KINEMATICS

Kine ⇒ Motion, Matics ⇒ Maths.

⇒ MOTION IN ONE DIMENSION (linear or rectilinear).

VARIABLE
* Distance
* Displacement
* Velocity

* Acceleration
* Speed
* Jerk.

DISTANCE AND DISPLACEMENT

* Distance

⇒ It is a scalar quantity

Path 1, Path 2, Path 3 are the distance between A and B.
Displacement.

- It is a vector quantity.

Path is the displacement. (It is the shortest distance from initial position to final position.)

Relation between distance and displacement.

1. Distance > displacement
2. Distance = displacement
3. Distance < displacement
4. Distance < displacement

Quest: Find the ratio of distance to displacement in the following diagram. (The given diagram is a semi-circle)

\[
\text{Ratio of distance to displacement} = \frac{\pi R}{2R} = \frac{\pi}{2}
\]
Cube

Question: Consider a cube, where distance of a bee is shown in figure. Find the shortest distance between A and B.

\[ B(a_1 + a_2 + a_3) \]

\[ A(0,0,0) \]

\[ (a,0,0) \]

Shortest distance of a cube is \( AB \) (body diagonal).

Shortest distance \( AB = a_1^2 + a_2^2 - a_3^2 \)

\[ |AB| = \sqrt{a_1^2 + a_2^2 + a_3^2} \]

\[ |AB| = \sqrt{3} a \]

|AB| = \sqrt{3} \ a

Question: A particle crosses \( \sqrt{3} \) of cube, find the displacement of particle.
\[ (AB)^2 = (OA)^2 + (OB)^2 \]
\[ (AB)^2 = R^2 + R^2 \]
\[ (AB)^3 = 2R^2 \]

\[ AB = \sqrt{2}R \]

**Question:** Find the displacement of the particle if it makes 120° angle with its initial position.

We know that

\[ R_1 + d = R_2 \]
\[ d = R_2 - R_1 \]

\[ d_1 = \sqrt{R_1^2 + R_2^2 - 2R_1R_2 \cos 120°} \]
\[ |d_1| = \sqrt{R_1^2 + R_2^2 - 2R_1R_2 \cos 120°} \]
\[ d = \sqrt{2R^2\left(1 - \cos \theta\right)} \]
\[ d = \sqrt{2R^2 \cdot 2 \sin^2 \theta / 2} \]
\[ d = 2R \sin \theta / 2 \]

**Question:** A wheel of radius \( R \) is rolling on a road. What will be the displacement of a point \( P \) in half a revolution of wheel on road.

\[ PP' = \sqrt{(2R)^2 + (\pi R)^2} \]
\[ PP' = \sqrt{4R^2 + \pi^2 R^2} \]
\[ PP' = R \left( \sqrt{4 + \pi^2} \right) \]

**Direction:**
\[ \tan \theta = \frac{P}{B} = \frac{2R}{\pi R} = \frac{2}{\pi} \]
**SPEED AND VELOCITY**

- **SPEED**

  \[
  \text{speed} = \frac{\text{distance}}{\text{time}}
  \]

  Speed is a scalar quantity.

  - **Average speed**
    - \[ \text{Average speed} = \frac{\text{Total distance}}{\text{Total time}} \]

  । Average speed is for large time (बड़े समय के लिए) \((\Delta t)\)

- **VELOCITY**

  \[
  \text{Velocity} = \frac{\text{Displacement}}{\text{Time}}
  \]

  Velocity is a vector quantity.

  - **Average velocity** \((\Delta t)\)
  - **Instantaneous velocity** (At particular instant \((dt)\))
Relation between average speed and average velocity.

Average speed ≥ average velocity.

Quest: A boy walks 3 m east in 2 sec then 2 m west in 3 sec. Find his average speed and average velocity.

\[\text{Average speed} = \frac{\text{Total Distance}}{\text{Total Time}}\]

\[= \frac{3 + 2}{5} = 1 \text{ m/sec.}\]

\[\text{Average velocity} = \frac{\text{Total Displacement}}{\text{Total Time}}\]

\[= \frac{1}{5} = 0.2 \text{ m/sec}\]

Quest: A man goes from A to B at \(v\) speed and returns from B to A at \(\frac{v}{2}\) speed. Find

(1) Average velocity.
(2) Average speed.
Ans (i) Average velocity = \[ \frac{0}{t} = 0 \text{ m/sec} \]

(ii) Average speed = \[ \frac{\text{total distance}}{\text{total time}} \]

\[ t_1 = \frac{d}{v_1} \]
\[ t_2 = \frac{d}{v_2} \]

\[ \text{av. speed} = \frac{2d}{\frac{d}{v_1} + \frac{d}{v_2}} \]

\[ \text{av. speed} = \frac{2d}{\frac{d}{v_1} + \frac{d}{v_2}} \]

\[ \text{av. speed} = \frac{2d}{\frac{dv_2 + dv_1}{v_1 v_2}} \]

\[ \text{av. speed} = \frac{2d}{\frac{d(v_1 + v_2)}{v_1 v_2}} \]

\[ \text{av. speed} = \frac{2v_1 v_2}{v_1 + v_2} \]

Quest - A man covers half distance at \( v_1 \) speed and half at \( v_2 \) speed. Find average speed.

\[ t_1 = \frac{d}{v_1}, \quad t_2 = \frac{d}{v_2} \]

Ans -

\[ \text{av. speed} = \frac{2d}{t_1 + t_2} \]
Question: A man covers 3 equal distances with speeds $v_1$, $v_2$, $v_3$. Find his average speed.

