
UNITS AND DIMENSIONS

PHYSICAL QUANTITIES

All quantities that can be measured are called physical quantities. eg. time, length, mass, force, work done, etc. In physics we study about physical quantities and their inter relationships.

MEASUREMENT

Measurement is the comparison of a quantity with a standard of the same physical quantity.

UNITS

All physical quantities are measured w.r.t. standard magnitude of the same physical quantity and these standards are called UNITS. eg. second, meter, kilogram, etc.

So the four basic properties of units are:—

1. They must be well defined.
2. They should be easily available and reproducible.
3. They should be invariable e.g. step as a unit of length is not invariable.
4. They should be accepted to all.

SET OF FUNDAMENTAL QUANTITIES

A set of physical quantities which are completely independent of each other and all other physical quantities can be expressed in terms of these physical quantities is called Set of Fundamental Quantities.

Physical Quantity	Units(SI)	Units(CGS)	Notations
Mass	kg (kilogram)	g	M
Length	m (meter)	cm	L
Time	s (second)	s	T
Temperature	K (kelvin)	°C	θ
Current	A (ampere)	A	I or A
Luminous intensity	cd (candela)	—	cd
Amount of substance	mol	—	mol

Physical Quantity (SI Unit)

Definition

Length (m)

The distance travelled by light in vacuum in $\frac{1}{299,792,458}$ second is called 1 metre.

Mass (kg)

The mass of a cylinder made of platinum-iridium alloy kept at International Bureau of Weights and Measures is defined as 1 kilogram.

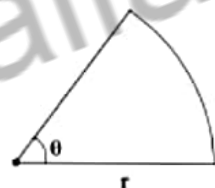
Time (s)

The second is the duration of 9,192,631,770 periods of

Electric Current (A)	<p>the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom.</p> <p>If equal currents are maintained in the two parallel infinitely long wires of negligible cross-section, so that the force between them is 2×10^{-7} newton per metre of the wires, the current in any of the wires is called 1 Ampere.</p>
Thermodynamic Temperature (K)	<p>The fraction $\frac{1}{273.16}$ of the thermodynamic temperature of triple point of water is called 1 Kelvin</p>
Luminous Intensity (cd)	<p>1 candela is the luminous intensity of a blackbody of surface area $\frac{1}{600,000} \text{ m}^2$ placed at the temperature of freezing platinum and at a pressure of $101,325 \text{ N/m}^2$, in the direction perpendicular to its surface.</p>
Amount of substance (mole)	<p>The mole is the amount of a substance that contains as many elementary entities as there are number of atoms in 0.012 kg of carbon-12.</p>

There are two supplementary units too:

1. Plane angle (radian) angle = arc / radius
 $\theta = \ell / r$
2. Solid Angle (steradian)



DERIVED PHYSICAL QUANTITIES

The physical quantities those can be expressed in terms of fundamental physical quantities are called derived physical quantities. eg. speed = distance/time.

DIMENSIONS AND DIMENSIONAL FORMULA

All the physical quantities of interest can be derived from the base quantities.

DIMENSION

The power (exponent) of base quantity that enters into the expression of a physical quantity, is called the dimension of the quantity in that base.

To make it clear, consider the physical quantity "force".

Force = mass \times acceleration

$$= \text{mass} \times \frac{\text{length} / \text{time}}{\text{time}}$$

$$= \text{mass} \times \text{length} \times (\text{time})^{-2}$$

So the dimensions of force are 1 in mass, 1 in length and -2 in time. Thus

$$[\text{Force}] = \text{MLT}^{-2}$$

Similarly energy has dimensional formula given by

$$[\text{Energy}] = ML^2T^{-2}$$

i.e. energy has dimensions, 1 in mass, 2 in length and -2 in time.

Such an expression for a physical quantity in terms of base quantities is called dimensional formula.

DIMENSIONAL EQUATION

Whenever the dimension of a physical quantity is equated with its dimensional formula, we get a dimensional equation.

PRINCIPLE OF HOMOGENEITY

According to this principle, we can multiply physical quantities with same or different dimensional formulae at our convenience, however no such rule applies to addition and subtraction, where only like physical quantities can only be added or subtracted. e.g. If $P + Q \Rightarrow P$ & Q both represent same physical quantity.

