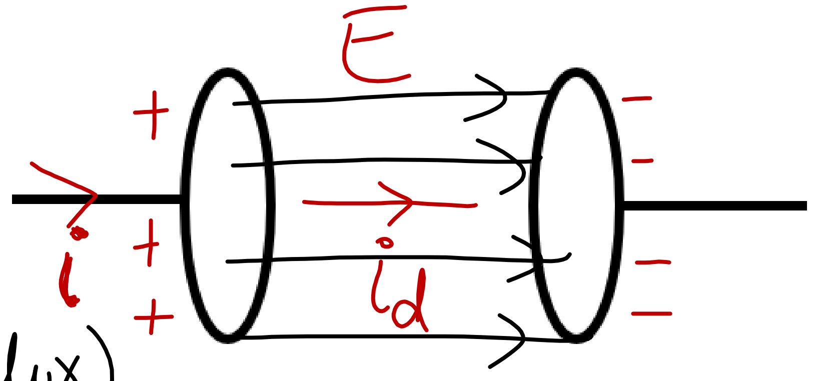
Idea of Displacement Current

- **Maxwell said that not only current produces magnetic field but a changing Electric Field in vacuum/free space also produces magnetic field.**

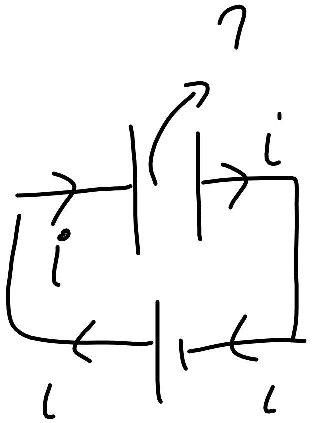
$i = i_c$
Conduction Current
$i = \frac{dq}{dt}$

Displacement Current
→ changing Electric Field (or Electric Flux)

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

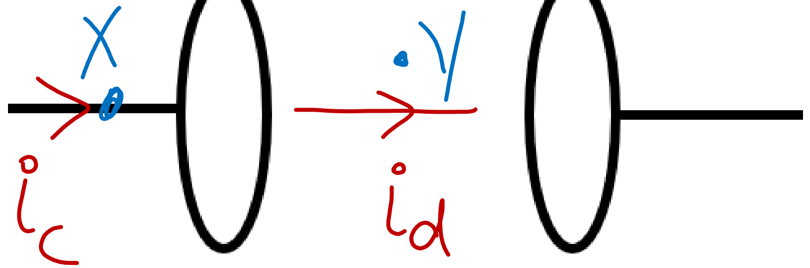


Displacement Current : A current due to changing Electric Field (or electric flux)