Answer:

\[
\begin{align*}
\text{Average speed} &= \frac{3d}{\frac{d}{v_1} + \frac{d}{v_2} + \frac{d}{v_3}} \\
&= \frac{3}{\frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3}}
\end{align*}
\]

Question: A man covers half distance with speed $v$ and remaining half with speed $v_1$ and $v_2$, for equal time interval. Find his average speed.
\[ 2t \Rightarrow 2 \left( \frac{d}{v_1 + v_2} \right) \]

\[ t = \frac{d}{v_1 + v_2} \]

Av. speed = \( 2 \frac{d}{d + 2d} \)

\[ = \frac{2}{d + 2d} \]

\[ = \frac{2}{v_1 + v_2} \]

Av. speed = \( \frac{v_1 + v_2 + 2v}{v(v_1 + v_2)} \)

\[ = \frac{2v(v_1 + v_2)}{v_1 + v_2 + 2v} \]
Ques: A bus covers half distance at speed \( v \) and remaining half at speed \( v_1 \) for 1 time and \( v_2 \) for \( \frac{2}{3} \) time find average speed.

\[ \text{Ans:} \quad \frac{v + v_1 + \frac{2v}{3}}{3} \]

For AC, \( t_1 = \frac{d}{v} \)

For CD, \( d = s \times t \)

\[ d = v_1 \times t \]

\[ d = v_2 \times \frac{2t}{3} \]

Total from C to B

\[ d = \frac{v_1 t + \frac{3}{2} v_2 t}{3} \]

\[ t = \frac{3d}{v_1 + 2v_2} \]

\[ \text{Av. velocity} = \frac{2d}{\frac{d}{v} + \frac{3d}{v_1 + 2v_2}} \]
\[ \begin{align*} 
\frac{1}{V} + \frac{3}{V_1 + 2V_2} &= \frac{2}{V_1 + 2V_2 + 3V} \\
\text{Av. speed} &= \frac{2V(V_1 + 3V_2)}{V_1 + 2V_2 + 3V} \\
\end{align*} \]

**Question:** A man covers a distance with speeds \( V_1 \) and \( V_2 \) for equal time periods. Find the average speed?

**Answer:**

\[ \begin{align*} 
d &= V_1 t + V_2 t \\
2t &= \frac{d}{V_1 + V_2} \\
\text{Av. speed} &= \frac{\text{total distance}}{\text{total time}} \\
&= \frac{d}{2t} \\
&= \frac{d}{V_1 + V_2} \\
\text{Av. speed} &= \frac{V_1 + V_2}{2} \\
\end{align*} \]
Instantaneous Velocity and Instantaneous speed.

Instantaneous velocity

\[ \text{(time should be very small)} \]
\[ \Delta t \to 0 \]
\[ \frac{\Delta s}{\Delta t} \to \frac{ds}{dt} \]

\[ v_{\text{inst}} = \frac{\Delta s}{\Delta t} = ds \frac{1}{dt} \]

\[ ds \Rightarrow \text{slope of s v/s t graph} \]
\[ \frac{dt}{} \]
\[ (\text{Displacement-time graph}) \]

Let \( s = t^2 \).

<table>
<thead>
<tr>
<th>t</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
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<tbody>
<tr>
<td>S</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
</tr>
</tbody>
</table>

Graph

Here, speed is not constant
slope = \frac{\Delta y}{\Delta t} = \frac{\Delta s}{\Delta t} = \text{velocity}

slope of displacement time graph gives velocity

given:
\[ s = t^2 \]

Question: Find velocity at \( t = 1 \text{sec} \) and \( t = 2 \text{sec} \)

Answer: we know that \( v = \frac{ds}{dt} \)

\[ v = 2t \]

\[ v_{inst} = 2t \]

at \( t = 3 \text{sec} \)
\[ v_{inst} = 2 \times 3 = 6 \text{ m/sec} \]

at \( t = 2 \text{sec} \)
\[ v_{inst} = 2 \times 2 = 4 \text{ m/sec} \]

at \( t = 1 \text{sec} \)
\[ v_{inst} = 2 \times 1 = 2 \text{ m/sec} \]
\[ V_{\text{average}} = \frac{S_2 - S_1}{t_2 - t_1} \]

What is the average velocity between 1 and 3 sec.

Average velocity = Total displacement / Total time

\[ = \frac{S_2 - S_1}{2} \]

\[ = \frac{9 - 1}{2} \]

Average velocity = 4 m/sec.

At what instant average velocity and instantaneous velocity is same.

At \( t = 2 \), average velocity and instantaneous velocity is same.

Question: Given, \( s = t^3 - 6t^2 \). Find, velocity at \( t = 1 \) sec, \( t = 3 \) sec, \( t = 2 \) sec.