Illustration :

Calculate the dimensional formula of energy from the equation $E = \frac{1}{2}mv^2$.

Sol. Dimensionally, $E = \text{mass} \times (\text{velocity})^2$.

Since $\frac{1}{2}$ is a number and has no dimension.

$$\text{or, } [E] = M \times \left(\frac{L}{T}\right)^2 = ML^2T^{-2}.$$

Illustration :

Kinetic energy of a particle moving along elliptical trajectory is given by $K = \alpha s^2$ where s is the distance travelled by the particle. Determine dimensions of α .

Sol. $K = \alpha s^2$

$$[\alpha] = \frac{(ML^2T^{-2})}{(L^2)}$$

$$[\alpha] = M^1 L^0 T^{-2}$$

$$[\alpha] = (M T^{-2})$$

Illustration :

The position of a particle at time t , is given by the equation, $x(t) = \frac{v_0}{\alpha}(1 - e^{-\alpha t})$, where v_0 is a constant and $\alpha > 0$. The dimensions of v_0 & α are respectively.

(A) $M^0 L^1 T^0$ & T^{-1}

(B) $M^0 L^1 T^{-1}$ & T

(C*) $M^0 L^1 T^1$ & T^1

(D) $M^1 L^1 T^1$ & LT^{-2}

Sol. $[V_0] = [x] [\alpha]$ & $[\alpha] [t] = M^0 L^0 T^0$
 $= M^0 L^1 T^{-1}$ $[\alpha] = M^0 L^0 T^{-1}$

Illustration :

The distance covered by a particle in time t is going by $x = a + bt + ct^2 + dt^3$; find the dimensions of a , b , c and d .

Sol. The equation contains five terms. All of them should have the same dimensions. Since $[x] = \text{length}$, each of the remaining four must have the dimension of length.

Thus, $[a] = \text{length} = L$

$$[bt] = L, \quad \text{or} \quad [b] = LT^{-1}$$

$$[ct^2] = L, \quad \text{or} \quad [c] = LT^{-2}$$

$$\text{and} \quad [dt^3] = L \quad \text{or} \quad [d] = LT^{-3}$$

USES OF DIMENSIONAL ANALYSIS

(I) TO CONVERT UNITS OF A PHYSICAL QUANTITY FROM ONE SYSTEM OF UNITS TO ANOTHER :

It is based on the fact that,

$$\text{Numerical value} \times \text{unit} = \text{constant}$$

So on changing unit, numerical value will also get changed. If n_1 and n_2 are the numerical values of a given physical quantity and u_1 and u_2 be the units respectively in two different systems of units, then

$$n_1 u_1 = n_2 u_2$$

$$n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$$

Illustration

Young's modulus of steel is $19 \times 10^{10} \text{ N/m}^2$. Express it in dyne/cm^2 . Here dyne is the CGS unit of force.

Sol. The unit of Young's modulus is N/m^2 .

This suggests that it has dimensions of $\frac{\text{Force}}{(\text{distance})^2}$.

$$\text{Thus, } [Y] = \frac{[F]}{L^2} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$$

N/m^2 is in SI units,

$$\text{So, } 1 \text{ N/m}^2 = (1 \text{ kg})(1 \text{ m})^{-1} (1 \text{ s})^{-2}$$

$$\text{and } 1 \text{ dyne/cm}^2 = (1 \text{ g})(1 \text{ cm})^{-1} (1 \text{ s})^{-2}$$

$$\text{so, } \frac{1 \text{ N/m}^2}{1 \text{ dyne/cm}^2} = \left(\frac{1 \text{ kg}}{1 \text{ g}} \right) \left(\frac{1 \text{ m}}{1 \text{ cm}} \right)^{-1} \left(\frac{1 \text{ s}}{1 \text{ s}} \right)^{-2} = 1000 \times \frac{1}{100} \times 1 = 10$$

$$\text{or, } 1 \text{ N/m}^2 = 10 \text{ dyne/cm}^2$$

$$\text{or, } 19 \times 10^{10} \text{ N/m}^2 = 19 \times 10^{11} \text{ dyne/m}^2.$$

Illustration :

The dimensional formula for viscosity of fluids is,

$$\eta = M^1 L^{-1} T^{-1}$$

Find how many poise (CGS unit of viscosity) is equal to 1 poiseuille (SI unit of viscosity).