Conduction Current $i_c = \frac{dq}{dt}$

displacement Current $i_d = \varepsilon_0 \frac{d\phi_E}{dt}$



$B_X = B_Y$ experiments

$i_c = i_d$

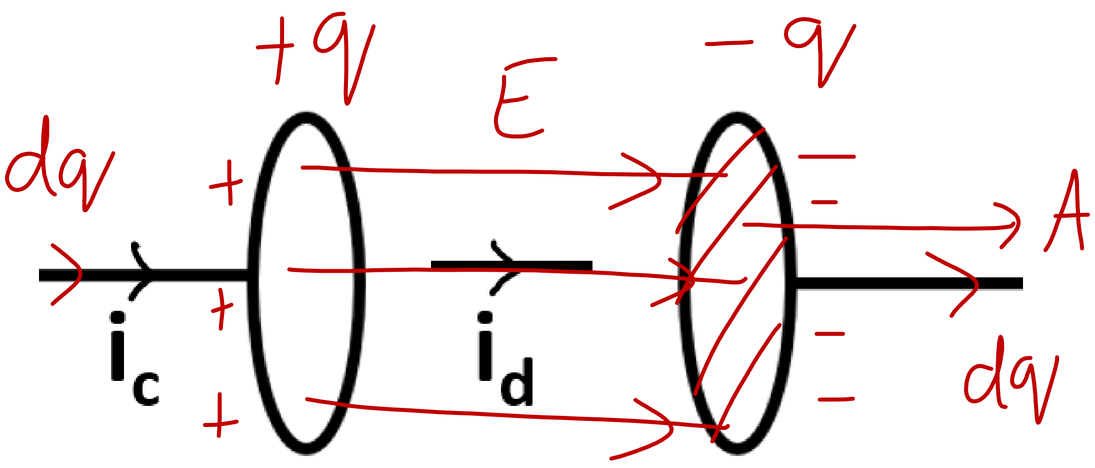
Continuity

Q1) Prove that the conduction current (i_c) and displacement current (i_d) are equal $\boxed{\text{(have property of continuity)}}$

Physics:

$$i_c = \frac{dq}{dt} \checkmark$$

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt} = \varepsilon_0 \frac{d(EA)}{dt}$$



$$\phi_E = EA\cos 0^\circ$$
$$= EA$$

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

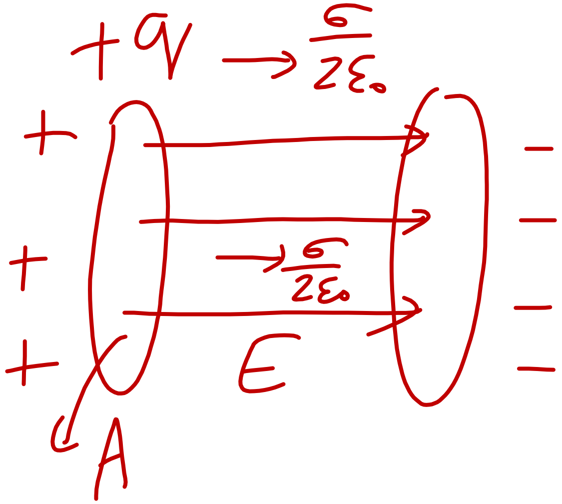
$$i_d = \varepsilon_0 \frac{d(EA)}{dt}$$

$$i_d = \varepsilon_0 \frac{d\left(\frac{q}{\cancel{A}\varepsilon_0}\cancel{A}\right)}{dt}$$

$$i_d = \frac{\cancel{\varepsilon_0}}{\cancel{\varepsilon_0}} \frac{dq}{dt}$$

$$i_d = \frac{dq}{dt}$$

$$= i_c$$



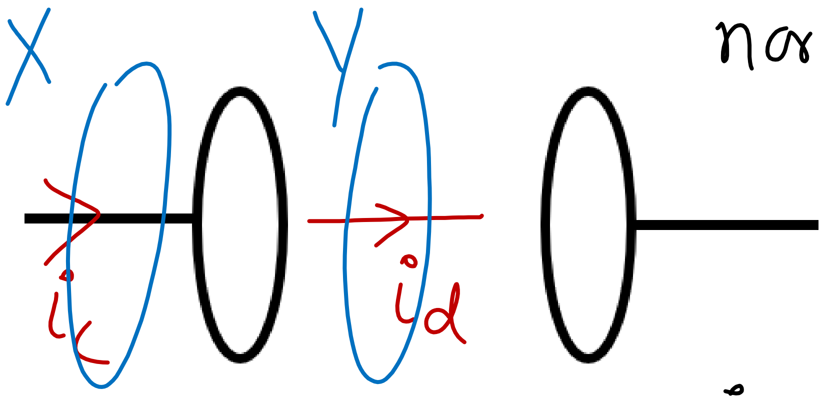
$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{q}{A\varepsilon_0}$$

Modification of Ampere's Circuital Law

$i_c = i$ normally

$\underline{\underline{X}}$ $\oint \vec{B}\cdot\vec{dl} = \mu_0 i_c$



$\underline{\underline{Y}}$ $\oint \vec{B}\cdot\vec{dl} = \mu_0 i_d$

$\underline{\underline{X}}$ $i_d = 0$

$\underline{\underline{Y}}$ $i_c = 0$

$\underline{\underline{\text{Modify}}}$ $\oint \vec{B}\cdot\vec{dl} = \mu_0 (i_c + i_d)$