Answer:

\[ V_{\text{inst}} = \frac{ds}{dt} = 3t^2 - 12t \]

At \( t = 1 \) sec

\[ V_{\text{inst}} = 3 - 12 \Rightarrow -9 \text{ m/sec} \]

At \( t = 3 \) sec

\[ V_{\text{inst}} = 27 - 36 \Rightarrow -9 \text{ m/sec} \]

At \( t = 2 \) sec.
\[ v_{\text{inst}} = 12 - 24 = -12 \text{ m/s} \]

**Question:** Find the time when the body is at rest.

\[ s = t^3 - 6t^2 \]

\[ v_{\text{inst}} = \frac{ds}{dt} = 3t^2 - 12t \]

Body is at rest \( (v = 0) \)

\[ 3t^2 - 12t = 0 \]

\[ 3t(t - 4) = 0 \]

\[ t = 0, t = 4 \]

**Answer:** Given, \( s = 3t^2 + 4t + 9 \).

Find velocity at \( t = 2 \text{ sec} \) and average velocity between 1 to 4 sec.

\[ v_{\text{inst}} = \frac{ds}{dt} = 6t + 4 \]

at \( t = 2 \text{ sec} \)\(, 12 + 4 = 16 \text{ m/sec} \)

**Average:**

\[ \text{Average} = \frac{s_4 - s_1}{t_4 - t_1} \]

\[ s_4 = 48 + 16 + 9 \]

\[ s_1 = 3 + 4 + 9 \]
\[ v_{\text{average}} = \frac{73 - 16}{3} = 5 \, \text{m/sec} \]

**Accelerating (Rate of change of velocity)**

\[ \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t} \quad (\text{m/sec}^2) \]

Let \( a = 2 \, \text{m/sec}^2 \).

\[
\begin{array}{cccccc}
V & 0 & 2 & 4 & 6 \\
T & 0 & 1 & 2 & 3 \\
\end{array}
\]

**Average acceleration**

\[ a_{\text{average}} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t} \]

**Instantaneous acceleration**

\[ a_{\text{instant}} = \frac{dv}{dt} \]

\( \Delta t \) is very small.

V-t graph gives acceleration.
Question: Consider a graph (velocity vs. time) A & B. In which graph, the acceleration is constant and in which the acceleration is changing?

Answer: In graph A, the slope is increasing, hence the acceleration is changing, whereas in graph B, the slope is constant, hence acceleration is constant.

Question: Given \( v = 2 + t^2 - 4t \), find acc at \( t = 3 \) sec, and average acc. from 0 to 3 sec.

Answer: \( a = \frac{dv}{dt} = 4t - 4 \)

at \( t = 3 \) sec

\( a = 12 - 4 = 8 \text{ m/sec}^2 \)
Average acc. from 0 to 3 sec
\[ a = \frac{V_f - V_i}{t} = \frac{6}{3} = 2 \text{ m/sec}^2 \]

Question: Given \( s = t^3 - 6t^2 \). Find acc. when body is at rest.

Answer: Given, \( V = 0 \)

\[ v = \frac{ds}{dt} \]
\[ v = 3t^2 - 12t \]

\[ 3t^2 - 12t = 0 \]
\[ 3t(t - 4) = 0 \]
\[ t = 0, t = 4 \]

\[ a = \frac{dv}{dt} = 6t - 12 \]

At \( t = 0 \) sec
\[ a = -12 \text{ m/sec}^2 \]

At \( t = 4 \) sec
\[ a = 6\times4 - 12 \]
\[ a = 12 \text{ m/sec}^2 \]
Given:
\[ s = 3t^3 + 4t^4 \]

Find velocity at 1 sec.
Speed at 1 sec.
Acceleration at 2 sec.

Answer:
\[ \vec{i} \Rightarrow \text{displacement in } x \text{ direction} \]
\[ \vec{j} \Rightarrow \text{displacement in } y \text{ direction} \]

\[ s_x = 3t \quad s_y = 4t \]

\[ v_x = \frac{ds_x}{dt} = 3 \quad v_y = \frac{ds_y}{dt} = 4 \]

\[ v = 3\vec{i} + 4\vec{j} \]

(Here, velocity is constant, it does not depend upon time)

\[ v = 3 \hat{i} + 4 \hat{j} \]

(ii) \[ |v| = \sqrt{3^2 + 4^2} \]

Speed = Magnitude of Velocity

\[ |v| = \sqrt{25} \]
\[ |v| = 5 \text{ m/sec} \]

(iii) \[ a_x = \frac{dv_x}{dt} = 0 \quad a_y = \frac{dv_y}{dt} = 0 \]

\[ a = 0. \]
Quesc. Given \( s = t^3 + (3t+1)^2 \), find \( \vec{v} \), speed, \( \vec{a} \) at \( t = 1 \) sec.

Ans. \( s_x = t^3 \), \( s_y = 3t - 1 \)

\[
\vec{v}_x = \frac{ds}{dt} = 3t^2 - 2t
\]
\[
\vec{v}_y = \frac{ds}{dt} = 3
\]

\( \vec{v} = \vec{v}_x + \vec{v}_y = 3t^2 - 2t + 3 \)

at \( t = 1 \) sec.

