1. A steel scale is to be prepared such that the millimeter intervals are to be accurate within $6 \times 10^{-3}$ mm. The maximum temperature variation from the temperature of calibration during the reading of the millimeter marks is $\alpha = 12 \times 10^{-6}$ k$^{-1}$.

(A) 4.0 °C  (B) 4.5 °C  (C) 5.0 °C  (D) 5.5 °C

Sol.

2. A steel rod 25 cm long has a cross-sectional area of 0.8 cm$^2$. Force that would be required to stretch this rod by the same amount as the expansion produced by heating it through 10°C is:

(Coefficient of linear expansion of steel is $10^{-5}$/°C and Young’s modulus of steel is $2 \times 10^{11}$ N/m$^2$.)

(A) 160 N  (B) 360 N  (C) 106 N  (D) 260 N

Sol.

3. Two rods of different materials having coefficients of thermal expansion $\alpha_1$, $\alpha_2$, and Young’s moduli $Y_1$, $Y_2$ respectively are fixed between two rigid massive walls. The rods are heated such that they undergo the same increase in temperature. There is no bending of the rods. If $\alpha_1 : \alpha_2 = 2 : 3$, the thermal stresses developed in the two rods are equal provided $Y_1 : Y_2$ is equal to

(A) 2 : 3  (B) 1 : 1  (C) 3 : 2  (D) 4 : 9

Sol.

4. If $I$ is the moment of inertia of a solid body having $\alpha$-coefficient of linear expansion then the change in $I$ corresponding to a small change in temperature $\Delta T$ is

(A) $\alpha I \Delta T$  (B) $I \frac{1}{2} \alpha I \Delta T$

(C) $2 \alpha I \Delta T$  (D) $3 \alpha I \Delta T$

Sol.
5. A metallic wire of length \( L \) is fixed between two rigid supports. If the wire is cooled through a temperature difference \( \Delta T \) (\( Y = \) young's modulus, \( \rho = \) density, \( \alpha = \) coefficient of linear expansion) then the frequency of transverse vibration is proportional to:

\[
\begin{align*}
(A) & \quad \frac{\alpha}{\sqrt{\rho Y}} \\
(B) & \quad \frac{Y_0}{\rho} \\
(C) & \quad \frac{\alpha}{\sqrt{Y_0}} \\
(C) & \quad \frac{\sigma}{\sqrt{Y}} \\
\end{align*}
\]

\text{Sol.}

7. A steel tape gives correct measurement at 20°C. A piece of wood is being measured with the steel tape at 0°C. The reading is 25 cm on the tape, the real length of the given piece of wood must be:

(A) 25 cm  \\
(B) < 25 cm  \\
(C) > 25 cm  \\
(D) can not say

\text{Sol.}

6. A metal wire is clamped between two vertical walls. At 20°C the unstrained length of the wire is exactly equal to the separation between walls. If the temperature of the wire is decreased the graph between elastic energy density \( (u) \) and temperature \( (T) \) of the wire is

\text{Sol.}

8. A rod of length 20 cm is made of metal. It expands by 0.075 cm when its temperature is raised from 0°C to 100°C. Another rod of a different metal B having the same length expands by 0.045 cm for the same change in temperature, a third rod of the same length is composed of two parts one of metal A and the other of metal B. Thus rod expand by 0.06 cm for the same change in temperature. The portion made of metal A has the length.

(A) 20 cm  \\
(B) 10 cm  \\
(C) 15 cm  \\
(D) 18 cm

\text{Sol.}
9. A sphere of diameter 7 cm and mass 266.5 gm floats in a bath of a liquid. As the temperature is raised, the sphere just begins to sink at a temperature 35°C. If the density of a liquid at 0°C is 1.527 gm/cc, then neglecting the expansion of the sphere, the coefficient of cubical expansion of the liquid is f:
(A) \(8.486 \times 10^{-4}\) per °C
(B) \(8.486 \times 10^{-3}\) per °C
(C) \(8.486 \times 10^{-4}\) per °C
(D) \(8.486 \times 10^{-1}\) per °C
**Sol.**