Ampere Maxwell Law

$$\boxed{\oint \vec{B}\cdot\vec{dl} = \mu_0 \left(i + \varepsilon_0 \frac{d\phi_E}{dt}\right)}$$

Properties of displacement Current

$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$

1. It is not conventional current, As it produces Magnetic Field, so it is called a current.
2. It exists only when there is a change in Electric Field (electric Flux)
3. It does not exist under steady conditions
4. Together with the conduction current, displacement current satisfies the property of continuity.

Q2) A parallel plate capacitor with plate area A and separation between the plates d, is charged by a constant current i. Consider a plane surface area A/2 parallel to the plates and drawn simultaneously between the plates. The displacement current through this area is

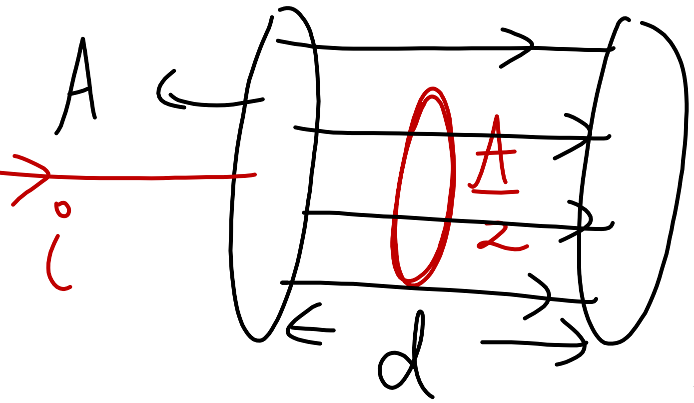
a) i

b) i/2

c) i/4

d) i/8

displacement Current is Uniformly distributed in whole area

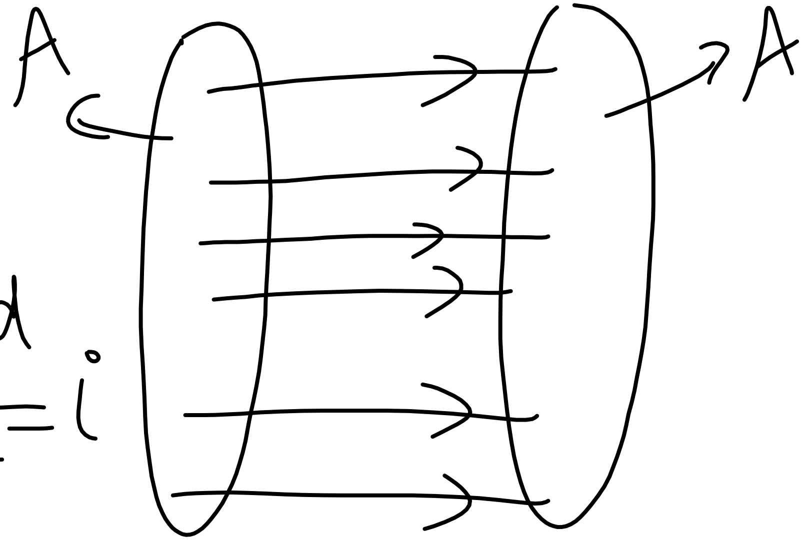


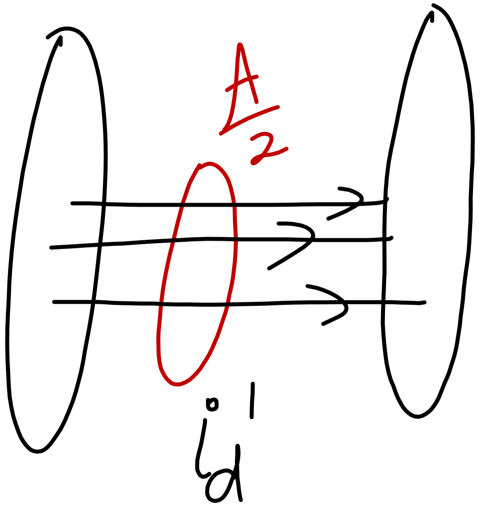
Displacement Current → uniformly distributed