\( \vec{v} = 3(1)^2 + 3 \)

(ii) speed \( = |\vec{v}| = \sqrt{(2)^2 + (3)^2} \)

\( |\vec{v}| = \sqrt{4 + 9} \)

\( |\vec{v}| = \sqrt{13} \)

(iii) \( \vec{a} \) at \( t = 1 \) sec.

\( a_x = \frac{d\vec{v}_x}{dt} = 6t - 2 \)

\( a_y = 0 \)

\( \vec{a} = 6 \hat{i} \)
If velocity is positive \( v > 0 \) → direction is positive.

If velocity is negative \( v < 0 \) → direction is negative.

**DIRECTION OF MOTION**

Direction of motion is explained by velocity, direction of motion cannot be explained by displacement.

Given: \( s = t^3 - 6t^2 + 9t \)

Find velocity at \( t = 2 \) sec.
Find acc. \( a \) at \( t = 3 \) sec.

**Ans:**

\[
\begin{align*}
\frac{ds}{dt} &= 3t^2 - 12t + 9 \\
\frac{d}{dt} &= 12 - 24 + 9 \\
\frac{d}{dt} &= -3 \text{ m/sec}.
\end{align*}
\]

\[
\begin{align*}
a &= \frac{dv}{dt} = 6t - 12 \\
\frac{d}{dt} &= 0 - 18 + 12 \\
a &= 6 \text{ m/sec}^2.
\end{align*}
\]
Question: Find the time at which body is at rest.
\[ s = t^3 - 6t^2 + 9t. \]

Answer:
\[ v = 0 \]

\[ 3t^3 - 12t + 9 = 0 \]
\[ t^3 - 4t + 3 = 0 \]
\[ t(t-3)(t-1) = 0 \]
\[ t = 1, \ t = 3 \]

Question: Find the time when body is moving in negative direction.
\[ s = t^3 - 6t^2 + 9t \]

Answer:
\[ v < 0. \]

\[ v = \frac{ds}{dt} = 3t^2 - 12t + 9 \]
\[ 3t^2 - 12t + 9 < 0 \]
\[ t^2 - 4t + 3 < 0 \]
\[ (t-1)(t-3) < 0 \]

(i) Let \( t < 1 \)
\[ t = 0 \]
\[!(-1)(0-3) \]
\[ (-1)(-3) \]
\[ \Rightarrow +3 \] (Assumption is wrong)
(b) \(1 < t < 3\)
\[ t = 2 \]
\[
\begin{pmatrix}
2 & -1 \\
1 & -1
\end{pmatrix}
\begin{pmatrix}
2 \\
-1
\end{pmatrix}
= -1
\]
\[ t = -1 \] (Assumption is right)

at \( t = -1 \), the body is moving in negative direction.

At \( 1 < t < 3 \), the body is moving in negative direction.

**DISPLACEMENT IN \( n \)th SECOND**

\[ \Rightarrow \text{Displacement in 3rd second} \]
\[ S_2 \to \to S_{3\text{rd second}} \]
\[ S_3 \to \to S_2 \]

\[ S_{3\text{rd}} = S_3 - S_2 \]

\[ \Rightarrow \text{Displacement in} \ n^{\text{th}} \text{second} \]
\[ S_{n^{\text{th}}} = S_n - S_{n-1} \]
DERIVATION OF EQUATION OF MOTION UNDER CONSTANT ACCELERATION.

1. \( v = u + at \)
2. \( s = ut + \frac{1}{2} at^2 \)
3. \( v^2 = u^2 + 2as \)

where, \( v \) = final velocity, 
\( u \) = initial velocity 
\( a \) = acceleration 
\( t \) = time period of acceleration 
\( s \) = displacement

DERIVATION:

1. \( v = u + at \)

\[ a = \frac{\Delta v}{\Delta t} \] (change in velocity)

\[ a = \frac{dv}{dt} \]

\[ dv = adt \]

\[ \int dv = \int adt \]
\[ v = u + at \]

\[ s = ut + \frac{1}{2}at^2 \]

\[ v = \frac{ds}{dt} \]

\[ ds = vdt \]

\[ \int ds = \int vdt = \int (u + at)dt \]

\[ \int_0^s ds = \int_0^t (u + at)dt \]

\[ (s)_0^s = u[\frac{t}{2}] + a[\frac{t^2}{2}] \]

\[ s = ut + \frac{1}{2}at^2 \]
\[ v^2 - u^2 = 2as \]

\[
a = \frac{dv}{dt}
\]

\[
a = \frac{dv}{ds} \times \frac{ds}{dt}
\]

\[
a = \left( \frac{dv}{ds} \right) v
\]

\[
a = -\gamma \frac{dv}{ds}
\]

\[
ads = vdv
\]

\[
\int ads = \int vdv
\]

\[
a[s]_0 = \left[ \frac{v^2}{2} \right]_u
\]

\[
0s = \frac{v^3 - u^3}{a}
\]

\[v^2 - u^2 = 2as\]
Graph

- Displacement-time graph, Velocity-time graph, Acceleration-time graph

- Displacement-time graph

The angle made by graph with positive x-axis denotes slope.