10. The volume of the bulb of a mercury thermometer at 0°C is \(V_0\) and cross section of the capillary is \(A_0\). The coefficient of linear expansion of glass is \(\alpha_g\) per °C and the cubical expansion of mercury \(\gamma_m\) per °C. If the mercury just fills the bulb at 0°C, what is the length of mercury column in capillary at T°C:
(A) \(\frac{V_0 T(\gamma_m + 3\alpha_g)}{A_0(1 - 2\alpha_g T)}\)
(B) \(\frac{V_0 T(\gamma_m - 3\alpha_g)}{A_0(1 + 2\alpha_g T)}\)
(C) \(\frac{V_0 T(\gamma_m + 2\alpha_g)}{A_0(1 + 3\alpha_g T)}\)
(D) \(\frac{V_0 T(\gamma_m - 2\alpha_g)}{A_0(1 + 3\alpha_g T)}\)
**Sol.**

11. A metallic rod 1 cm long with a square cross-section is heated through 1°C. If Young’s modulus of elasticity of the metal is E and the mean coefficient of linear expansion is \(\alpha\) per degree Celsius, then the compressional force required to prevent the rod from expanding along its length is:
(Neglect the change of cross-sectional area)
(A) \(EA\alpha t\)
(B) \(EA\alpha t / (1 + \alpha t)\)
(C) \(EA\alpha t / (1 - \alpha t)\)
(D) \(E / \alpha t\)
**Sol.**

12. The loss in weight of a solid when immersed in a liquid at 0°C is \(W_o\) and at T°C is \(W\). If cubical coefficient of expansion of the solid and the liquid by \(\gamma_s\) and \(\gamma_l\) respectively, then \(W\) is equal to:
(A) \(W_o [1 + (\gamma_s - \gamma_l) \times T]\)
(B) \(W_o [1 - (\gamma_s - \gamma_l) \times T]\)
(C) \(W_o [\gamma_s - \gamma_l] \times T]\)
(D) \(W_o / (\gamma_s - \gamma_l)\)
**Sol.**
13. A thin walled cylindrical metal vessel of linear coefficient of expansion $10^{-3} \text{oC}^{-1}$ contains benzene of volume expansion coefficient $10^{-3} \text{oC}^{-1}$. If the vessel and its contents are now heated by $10^\circ\text{C}$, the pressure due to the liquid at the bottom.

(A) increases by 2%  
(B) decreases by 1%  
(C) decreases by 2%  
(D) remains unchanged

Sol.

14. A rod of length 2m at $0^\circ\text{C}$ and having expansion coefficient $\alpha = (3x + 2) \times 10^{-4} \text{oC}^{-1}$ where $x$ is the distance (in cm) from one end of rod. The length of rod at $20^\circ\text{C}$ is:

(A) 2.124 m  
(B) 3.24 m  
(C) 2.0120 m  
(D) 3.124 m

Sol.

15. A copper ring has a diameter of exactly 25 mm at its temperature of $0^\circ\text{C}$. An aluminium sphere has a diameter of exactly 25.05 mm at its temperature of $100^\circ\text{C}$. The sphere is placed on top of the ring and two are allowed to come to thermal equilibrium, no heat being lost to the surrounding. The sphere just passes through the ring at the equilibrium temperature. The ratio of the mass of the sphere & ring is:

(given : $\alpha_{Cu} = 17 \times 10^{-6}/\text{oC}$, $\alpha_{Al} = 2.3 \times 10^{-5}/\text{oC}$, specific heat of Cu = 0.0923 Cal/g°C and specific heat of Al = 0.215 cal/g°C)

(A) $1/5$  
(B) $23/108$  
(C) $23/54$  
(D) $216/23$

Sol.