Sol. $\eta = M^1 L^{-1} T^{-1}$

1 CGS units = $g\ cm^{-1}\ s^{-1}$

1 SI units = $kg\ m^{-1}\ s^{-1}$

= $1000\ g\ (100\ cm)^{-1}\ s^{-1}$

= $10\ g\ cm^{-1}\ s^{-1}$

Thus, 1 Poiseuille = 10 poise

Illustration :

A calorie is a unit of heat or energy and it equals about 4.2 J, where $1\ J = 1\ kg\ m^2/s^2$. Suppose we employ a system of units in which the unit of mass equals $\alpha\ kg$, the unit of length equals β metre, the unit of time is γ second. Show that a calorie has a magnitude $4.2\ \alpha^{-1}\beta^{-2}\gamma^2$ in terms of the new units.

Sol. $1\ cal = 4.2\ kg\ m^2s^{-2}$

SI

$n_1 = 4.2$

$M_1 = 1\ kg$

$L_1 = 1\ m$

$T_1 = 1\ s$

New system

$n_2 = ?$

$M_2 = \alpha\ kg$

$L_2 = \beta\ metre$

$T_2 = \gamma\ second$

Dimensional formula of energy is $[ML^2T^{-2}]$

Comparing with $[M^aL^bT^c]$, we find that $a = 1, b = 2, c = -2$

Now, $n_2 = n_1 \left[\frac{M_1}{M_2} \right]^a \left[\frac{L_1}{L_2} \right]^b \left[\frac{T_1}{T_2} \right]^c$

= $4.2 \left[\frac{1\ kg}{\alpha\ kg} \right]^1 \left[\frac{1\ m}{\beta\ m} \right]^2 \left[\frac{1\ s}{\gamma\ s} \right]^{-2} = 4.2\ \alpha^{-1}\beta^{-2}\gamma^2$

(II) TO CHECK THE DIMENSIONAL CORRECTNESS OF A GIVEN PHYSICAL RELATION:

It is based on principle of homogeneity, which states that a given physical relation is dimensionally correct if the dimensions of the various terms on either side of the relation are the same.

(i) Powers are dimensionless

(ii) $\sin\theta, e^\theta, \cos\theta, \log\theta$ gives dimensionless value and in above expression θ is dimensionless

(iii) We can add or subtract quantity having same dimensions.

Illustration :

Let us check the dimensional correctness of the relation $v = u + at$.

Here 'u' represents the initial velocity, 'v' represents the final velocity, 'a' the uniform acceleration and 't' the time.

Dimensional formula of 'u' is $[M^0LT^{-1}]$

Dimensional formula of 'v' is $[M^0LT^{-1}]$

Dimensional formula of 'at' is $[M^0LT^{-2}][T] = [M^0LT^{-1}]$

Here dimensions of every term in the given physical relation are the same, hence the given physical relation is dimensionally correct.

Illustration :

Let us check the dimensional correctness of the relation

$$x = ut + \frac{1}{2}at^2$$

Here 'u' represents the initial velocity, 'a' the uniform acceleration, 'x' the displacement and 't' the time.

Sol.

$$[x] = L$$

$$[ut] = \text{velocity} \times \text{time} = \frac{\text{length}}{\text{time}} \times \text{time} = L$$

$$\left[\frac{1}{2}at^2\right] = [at^2] = \text{acceleration} \times (\text{time})^2$$

$$\left(\because \frac{1}{2} \text{ is a number hence dimensionless}\right)$$

$$= \frac{\text{velocity}}{\text{time}} \times (\text{time})^2 = \frac{\text{length/time}}{\text{time}} \times (\text{time})^2 = L$$

Thus, the equation is correct as far as the dimensions are concerned.

(III) TO ESTABLISH A RELATION BETWEEN DIFFERENT PHYSICAL QUANTITIES :

If we know the various factors on which a physical quantity depends, then we can find a relation among different factors by using principle of homogeneity.