\[ \theta \rightarrow \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{s_2 - s_1}{t_2 - t_1} = \frac{\Delta S}{\Delta t} \rightarrow \text{velocity} \]

\( \theta \) increases, velocity increases

slope = tan\( \theta \) = velocity
Question: Find velocity in region OA, AB, BC?

\[ S \text{ (m)} \]

\[ \uparrow \]

\[ 15 \]

\[ 10 \]

\[ 0 \]

\[ 10 \]

\[ 15 \]

\[ 20 \]

\[ t \text{ (sec)} \rightarrow \]

\[ A \]

\[ B \]

\[ C \]

Answer:

Velocity in OA Region

\[ v \rightarrow \text{slope} = \frac{y_2 - y_1}{x_2 - x_1} \]

\[ V_{oa} = \frac{10 - 0}{10 - 0} = 1 \text{ m/sec} \]

Velocity in AB Region,

Slope is zero, so \[ V_{ab} = 0 \text{ m/sec} \]
velocity in BC region

\[ \text{V}_{\text{BC}} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{15 - 10}{25 - 15} = 1 \text{ m/sec} \]

Question: Find velocity in region OA, AB, BC?

Ans.: Velocity in OA region

\[ \text{V}_{\text{OA}} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{10 - 0}{5 - 0} = 2 \text{ m/sec} \]

Velocity in AB region

\[ \text{V}_{\text{AB}} = 0 \text{ m/sec} \]

Velocity in BC region

\[ \text{V}_{\text{BC}} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{0 - 10}{25 - 15} = -1 \text{ m/sec} \]
If $0 < 90^\circ$ (with positive $x$-axis), slope is positive.

If $0 > 90^\circ$ (with positive $x$-axis), slope is negative.

Guess: The displacement-time graph is given as:

\[ \text{Find the corresponding velocity-time graph.} \]

\[ \text{Ans:} \quad \text{slope is negative.} \]

\[ \frac{0 - y_2}{t_2 - t_1} = \frac{-10}{5} = -2 \text{ m/s}. \]

\[ v = \frac{y_2 - y_1}{t_2 - t_1} = \frac{0 - 10}{5} = -2 \text{ m/s}. \]
* If curve upward, acceleration positive
* If curve downwards, acceleration negative

* If v and a has same sign, velocity speed increases
* If v and a has opposite sign, velocity speed decreases
Find v, a, speed?

Ans: Here θ is greater than 90°, which means v → -ive and curve is upwards, acc. is positive

Hence, speed → decreases
Velocity-time graph:

\[ \text{slope} = \frac{y_2 - y_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} = a \]

slope of v-t graph represents acceleration.

Area under v-t graph represents displacement (area with time axis)

Question: Given v-t graph, find acceleration and displacement of the graph.
Ans:
- Acceleration in OA region

\[
\text{slope} = \frac{10 - 0}{5 - 0} = \frac{10}{5} = 2 \text{ m/s}^2
\]

Acceleration in AB region is zero.

⇒ Displacement in OA
\[
\text{Area} = \frac{1}{2} \times 5 \times 10 = 25 \text{ m}^2
\]

⇒ Displacement in AB
\[
\text{Area} = 5 \times 10 = 50 \text{ cm}^2
\]
**Acc - time Graph**

\[
\text{Slope } = \frac{\Delta a}{\Delta t} = \text{ Jerk, m/sec}^3
\]

Area of \(a-t\) curve = change in velocity

**MOTION UNDER GRAVITY**

\(\Rightarrow\) Motion under gravity is an example of uniform acceleration

\(\Rightarrow\) \(a = g \leq 9.8\ m/s^2\)

For uniform acceleration, we know that

\[s = ut + \frac{1}{2}at^2\]

\[v^2 - u^2 = 2as\]

\(\Rightarrow v = u + at\)
CONVENTION

When the object is moving upward
⇒ the velocity is positive
⇒ \( a = -g \)
⇒ \( s = +\text{ind} \)

When the object is moving downward
⇒ the velocity is taken as negative
⇒ \( a = -g \)
⇒ \( s = \text{free} \)

Question: A body is thrown vertically upward with speed \( u \). Find:
(i) time taken by it to reach max. ht.
(ii) max. height.
(iii) time taken to reach initial position

max. height \( (v = 0) \)

\[ a = -g \quad u = +u \]

\[ v = u + at \]
\[ 0 = u + (-g)t \]
\[ u = gt \]
\[ t = \frac{u}{g} \]
(ii) Let the max. height be 'h'

$$ s = +h $$

$$ v^2 = u^2 + 2as $$

$$ 0 = u^2 + 2(-g)(h) $$

$$ u^2 = 2gh $$

$$ h = \frac{u^2}{2g} $$

(iii) Total time = 2t

Total time = \(2 \left( \frac{u}{g} \right) \)

Que: A stone is dropped from a tower of height 20 m. Find

(i) time taken to reach ground.