16. A cuboid ABCDEFGH is anisotropic with $\alpha_x = 1 \times 10^{-5}/\text{oC}$, $\alpha_y = 2 \times 10^{-5}/\text{oC}$, $\alpha_z = 3 \times 10^{-5}/\text{oC}$. Coefficient of superficial expansion of faces can be

(A) $\beta_{ABCD} = 5 \times 10^{-5}/\text{oC}$  
(B) $\beta_{BFGH} = 4 \times 10^{-5}/\text{oC}$  
(C) $\beta_{CDEH} = 3 \times 10^{-5}/\text{oC}$  
(D) $\beta_{EFGH} = 2 \times 10^{-5}/\text{oC}$

Sol.
17. An open vessel is filled completely with oil which has same coefficient of volume expansion as that of the vessel. On heating both oil and vessel,
(A) the vessel can contain more volume and more mass of oil
(B) the vessel can contain same volume and same mass of oil
(C) the vessel can contain same volume but more mass of oil
(D) the vessel can contain more volume but same mass of oil

*Sol.*

18. A metal ball immersed in Alcohol weighs \( W_1 \) at 0°C and \( W_2 \) at 50°C. The coefficient of cubical expansion of the metal (\( \gamma_m \)) is less than that of alcohol (\( \gamma_a \)). Assuming that density of metal is large compared to that of alcohol, it can be shown that
(A) \( W_1 > W_2 \)
(B) \( W_1 = W_2 \)
(C) \( W_1 < W_2 \)
(D) any of (A), (B) or (C)

*Sol.*

19. A solid ball is completely immersed in a liquid. The coefficients of volume expansion of the ball and liquid are \( 3 \times 10^{-4} \) and \( 8 \times 10^{-4} \) per °C respectively. The percentage change in upthrust when the temperature is increased by 100°C is
(A) 0.5 %
(B) 0.11 %
(C) 1.1 %
(D) 0.05 %

*Sol.*

20. A thin copper wire of length \( L \) increase in length by 1% when heated from temperature \( T_1 \) to \( T_2 \). What is the percentage change in area when a thin copper plate having dimensions \( 2L \times L \) is heated from \( T_1 \) to \( T_2 \)?
(A) 1%
(B) 2%
(C) 3%
(D) 4%

*Sol.*
21. If two rods of length L and 2L having coefficients of linear expansion $\alpha$ and $2\alpha$ respectively are connected so that total length becomes 3L, the average coefficient of linear expansion of the composition rod equals:

(A) $\frac{3}{2}\alpha$  (B) $\frac{5}{2}\alpha$

(C) $\frac{5}{3}\alpha$  (D) none of these

Sol.

22. The bulk modulus of copper is $1.4 \times 10^{11}$ Pa and the coefficient of linear expansion is $1.7 \times 10^{-5}$ $(^\circ\text{C})^{-1}$. What hydrostatic pressure is necessary to prevent a copper block from expanding when its temperature is increased from $20^\circ\text{C}$ to $30^\circ\text{C}$?

(A) $6.0 \times 10^5$ Pa  (B) $7.1 \times 10^7$ Pa

(C) $5.2 \times 10^4$ Pa  (D) 40 atm

Sol.

23. The coefficients of thermal expansion of steel and a metal X are respectively $12 \times 10^{-6}$ and $2 \times 10^{-5}$ per $^\circ\text{C}$. At $40^\circ\text{C}$, the side of a cube of metal X was measured using a steel vernier calipers. The reading was 100 mm. Assuming that the calibration of the vernier was done at $0^\circ\text{C}$, then the actual length of the side of the cube at $0^\circ\text{C}$ will be

(A) > 100 mm  (B) < 100 mm

(C) = 100 mm  (D) data insufficient to conclude

Sol.

24. A glass flask contains some mercury at room temperature. It is found that at different temperature the volume of air inside the flask remains the same. If the volume of mercury in the flask is 300 cm$^3$, then volume of the flask is (given that coefficient of volume expansion of mercury and coefficient of linear expansion of glass are $1.8 \times 10^{-1}(^\circ\text{C})^{-1}$ and $9 \times 10^{-3}(^\circ\text{C})^{-1}$ respectively)

(A) 4500 cm$^3$  (B) 450 cm$^3$

(C) 2000 cm$^3$  (D) 6000 cm$^3$

Sol.
Question No. 25 to 29 (5 questions)

Solids and liquids both expand on heating. The density of substance decreases on expanding according to the relation
\[ \rho_2 = \frac{\rho_1}{1 - \gamma(T_2 - T_1)} \]
where, \( \rho_1 \rightarrow \text{density at } T_1 \)
\( \rho_2 \rightarrow \text{density at } T_2 \)
\( \gamma \rightarrow \text{coeff. of volume expansion of substances} \)
when a solid is submerged in a liquid, liquid exerts an upward force on solid which is equal to the weight of liquid displaced by submerged part of solid.