J → Current density same everywhere

Total → i_d

$i_d = i_c = i$





$$J = \frac{i_d}{A} = \frac{i}{A}$$

$$J = \frac{i_d'}{\frac{A}{2}} = \frac{2i_d'}{A}$$

$$\frac{i}{\cancel{A}} = \frac{2i_d'}{\cancel{A}}$$

$$i_d' = \frac{i}{2}$$

Q3) The voltage between the plates of a parallel-plate capacitor of capacitance 1.0μF is changing at the rate of 5 V/s. What is the displacement current in the capacitor?

a) 5μA

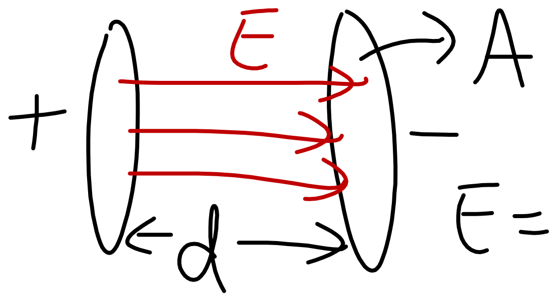
b) 10μA

c) 15μA

d) ZERO

$$i_d = \varepsilon_0 \frac{d\phi_E}{dt}$$

$$= \varepsilon_0 \frac{d(EA)}{dt}$$



$$E = \frac{V}{d}$$

$$i_d = \varepsilon_0 \frac{d\left(\frac{V}{d}A\right)}{dt}$$

$$i_d = \frac{\varepsilon_0 A}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

$$i_d = C\frac{dV}{dt}$$

$$= 1 \times 10^{-6} \times 5$$

$$= 5 \times 10^{-6}\ A$$

$$= 5\mu A$$

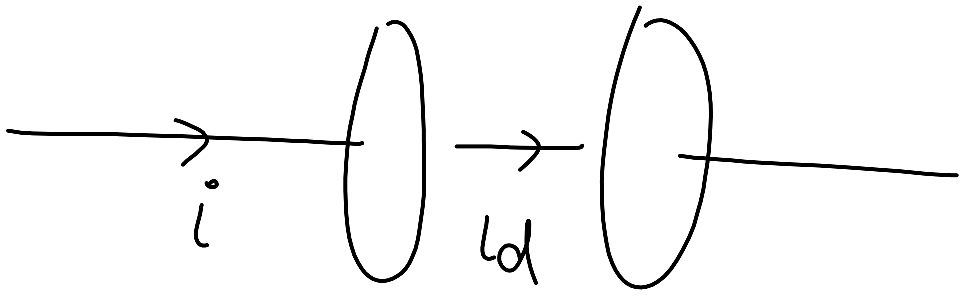
Q4) A parallel plate capacitor of Capacitance 5μF is being charged so that the current has a steady value of 100 mA. Calculate: Rate of change of Potential difference between the plates

a) $200\,\frac{KV}{s}$

b) $100\,\frac{KV}{s}$

c) $40\,\frac{KV}{s}$

d) $20\,\frac{KV}{s}$



$i_d = i = 100\,mA$

$i_d = C\frac{dV}{dt} \Rightarrow \overset{20}{\cancel{100}}\times10^{-3} = \cancel{5}\times10^{-6}\,\frac{dV}{dt}$