(ii) speed just before hitting ground.

\[
\begin{align*}
&u = 0 \\
&v = ? \\
&a = -g \\
&s = -20 \\
&t = ?
\end{align*}
\]
\[ s = ut + \frac{1}{2} at^2 \]
\[-20 = 0 \cdot t + \frac{1}{2} (-10) \cdot t^2. \]
\[-20 = -5t^2. \]

\[ 4 = t^2 \]
\[ t = \pm 2 \text{ sec.} \]

(i) \[ v = u + at \]
\[ v = 0 - gt \]
\[ v = -10 \cdot x^2 \]
\[ v = -20 \text{ m/sec} \]

(ii) A ball is projected vertically upwards from a tower of height 20 m at a speed of 5 m/sec. Find:

(i) Max. height reached by ball (from ground)
(ii) time when it passes from its mean position

\[ u = +5 \text{ m/sec} \]
\[ g = -g \]
\[ v = 0 \]
\[ \Delta h = -5 \text{ m} \]

\[ \text{Ans:} h = -5 \text{ m} \]
\[ v^2 = u^2 + 2as \]
\[ 0 = (5^2) + 2(-10)s \]
\[ 20s = 25 \]
\[ s = \frac{25}{20} = 1.25 \text{ m} \]

(ii) \[ v = 0 \]

\[ u = 5 \text{ m/sec} \]

\[ v = u + at \]
\[ 0 = 5 - 10xt \]
\[ t = 0.5 \text{ sec} \]

**Question:** A ball is thrown upwards at a speed of 28 m/sec. Find the speed of ball 1 sec before reaching maximum height.

**Answer:**
\[ a = g \]
\[ u = 28 \text{ m/sec} \]
At maximum height \( v = 0 \).
\[ v = u + at \]
\[ v = u + at \]
\[ 0 = 28 + (-10)t \]
\[ t = 2.8 \text{ sec} \]

At 2.8 sec, the ball will reach its max height.

Before 1 sec i.e., 1.8 sec

\[ u = 28 \text{ m/sec} \]
\[ a = -10 \]
\[ t = 1.8 \]
\[ v = u + at \]
\[ v = 28 - 10 \times (1.8) \]
\[ v = 28 - 18 \]
\[ v = 10 \text{ m/sec} \]

Question: A tap drops water drops at one second intervals. Find the distance between 3rd and 5th drop when 6th drop is just coming out from tap.

3rd drop
\[ u = 0 \]
\[ a = -g = 10 \]
\[ t = 3 \text{ sec} \]
\[ s = ut + \frac{1}{2} at^2 \]

\[ = 0 + \frac{1}{2} \times (-10) \times 1 \]

\[ s = -5 \text{ m} \]

\[ s = -45 \text{ m} \]

5th drop:
\[ u = 0 \]
\[ a = -10 \]
\[ t = 1 \text{ sec} \]

Distance between them is 40 m.
These all are described by frame of reference.

Position of observers.

Consider a person A in trolley moving with the speed of 10 m/sec. Another person B watching A at some distance away.

\[ \vec{v}_{AB} = \vec{v}_A - \vec{v}_B \]

\[ \vec{v}_{AB} = 10 - 0 \]

\[ \vec{v}_{AB} = 10 \text{ m/sec} \]

(Velocity of A w.r.t B)

Now, when B joins A

Velocity of A w.r.t B

\[ \vec{v}_{AB} = \vec{v}_A - \vec{v}_B \]

\[ \vec{v}_{AB} = 10 - 10 \]

\[ \vec{v}_{AB} = 0 \text{ m/sec} \]
Time does not depend upon frame of reference.

Question: Consider 2 people A and B. A is travelling with the speed of 10 m/sec, and B is travelling at 2 m/sec. When B observes A, what is the speed of A as seen by B?

Answer: \[ \vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG} \]
\[ V_{AG} = 10 - 20 \]
\[ V_{AB} = -10 \text{ m/sec} \]

\[ \Rightarrow \text{speed of B as seen by A} \]
\[ \vec{V}_{BA} = \vec{V}_{BG} - \vec{V}_{AG} \]
\[ V_{BA} = 20 - 10 \]
\[ V_{BA} = 10 \text{ m/sec} \]

Question: Consider 2 travelling travelling off to each other on same way. A by 30 m/sec, B by 10 m/sec, what is the velocity of B as seen by A.
Ans. $\overrightarrow{V_{BA}} = \overrightarrow{V_R} - \overrightarrow{V_A}$

$= 10 - (-30)$

$\overrightarrow{V_{BA}} = 40 \text{ m/ sec}$

The velocity of $A$ as seen by $B$

$\overrightarrow{V_{AB}} = \overrightarrow{V_A} - \overrightarrow{V_B}$

$= -30 - 10$

$\overrightarrow{V_{AB}} = -40 \text{ m/ sec}$

Quest: What is the velocity of $A$ w.r.t. $P$, the speed of trolley w.r.t. ground is $10 \text{ m/ sec}$ and an object $A$ is moving at the speed of $2 \text{ m/ sec}$ w.r.t. trolley.