Solid will float or sink depends on relative densities of solid and liquid.

A cubical block of solid floats in a liquid with half of its volume submerged in liquid as shown in figure (at temperature T)

25. The relation between densities of solid and liquid at temperature T is
   (A) \( \rho_s = 2\rho_l \)
   (B) \( \rho_s = \frac{1}{2}\rho_l \)
   (C) \( \rho_s = \rho_l \)
   (D) \( \rho_s = \frac{1}{4}\rho_l \)

Sol.

26. If temperature of system increases, then fraction of solid submerged in liquid
   (A) increases
   (B) decreases
   (C) remains the same
   (D) inadequate information

Sol.

27. Imagine fraction submerged does not change on increasing temperature the relation between \( \gamma \) and \( \alpha_s \) is
   (A) \( \gamma = 3\alpha_s \)
   (B) \( \gamma = 2\alpha_s \)
   (C) \( \gamma = 4\alpha_s \)
   (D) \( \gamma = \frac{3}{2}\alpha_s \)

Sol.
28. Imagine the depth of the block submerged in the liquid does not change on increasing temperature then

(A) \( \gamma_l = 2\alpha \)  
(B) \( \gamma_l = 3\alpha \)  
(C) \( \gamma_l = (3/2)\alpha \)  
(D) \( \gamma_l = (4/3)\alpha \)

**Sol.**

29. Assume block does not expand on heating. The temperature at which the block just begins to sink in liquid is

(A) \( T + \frac{1}{\gamma_l} \)  
(B) \( T + \frac{1}{2\gamma_l} \)  
(C) \( T + \frac{2}{\gamma_l} \)  
(D) \( T + \frac{\gamma_l}{2} \)

**Sol.**

30. The coefficient of apparent expansion of a liquid in a copper vessel is \( C \) and in a silver vessel is \( S \). The coefficient of volume expansion of copper is \( \gamma_c \). What is the coefficient of linear expansion of silver?

(A) \( \frac{C + \gamma_c + S}{3} \)  
(B) \( \frac{C - \gamma_c + S}{3} \)  
(C) \( \frac{C + \gamma_c - S}{3} \)  
(D) \( \frac{C - \gamma_c - S}{3} \)

31. An aluminium container of mass 100 gm contains 200 gm of ice at \(-20°C\). Heat is added to the system at the rate of 100 cal/s. The temperature of the system after 4 minutes will be (specific heat of ice = 0.5 and \( L = 80 \) cal/gm, specific heat of \( A \) = 0.2 cal/gm/°C).

(A) 40.5°C  
(B) 25.5°C  
(C) 30.3°C  
(D) 35.0°C

**Sol.**

32. Two vertical glass tubes filled with a liquid are connected by a capillary tube as shown in the figure. The tube on the left is put in an ice bath at 0°C while the tube on the right is kept at 30°C in a water bath. The difference in the levels of the liquid in the two tubes is 4 cm while the height of the liquid column at 0°C is 120 cm. The coefficient of volume expansion of liquid is (Ignore expansion of glass tube)
33. A difference of temperature of 25°C is equivalent to a difference of:
(A) 45°F  (B) 72°F  (C) 32°F  (D) 25°F  
Sol.

34. Two thermometers x and y have fundamental intervals of 80° and 120°. When immersed in ice, they show the reading of 20° and 30°. If y measures the temperature of a body as 120°, the reading of x is:
(A) 59°  (B) 65°  (C) 75°  (D) 80°  
Sol.