Illustration :

Let us find an expression for the time period t of a simple pendulum. The time period t may depend upon (i) mass m of the bob of the pendulum, (ii) length ℓ of pendulum, (iii) acceleration due to gravity g at the place where the pendulum is suspended.

Sol. Let (i) $t \propto m^a$ (ii) $t \propto \ell^b$ (iii) $t \propto g^c$

Combining all the three factors, we get

$$t \propto m^a \ell^b g^c \quad \text{or} \quad t = Km^a \ell^b g^c$$

where K is a dimensionless constant of proportionality.

Writing down the dimensions on either side of equation (i), we get

$$[T] = [M^a][L^b][LT^{-2}]^c = [M^a L^{b+c} T^{-2c}]$$

Comparing dimensions, $a = 0$, $b + c = 0$, $-2c = 1$

$$\therefore a = 0, c = -1/2, b = 1/2$$

$$\text{From equation (i) } t = Km^0 \ell^{1/2} g^{-1/2} \quad \text{or} \quad t = K \left(\frac{\ell}{g}\right)^{1/2} = K \sqrt{\frac{\ell}{g}}$$

Illustration :

When a solid sphere moves through a liquid, the liquid opposes the motion with a force F . The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.

Sol. Suppose the formula is $F = k \eta^a r^b v^c$

$$\begin{aligned} \text{Then, } MLT^{-2} &= [ML^{-1} T^{-1}]^a L^b \left(\frac{L}{T}\right)^c \\ &= M^a L^{-a+b+c} T^{-a-c} \end{aligned}$$

Equating the exponents of M, L and T from both sides,

$$\begin{aligned} a &= 1 \\ -a + b + c &= 1 \\ -a - c &= -2 \end{aligned}$$

Solving these, $a = 1$, $b = 1$ and $c = 1$

Thus, the formula for F is $F = k\eta r v$.

Illustration :

If P is the pressure of a gas and ρ is its density, then find the dimension of velocity in terms of P and ρ .

(A) $P^{1/2} \rho^{-1/2}$ (B) $P^{1/2} \rho^{1/2}$ (C) $P^{-1/2} \rho^{1/2}$ (D) $P^{-1/2} \rho^{-1/2}$

[Sol. $v \propto P^a \rho^b$

$$v = k P^a \rho^b$$

$$[LT^{-1}] = [ML^{-1}T^{-2}]^a [ML^{-3}]^b \quad (\text{Comparing dimensions})$$

$$a = \frac{1}{2}, \quad b = -\frac{1}{2} \quad \Rightarrow \quad [V] = [P^{1/2} \rho^{-1/2}]$$

UNITS AND DIMENSIONS OF SOME PHYSICAL QUANTITIES

Quantity	SI Unit	Dimensional Formula
Density	kg/m ³	M/L ³
Force	Newton (N)	ML/T ²
Work	Joule (J)(=N-m)	ML ² /T ²
Energy	Joule(J)	ML ² /T ²
Power	Watt (W) (=J/s)	ML ² /T ³
Momentum	kg-m/s	ML/T
Gravitational constant	N-m ² /kg ²	L ³ /MT ²
Angular velocity	radian/s	T ⁻¹
Angular acceleration	radian/s ²	T ⁻²
Angular momentum	kg-m ² /s	ML ² /T
Moment of inertia	kg-m ²	ML ²
Torque	N-m	ML ² /T ²
Angular frequency	radian/s	T ⁻¹
Frequency	Hertz (Hz)	T ⁻¹
Period	s	T
Surface Tension	N/m	M/T ²
Coefficient of viscosity	N-s/m ²	M/LT
Wavelength	m	L
Intensity of wave	W/m ²	M/T ³