$\overrightarrow{V_{AP}} = \overrightarrow{V_{AG}} - \overrightarrow{V_{PG}}$

$\overrightarrow{V_{AG}} = ?$
\[ V_{AT} = V_{AG} - V_{TG} \]

\[ 2 = V_{AG} - 10 \]

\[ V_{AG} = 12 \text{ m/sec} \]

\[ V_{AP} = 12 - 0 \]

\[ V_{AP} = 12 \text{ m/sec} \]

**POLICE - THIEF PROBLEM**

Find the time at which police caught the thief, if speed of thief is 10 m/sec and police is 20 m/sec and the distance between them is 100 m.

\[ V_{PC} = V_{PG} - V_{TG} \]

\[ = 20 - 10 \]

\[ V_{PC} = 10 \text{ m/sec} \]

Suppose T is at rest.

"No religion has mandated killing others as a requirement for its sustenance or promotion." — Dr. A.P.J. Abdul Kalam
SPT (Relative distance between Person and Thief Thief)

SPT = 100.

Relative distance does not depend upon frame of reference.

\[ t = \frac{S}{v} = \frac{100}{10} = 10 \text{ sec.} \]

Discovered by thief in 10 sec

\[ S = v \times t \]
\[ S = 10 \times 10 \]
\[ S = 100 \text{ m,} \]

Police caught thief at a distance of 200 m.

Question: What is the time and position where thief & caught, if the acc of thief is 1 m/sec², and the speed of police is 20 m/sec, and distance between them is 150 m.

[Diagram with stick figures showing movement and distances]
\[ v_{pc} = v_p - v_c = 20 - 0 = 20. \]

\[ v_{pc} = 0. \]

\[ a_{pc} = a_p - a_c = 0 - 1 = -1 \text{ m/sec}^2. \]

\[ s_{pc} = 150 \text{ m}. \]

\[ s = ut + \frac{1}{2} at^2. \]

\[ 150 = 20t + \frac{1}{2}(-1)t^2. \]

\[ t^2 - 40t + 300 = 0. \]

\[ t^2 - 30t - 10t + 300 = 0. \]

\[ (t - 30)(t - 10) = 0. \]

\[ t = 30 \text{ sec}, t = 10 \text{ sec}. \]

\[ \Rightarrow \text{ After 10 sec from Police}. \]

\[ s = 20 \times 10 = 200 \text{ m}. \]

\[ \Rightarrow \text{ After Thief}. \]

\[ s = ut + \frac{1}{2} at^2 \]

\[ s = 0 \times t + \frac{1}{2} \times 1 \times (1) \times 100. \]

\[ s = 50 \text{ m}. \]
Question: Find the time when boy catches the girl, or they are at their closest separation.

\[ v_{BG}' = v_{BG} - v_{G}'G \]
\[ = 50 - 30 = 20 \text{ m/s}\]

\[ a_{BG}' = a_{BG} - a_{G}'G \]
\[ = 1 - 2 = -1 \text{ m/s}^2 \]

\[ s_{BG}' = 150 \text{ m} \]

\[ s = ut + \frac{1}{2}at^2 \]
MOTION UNDER GRAVITY

Question: Find the separation between A and B after 3 sec. If a ball with initial speed 0 and 3 m/sec respectively are thrown from building A and B.

\[ u = 0 \quad u = 3 \text{ m/sec} \]

\[ v_{AB} = v_A - v_B \]
\[ = 0 - (-3) \]
\[ v_{AB} = 3 \]

\[ a_{AB} = a_A - a_B \]
\[ = -g - (-g) \]
\[ a_{AB} = 0 \]

\[ t = 3 \text{ sec} \]
\[ s = v \times t \]
\[ s = 9 \text{ m} \]

Separation
Ques:- When the two ball will meet,

\[ A \uparrow \]

\[ \downarrow \]

\[ 36 \text{ m} \]

\[ \downarrow \]

\[ \bullet B \]

2 m/sec

16 m/sec

\[ A = \begin{align*} \vec{v}_A &= \vec{v}_A^0 - \vec{g} \\ \vec{v}_A &= -2 \text{ m/sec} \end{align*} \]

\[ B = \begin{align*} \vec{v}_B &= \vec{v}_B^0 + \vec{g} \\ \vec{v}_B &= +16 \text{ m/sec} \end{align*} \]

\[ \vec{v}_{AB} = \vec{v}_A - \vec{v}_B = -2 - (16) = -18 \text{ m/sec} \]

\[ a_{AB} = -g - (-g) = 0 \]

\[ s_{AB} = 36 \text{ m} \]

\[ t = \frac{s}{\frac{1}{2} a} = \frac{36}{18} = 2 \text{ sec} \]

They will meet after 2 sec.

\[ \Rightarrow \text{ The distance travelled by } B \text{ is} \]

\[ s = ut + \frac{1}{2} at^2 \]

\[ s = 16 \times 2 + \frac{1}{2} \times (-10) \times 2^2 \]
RAIN-MAN PROBLEM.

Question 1: It's raining at the rate of 40 km/h. What is the velocity of rain as seen by man?

Rain

\[ v_{RM} = v_{RG} - v_{MG}. \]

\[ v_{RM} = 40 \hat{j}. \]

Question 2: It's raining at the rate of 60 km/h, and a man is moving in the trolley at the rate of 80 km/h. At what angle, the man will open his umbrella.

\[ v_{RM} = v_{RG} - v_{MG}. \]

\[ v_{RM} = -60 \hat{j} - 80 \hat{j}. \]

\[ \tan \theta = \frac{-60}{80} \]

\[ \theta = 53^\circ. \]
RIVER SWIMMER / RIVER BOAT
AEROPLANE WIND

Que: The river is flowing at the rate of 3 km/hr. A man tries to flow perpendicular to the flow of river at the velocity of 4 km/hr. According to the rule, \( V_{SR} \) (velocity of swimmer relative to river) is in the direction of the person outside the river (P) saw that the river drift's the swimmer. The speed of swimmer is given as 16 km.