35. When an enclosed perfect gas is subjected to an adiabatic process:
(A) Its total internal energy does not change
(B) Its temperature does not change
(C) Its pressure varies inversely as a certain power of its volume
(D) The product of its pressure and volume is directly proportional to its absolute temperature.
Sol.
36. Four rods A, B, C, D of same length and material but of different radii \( r, r\sqrt{2}, r\sqrt{3} \) and \( 2r \) respectively are held between two rigid walls. The temperature of all rods is increased by same amount. If the rods do not bend, then
(A) the stress in the rods are in the ratio \( 1 : 2 : 3 : 4 \)
(B) the force on the rod exerted by the wall are in the ratio \( 1 : 2 : 3 : 4 \)
(C) the energy stored in the rods due to elasticity are in the ratio \( 1 : 2 : 3 : 4 \)
(D) the strains produced in the rods are in the ratio \( 1 : 2 : 3 : 4 \)
**Sol.**

38. When the temperature of a copper coin is raised by 80°C, its diameter increases by 0.2%. (A) Percentage rise in the area of a face is 0.4% (B) Percentage rise in the thickness is 0.4% (C) Percentage rise in the volume is 0.6% (D) Coefficient of linear expansion of copper is \( 0.25 \times 10^{-5} \text{°C}^{-1} \).
**Sol.**

37. A body of mass M is attached to the lower end of a metal wire, whose upper end is fixed. The elongation of the wire is \( l \).
(A) Loss in gravitational potential energy of M is Mg/
(B) The elastic potential energy stored in the wire is Mg/
(C) The elastic potential energy stored in the wire is \( 1/2 \text{ Mg/} \)
(D) Heat produced is \( 1/2 \text{ Mg/} \)
**Sol.**
1. We have a hollow sphere and a solid sphere of equal radii and of the same material. They are heated to raise their temperature by equal amounts. How will the change in their volumes, due to volume expansions, be related? Consider two cases (i) hollow sphere is filled with air, (ii) there is vacuum inside the hollow sphere. Sol.

2. The time represented by the clock hands of a pendulum clock depends on the number of oscillation performed by pendulum every time it reach to its extreme position the second hand of the clock advances by one second that means second hand move by two second when one oscillation is complete
   (a) How many number of oscillations completed by pendulum of clock in 15 minutes at calibrated temperature 20°C
   (b) How many number of oscillations are completed by a pendulum of clock in 15 minute at temperature of 40°C if α = 2 x 10⁻⁴°C
   (c) What time is represented by the pendulum clock at 40°C after 15 minutes if the initial time shown by the clock is 12:00 pm?
   (d) If the clock gains two second in 15 minutes then find - (i) Number of extra oscillation (ii) New time period (iii) change in temperature.

Sol.
3. Consider a cylindrical container of cross section area 'A', length 'h' having coefficient of linear expansion $\alpha$. The container is filled by liquid of real expansion coefficient $\gamma$, up to height $h_1$. When temperature of the system is increased by $\Delta T$ then

(a) Find out new height, area and volume of cylindrical container and new volume of liquid.
(b) Find the height of liquid level when expansion of container is neglected.
(c) Find the relation between $\gamma$ and $\alpha$, for which volume of container above the liquid level.
   (i) increases (ii) decreases (iii) remains constant.
(d) If $\gamma > 3 \alpha_c$ and $h = h_1$ then calculate, the volume of liquid overflow
(e) What is the relation between $\gamma$, and $\alpha$, for which volume of empty space becomes independent of change of temp.
(f) If the surface of a cylindrical container is marked with numbers for the measurement of liquid level of liquid filled inside it. If we increase the temperature of the system be $\Delta T$ then
   (i) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of liquid
   (ii) Find height of liquid level as shown by the scale on the vessel. Neglect expansion of container
   (iii) Find relation between $\gamma$, and $\alpha$, so that height of liquid level with respect to ground
    (1) increases (2) decreases (3) remains constant.

Sol.

4. A loaded glass bulb weighs 156.25 g in air. When the bulb is immersed in a liquid at temperature 15°C, it weighs 56.25 g. On heating the liquid, for a temperature upto 52°C the apparent weight of the bulb becomes 66.25 g. Find the coefficient of real expansion of the liquid. (Given coefficient of linear expansion of glass = $9 \times 10^{-4}$/°C).