$20\times10^{3} = \frac{dV}{dt}$

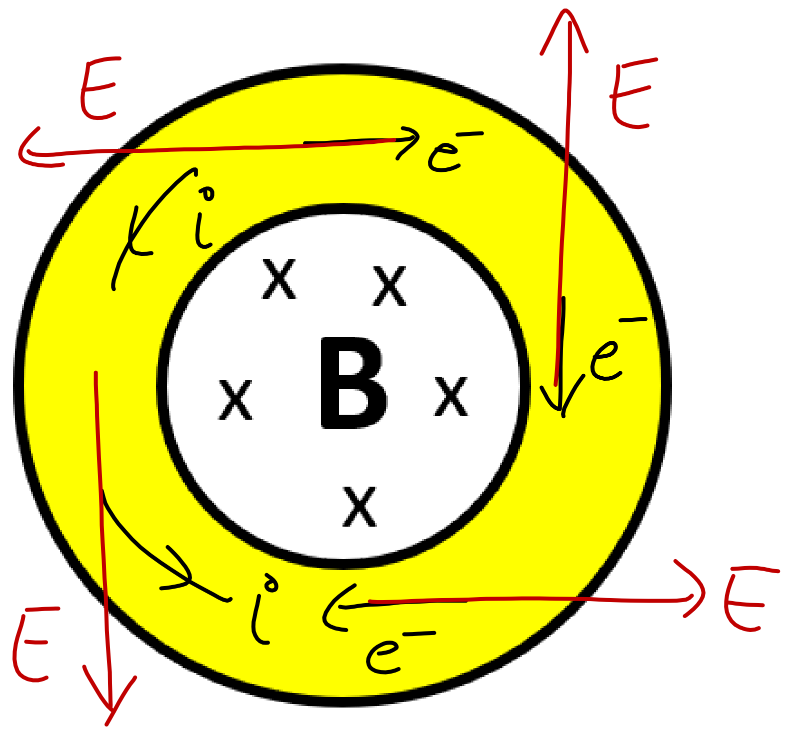
$\frac{dV}{dt} = 20\times10^{3} = 20\,\frac{KV}{s}$

06-10-2020

Important Note:

1

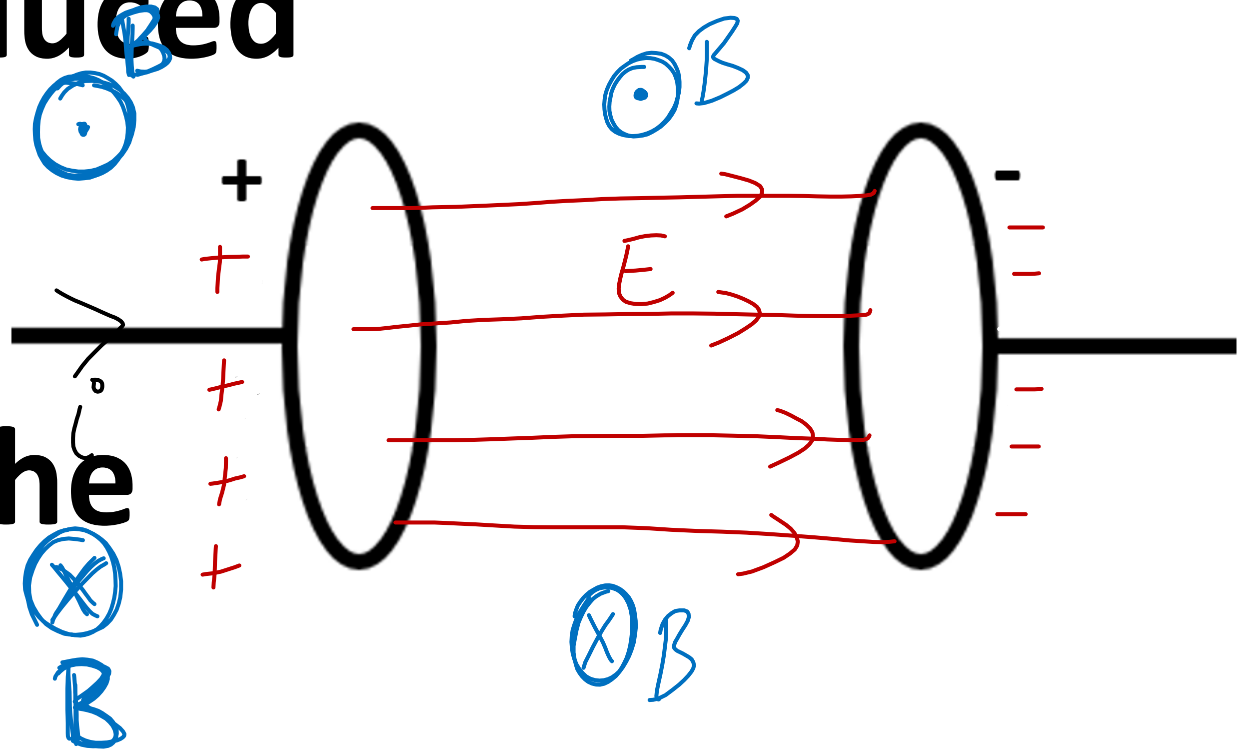
Electric Field induced due to changing Magnetic Field is Perpendicular to the Magnetic Field