Find:
(1) time taken by swimmer to cross river.
(2) At what point on the opposite of bank did the swimmer arrive.

\[ V_{SR} = 4\uparrow + 3\uparrow \]
\[ V_{SG} = V_{SR} + V_{RG} \]  
(The swimmer got 2 velocities)

**Vertical Motion**

\[ S = 16 \text{ km} \]
\[ V = 4 \text{ km/hr} \]
\[ t = \frac{16}{4} = 4 \text{ hr} \]

Drift is in x-direction.

**Horizontal**

\[ d = s \times t \]
\[ d = 5 \text{ km/hr} \times 4 \text{ hr} \]
\[ d = 20 \text{ km} \text{ (drift)} \]

Actually distance he swims is \[ \sqrt{(06)^2 + (02)^2} \]
\[ = 20 \text{ km} \]

**Vertical motion will be covered by vertical speed; horizontal motion will be covered by horizontal speed.**
Que 6: A river flows at an rate of 8 km/hr \((V_R)\), breadth of river is 30 km. A man swims at an angle of 37° at a speed of 10 km/hr. Find
(i) time taken to cross river
(ii) at what point does he receive \(\Delta \) at a speed of 9 km/hr.
(iii) actual distance he swims.

\[ V_R = 8 \text{ km/hr} \]

\[ V_1 = 10 \text{ km/hr} \]

\[ \theta = 37° \]

\[ 10 \cos 37° = 8 \]

\[ 10 \sin 37° = 6 \]

velocity of swimmer w.r.t \( \text{ground} = \)

\[ V_{SG} = V_{SR} + V_{RG} \]

Horizontal speed = 0

\[ V_{SG} = 6 \]

(i) \[ t = \frac{30}{6} = 5 \text{ hour} \]

"Whenever you take a step forward, you are bound to disturb something." — Indira Gandhi
(a) How much time will he take to cross a river of width 500m wide.

(b) In which direction should he try to swim.

(c) 4 km/h, he wants to cross a speed of 3 km/h. He wants to throw a stone that land at 3 km/h.

Question: A man can swim in still water at a speed of 3 km/h. He wants to swim vertically off a point. He decides vertically off.

Actual distance = 30 km.
Ans. To reach the point directly opposite to his starting point, the man should leave with some angle with its vertical position.

There should be no horizontal velocity so,

\[ 3 \sin \theta = 2 \]

\[ \sin \theta = \frac{2}{3} \]

\( \text{Ans} \)

\[ t = \frac{0.5}{3 \cos \theta} \]

\[ \cos \theta = \sqrt{1 - \sin^2 \theta} \]

\[ \cos \theta = \frac{\sqrt{5}}{3} \]

\[ t = \frac{1}{2\sqrt{5}} \text{ hours} \]

Question: A man can swim at a speed of 3 km/hr in still water. He wants to cross a 500 m wide river flowing at 2 km/hr. He keeps himself always at an angle of 120° with the current.

(a) Find the time he takes to cross the river.

(b) At which point on the opposite bank will he arrive?
\[ V_r = 2 \text{ km/h} \]

\[ V_s = 3 \text{ km/h} \]

\[ 3 \cos 60^\circ = 1.5 \]

\[ 3 \sin 60^\circ = 1.5 \sqrt{3} \]

\[ V_{sg} = V_{sr} + V_{rg} \]

\[ V_{sg} = 0.5 \uparrow + 1.5 \sqrt{3} \uparrow \]

\[ t = \frac{0.5}{1.5 \sqrt{3}} \]

\[ t = \frac{1}{3 \sqrt{3}} \]

The swimmer will go towards \( C \)

\[ B = 0.5 \times \frac{1}{3 \sqrt{3}} = \frac{1}{6 \sqrt{3}} \text{ km} \]

\[ \text{Horizontal} \]
MINIMUM DISTANCE \((\text{Drift} = 0)\)

TO CROSS RIVER

To travel minimum distance to cross river horizontal component should be zero.

\[ V_{SR} \sin \theta = V_{RG} \]

\[ \frac{\sin \theta}{V_{SR}} = \frac{V_{RG}}{V_{SR}} \]

As shown in figure

[Diagram showing vector components]

MINIMUM TIME TO CROSS RIVER

To take minimum time to cross river the swimmer should try to swim perpendicular to riverflow.
If the swimmer swims at some angle $\theta$.

\[ t = \frac{d}{V_{S\cos \theta}} \quad (\text{vertical component}) \]

For $t \to \text{min}$,

\[ \cos \theta \to \text{max} \]

Max. value of $\cos \theta = 1$

and $\theta = 0^\circ$

So to take minimum time to cross river, the swimmer should try to swim $\theta = 0^\circ$ to river flow.