Sol.

5. A body is completely submerged inside the liquid. It is in equilibrium and in rest condition at certain temperature. It is volumetric expansion coefficient of liquid $\alpha_v = $ linear expansion coefficient by of body. It we increases temperature by $\Delta T$ amount than find
   (a) New thrust force if initial volume of body is $V_o$ and density of liquid is $d_o$.
   (b) Relation between $\alpha_v$ and $\gamma$, so body will (i) move upward (ii) down ward (iii) remains are rest.
6. A clock pendulum made of invar has a period of 0.5 sec at 20°C. If the clock is used in a climate where average temperature is 30°C, approximately, how much fast or slow will the clock run in 10^4 sec.

(\alpha_{\text{invar}} = 1 \times 10^{-4}/^\circ\text{C})

Sol.

7. An iron bar (Young's modulus = 10^{11} \text{ N/m}^2, \alpha = 10^{-5}/^\circ\text{C}) 1 m long and 10^{-3} \text{ m}^2 in area is heated from 0°C to 100°C without being allowed to bend or expand. Find the compressive force developed inside the bar.

Sol.

8. Three aluminium rods of equal length form an equilateral triangle ABC. Taking O (mid point of rod BC) as the origin. Find the increase in Y-coordinate of center of mass per unit change in temperature of the system. Assume the length of each rod is

2m, and \alpha_{\text{al}} = 4\sqrt{3} \times 10^{-4}/^\circ\text{C}

Sol.
9. If two rods of length \( L \) and \( 2L \) having coefficients of linear expansion \( \alpha \) and \( 2\alpha \) respectively are connected so that total length becomes \( 3L \), determine the average coefficient of linear expansion of the composite rod.
   **Sol.**

11. The coefficient of volume expansion of mercury is 20 times the coefficient of linear expansion of glass. Find the volume of mercury that must be poured into a glass vessel of volume \( V \) so that the volume above mercury may remain constant at all temperature.
   **Sol.**

10. A thermostatted chamber at small height \( h \) above earth's surface maintained at 30°C has a clock fitted in it with an uncompensated pendulum. The clock designer correctly designs it for height \( h \), but for temperature of 20°C. If this chamber is taken to earth's surface, the clock in it would click correct time. Find the coefficient of linear expansion of material of pendulum. (earth's radius is \( R \))
   **Sol.**

12. A metal rod of 25 cm lengths expands by 0.050 cm. When its temperature is raised from 0°C to 100°C. Another rod \( B \) of a different metal of length 40 cm expands by 0.040 cm for the same rise in temperature. A third rod \( C \) of 50 cm length is made up of pieces of rods \( A \) and \( B \) placed end to end expands by 0.03 cm on heating from 0°C to 50°C. Find the lengths of each portion of the composite rod.
   **Sol.**
13. The figure shows three temperature scales with the freezing and boiling points of water indicated.

(a) Rank the size of a degree on these scales, greatest first.
(b) Rank the following temperatures, highest first: 50°C, 50°F and 50°C.

Sol.

14. What is the temperature at which we get the same reading on both the centigrade and Fahrenheit scales?
1. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. The elastic energy stored in the wire is \[ \text{[AIEEE 2003]} \]
   (a) 0.2 J  (b) 10 J  (c) 20 J  (d) 0.1 J  
   Sol.

2. A wire fixed at the upper end stretches by length \( l \) by applying a force \( F \). The work done in stretching is \[ \text{[AIEEE 2004]} \]
   (a) \( \frac{F}{2l} \)  (b) \( FL \)  (c) \( 2FL \)  (d) \( \frac{F^2}{2} \)  
   Sol.

3. If \( S \) is stress and \( Y \) is Young’s modulus of material of a wire, the energy stored in the wire per unit volume is \[ \text{[AIEEE 2005]} \]
   (a) \( 2SY \)  (b) \( \frac{S^2}{2Y} \)  (c) \( \frac{2Y}{S^2} \)  (d) \( \frac{S}{2Y} \)  
   Sol.