B increasing

②

Magnetic Field induced due to changing Electric Field is Perpendicular to the Electric Field



Maxwell's equation

Maxwell’s Equation

- **All the basic principles of ElectroMagnetism can be explained in terms of FOUR fundamental equations called Maxwell’s Equation.**

- **Maxwell did not discovered FOUR equations, but he worked on them & stated that these FOUR fundamental equations define complete ElectroMagnetism**

1. Gauss Law of Electrostatics

Electric Flux through a closed surface is $\frac{1}{\varepsilon_0}$ times the total charge 'q' enclosed by surface

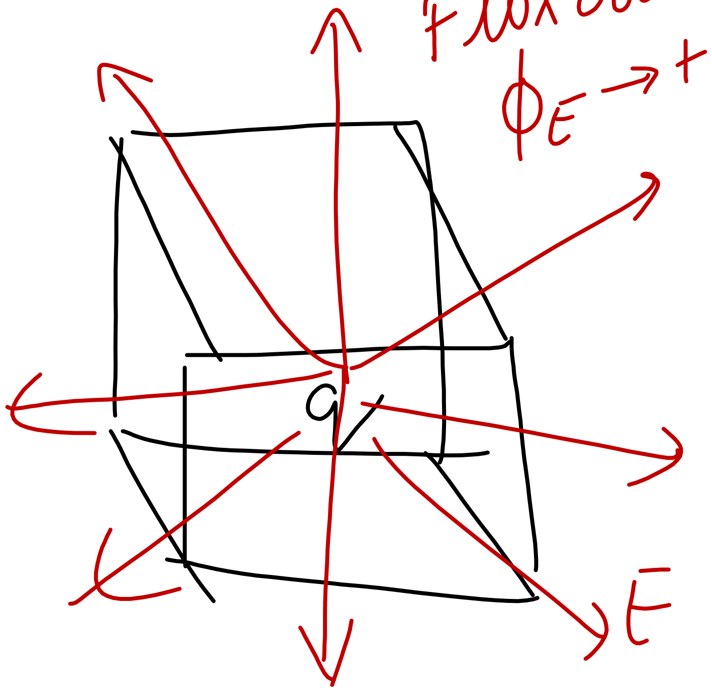
3-D

$$\phi_{E\,closed} = \frac{q_{in}}{\varepsilon_0}$$

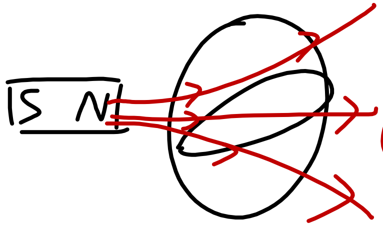
$$\boxed{\oint_{closed} \vec{E}\cdot\vec{dA} = \frac{q_{in}}{\varepsilon_0}}$$