Temperature	kelvin (K)	K
Specific heat capacity	J/(kg-K)	L^2/T^2K
Stefan's constant	$W/(m^2-K^4)$	M/T^3K^4
Heat	J	ML^2/T^2
Thermal conductivity	$W/(m-K)$	ML/T^3K
Current density	A/m^2	I/L^2
Electrical conductivity	$1/\Omega\text{-m}(= \text{mho/m})$	I^2T^3/ML^3
Electric dipole moment	C-m	LIT
Electric field	$V/m(=N/C)$	ML/IT^3
Potential (voltage)	volt (V) ($=J/C$)	ML^2/IT^3
Electric flux	V-m	ML^3/IT^3
Capacitance	farad (F)	I^2T^4/ML^2
Electromotive force	volt (V)	ML^2/IT^3
Resistance	ohm (Ω)	ML^2/I^2T^3
Permittivity of space	$C^2/N\text{-m}^2(=F/m)$	I^2T^4/ML^3
Permeability of space	N/A^2	ML/I^2T^2
Magnetic field	Tesla (T) ($=Wb/m^2$)	M/IT^2
Magnetic flux	Weber (Wb)	ML^2/IT^2
Magnetic dipole moment	N-m/T	IL^2
Inductance	Henry (H)	ML^2/I^2T^2

LIMITATIONS OF DIMENSIONAL ANALYSIS

- (i) Dimension does not depend on the magnitude. Due to this reason the equation $x = ut + at^2$ is also dimensionally correct. Thus, a dimensionally correct equation need not be actually correct.
- (ii) The numerical constants having no dimensions cannot be deduced by the method of dimensions.
- (iii) This method is applicable only if relation is of product type. It fails in the case of exponential and trigonometric relations.

SI Prefixes : The magnitudes of physical quantities vary over a wide range. The mass of an electron is 9.1×10^{-31} kg and that of our earth is about 6×10^{24} kg. Standard prefixes for certain powers of 10.

Table shows these prefixes :

Power of 10	Prefix	Symbol
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deka	da
-1	deci	d

-2	centi	c
-3	milli	m
-6	micro	μ
-9	nano	n
-12	pico	p
-15	fernto	f

ORDER-OF MAGNITUDE CALCULATIONS

If value of phycal quantity P satisfy

$$0.5 \times 10^x < P \leq 5 \times 10^x$$

x is an integer

x is called order of magnitude

Illustration :

The diameter of the sun is expressed as 13.9×10^9 m. Find the order of magnitude of the diameter ?

Sol. Diameter = 13.9×10^9 m

$$\text{Diameter} = 1.39 \times 10^{10} \text{ m}$$

order of magnitude is 10.

SYMBOLS AND THERE USUAL MEANINGS

The scientific group in Greece used following symbols.

θ	Theta
α	Alpha
β	Beta
γ	Gamma
δ	Delta
Δ	Delta
μ	Mu
λ	Lambda
ω, Ω	Omega
π	Pi
ϕ, φ	Phi
ϵ	epsilon

ψ	Psi
ρ	Roh
ν	Nu
η	Eta
σ	Sigma
τ	Tau
κ	Kappa
χ	chi
\cong	Approximately equal to

On solving $n = \frac{1}{5}, m = \frac{3}{5}$

Q.3 If energy (E), velocity (V) and time (T) are chosen as the fundamental quantities, then the dimensions of surface tension will be. (Surface tension = force / length)

- (A) $E V^{-2} T^{-1}$ (B) $E V^{-1} T^{-2}$ (C) $E^{-2} V^{-1} T^{-3}$ (D*) $E V^{-2} T^{-2}$

Sol. [surface tension] = [force/length] = $M^1 L^0 T^{-2}$

suppose [surface tension] = $E^a V^b T^c$

$\therefore M^1 L^0 T^{-2} = [M^1 L^2 T^{-2}]^a [L^1 T^{-1}]^b [T]^c$

Matching dimensions of M $\Rightarrow a = 1$

Matching dimensions of L $\Rightarrow 2a + b = 0 \Rightarrow b = -2$

Matching dimensions of T $\Rightarrow -2a - b + c = -2 \Rightarrow c = -2$

\therefore [surface tension] = $E V^{-2} T^{-2}$

Q.4 Given that $\ln(\alpha/p\beta) = \alpha z/K_B \theta$ where p is pressure, z is distance, K_B is Boltzmann constant and θ is temperature, the dimensions of β are (useful formula Energy = $K_B \times$ temperature)

- (A) $L^0 M^0 T^0$ (B) $L^1 M^{-1} T^2$ (C*) $L^2 M^0 T^0$ (D) $L^{-1} M^1 T^{-2}$

Sol. $\ln\left(\frac{\alpha}{p\beta}\right) = \frac{\alpha z}{k_B \theta}$

$[\alpha z] = [k_B \theta]$ Also $[\alpha] = [p\beta]$

$[p \beta z] = [k_B \theta]$

$[\beta] = \frac{(k_B \theta)}{(p z)} = \frac{ML^2 T^{-2} K^{-1} K}{ML^{-1} T^{-2} L} = L^2$

Q.5 The SI and CGS units of energy are joule and erg respectively. How many ergs are equal to one joule ?