Flux outward

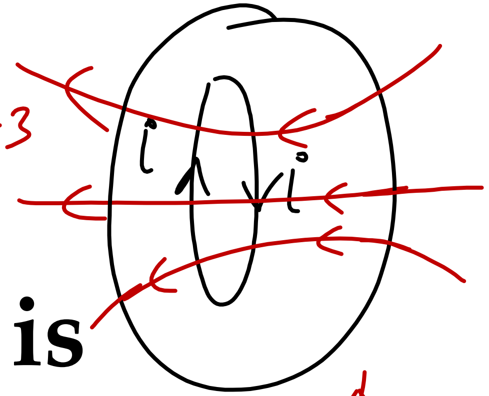
$\phi_E \rightarrow +ve$



2. Gauss Law of Magnetism



$\phi = +3$



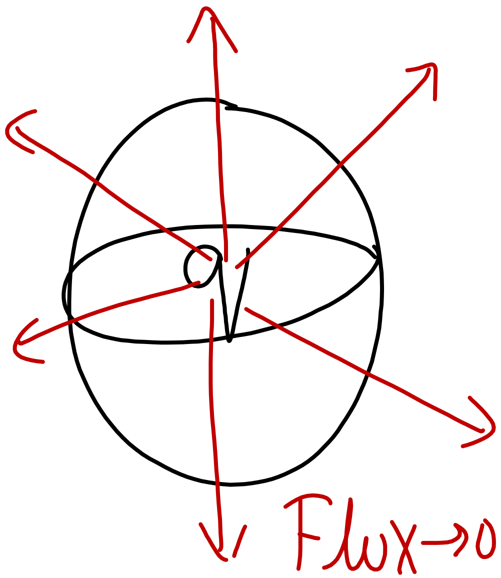
$\phi = -3$

Magnetic Flux through any closed surface is always ZERO

$\phi_B = 0$

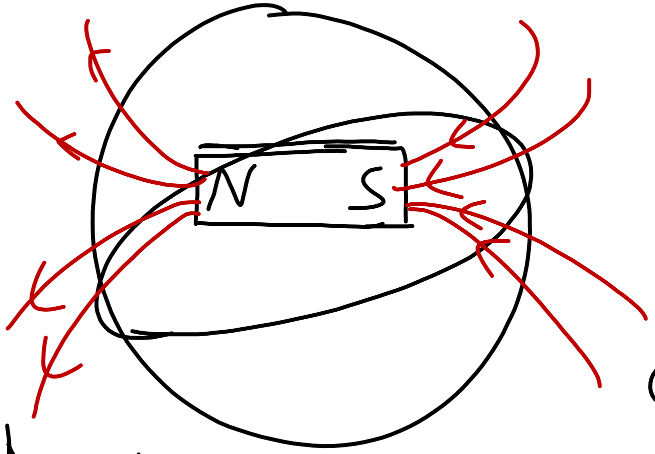
Closed

$$\boxed{\oint_{\text{Closed}} \vec{B} \cdot \vec{dA} = 0}$$



Flux→outward

$\phi_E \rightarrow +ve$



$\phi_{net} = 0$

$\phi = +4$

$\phi = -4$

$\phi_{net} = 0$

Magnetic Monopoles do not exist.

3. Faraday's Law of Electromagnetic Induction

Charge in Magnetic Flux induces an emf $e = -\frac{d\phi_B}{dt}$

OR

Imp

Changing Magnetic Field induces an Electric Field

$$\boxed{\oint \vec{E} \cdot \vec{dl} = -\frac{d\phi_B}{dt}}$$

line
Closed

4. Ampere-Maxwell Law

Changing Electric Field induces a Magnetic Field

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 (i_c + i_d)$$

line Closed

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 \left(i + \varepsilon_0 \frac{d\phi_E}{dt} \right)}$$

Maxwell's FOUR Fundamental Equation

① $$\oint \vec{E}\,\vec{dA} = \frac{q_{in}}{\varepsilon_0}$$

Closed surface

② $$\oint \vec{B}\cdot\vec{dA} = 0$$

Closed Surface

③ $$\oint \vec{E}\cdot\vec{dl} = -\frac{d\phi_B}{dt}$$

line closed

④ $$\oint \vec{B}\,\vec{dl} = \mu_0\left(i + \varepsilon_0 \frac{d\phi_E}{dt}\right)$$

Thank You

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