Sol. Dimensionally, Energy = mass \times (velocity)²

$$= \text{mass} \times \left(\frac{\text{length}}{\text{time}}\right)^2 = ML^2 T^{-2}$$

Thus, 1 joule = (1 kg) (1 m)² (1 s)⁻²

and 1 erg = (1 g) (1 cm)² (1 s)⁻²

$$\frac{1 \text{ joule}}{1 \text{ erg}} = \left(\frac{1 \text{ kg}}{1 \text{ g}}\right) \left(\frac{1 \text{ m}}{1 \text{ cm}}\right)^2 \left(\frac{1 \text{ s}}{1 \text{ s}}\right)^{-2}$$

$$= \left(\frac{1000 \text{ g}}{1 \text{ g}}\right) \left(\frac{1000 \text{ cm}}{1 \text{ cm}}\right)^2 = 1000 \times 10000 = 10^7.$$

So 1 joule = 10^7 erg.

- Q.6 Young's modulus of steel is $19 \times 10^{10} \text{ N/m}^2$. Express it in dyne/cm^2 . Here dyne is the CGS unit of force.
 Sol. The unit of Young's modulus is N/m^2 .

This suggests that it has dimensions of $\frac{\text{Force}}{(\text{area})}$.

$$\text{Thus, } [Y] = \left[\frac{F}{L^2} \right] = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$$

N/s^2 is in SI units.

$$\text{So, } 1 \text{ N/m}^2 = (1 \text{ kg}) (1 \text{ m})^{-1} (1 \text{ s})^{-2}$$

$$\text{and } 1 \text{ dyne/cm}^2 = (1 \text{ g}) (1 \text{ cm})^{-1} (1 \text{ s})^{-2}$$

$$\text{So, } \frac{1 \text{ N/m}^2}{1 \text{ dyne/cm}^2} = \left(\frac{1 \text{ kg}}{1 \text{ g}} \right) = \left(\frac{1 \text{ m}}{1 \text{ cm}} \right)^{-1} \left(\frac{1 \text{ s}}{1 \text{ s}} \right)^{-2}$$

$$= 1000 \times \frac{1}{100} \times 1 = 10$$

$$\text{or, } 1 \text{ N/s}^2 = 10 \text{ dyne/cm}^2$$

$$\text{or, } 19 \times 10^{10} \text{ N/m}^2 = 19 \times 10^{11} \text{ dyne/cm}^2$$

- Q.7 If velocity, time and force were chosen as basic quantities, find the dimensions of mass.

Sol. Dimensionally, Force = mass \times acceleration

$$\text{Force} = \text{mass} \times \frac{\text{velocity}}{\text{time}}$$

$$\text{or, } \text{mass} = \frac{\text{Force} \times \text{time}}{\text{velocity}}$$

$$\text{or, } [\text{mass}] = \text{FTV}^{-1}$$

- Q.8 The dimension of $\frac{a}{b}$ in the equation $P = \frac{a - t^2}{bx}$ where P is pressure, x is distance and t is time are _____?

$$\text{Sol. } P = \frac{a - t^2}{bx}$$

$$\Rightarrow Pbx = a - t^2$$

$$\Rightarrow [Pbx] = [a] = [T^2]$$

$$\text{or } [b] = \frac{[T^2]}{[P][x]} = \frac{[T^2]}{[ML^{-1}T^{-2}][L]} = [M^{-1}T^4]$$

$$\therefore \left[\frac{a}{b} \right] = \frac{[T^2]}{[M^{-1}T^4]} = [MT^{-2}]$$