4. A wire elongates by \( \frac{l}{2} \) mm when a load \( w \) is hanged from it. If the wire goes over a pulley and two weights \( w \) each are hung at the two ends, the elongation of the wire will be \( \text{(in mm)} \) \[ \text{[AIEEE 2006]} \]
   (a) \( l \)  (b) \( 2l \)  (c) zero  (d) \( \frac{l}{2} \)  
   Sol.

5. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area \( A \) and wire-2 has cross-sectional area \( 3A \). If the length of wire 1 increases by \( \Delta x \) on applying force \( F \), how much force is needed to stretch wire 2 by the same amount? \[ \text{[AIEEE 2009]} \]
   (a) \( F \)  (b) \( 4F \)  (c) \( 6F \)  (d) \( 9F \)  
   Sol.
6. A metal rod of Young's modulus $Y$ and coefficient of thermal expansion $\alpha$ is held at its two ends such that its length remains invariant. If its temperature is raised by $t^\circ C$, the linear stress developed in it is \[ \text{[AIEEE 2011]} \]

(a) $\frac{ut}{Y}$  \hspace{1cm} (b) $Yut$  \hspace{1cm} (c) $\frac{Y}{\alpha t}$  \hspace{1cm} (d) $\frac{1}{Yut}$

Sol.

7. An aluminium sphere of 20 cm diameter is heated from 0$^\circ$C to 100$^\circ$C. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_a = 23 \times 10^{-6} \, ^\circ \text{C}^{-1}$) \[ \text{[AIEEE 2011]} \]

(a) 28.9 cc \hspace{1cm} (b) 2.89 cc \hspace{1cm} (c) 9.28 cc \hspace{1cm} (d) 49.8 cc

Sol.

8. A wooden wheel of radius $R$ is made of two semicircular parts (see figure). The two parts are held together by a ring made of a metal strip of cross-sectional area $S$ and length $L$. $L$ is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by $\Delta T$ and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semicircular parts together. If the coefficient of linear expansion of the metal is $\alpha$ and its Young's modulus is $Y$, the force that one part of the wheel applies on the other part is \[ \text{[AIEEE 2012]} \]

(a) $2\pi Sy\alpha \Delta T$ \hspace{1cm} (b) $Sy\alpha \Delta T$ \hspace{1cm} (c) $Sy\alpha \Delta T$ \hspace{1cm} (d) $2Sy\alpha \Delta T$

Sol.
**Exercise - IV**

1. The apparatus shown in the figure consists of four glass columns connected by horizontal sections. The height of two central columns B & C are 49 cm each. The two outer columns A & D are open to the atmosphere. A & C are maintained at a temperature of 95° C while the columns B & D are maintained at 5°C. The height of the liquid in A & D measured from the base line are 52.8 cm & 51 cm respectively. Determine the coefficient of thermal expansion of the liquid. [JEE '97]

![Diagram](image)

**Sol.**

3. Two rods one of aluminium of length \( l \), having coefficient of linear expansion \( \alpha_a \) and other steel of length \( l \), having coefficient of linear expansion \( \alpha_s \) are joined end to end. The expansion in both the rods is same on variation of temperature. Then the value of \( \frac{l_1}{l_1 + l_2} \) is

[JEE' (Scr) 2003]

(A) \( \frac{\alpha_a}{\alpha_a + \alpha_s} \)  
(B) \( \frac{\alpha_a}{\alpha_a - \alpha_s} \)  
(C) \( \frac{\alpha_s + \alpha_a}{\alpha_s} \)  
(D) None of these

**Sol.**

2. A bimetallic strip is formed out of two identical strips one of copper and the other of brass. The coefficient of linear expansion of the two metals are \( \alpha_c \) and \( \alpha_b \). On heating, the temperature of the strip goes up by \( \Delta T \) and the strip bends to form an arc of radius of curvature \( R \). Then \( R \) is:

[JEE '99]

(A) proportional at \( \Delta T \)  
(B) inversely proportional to \( \Delta T \)  
(C) proportional to \( |\alpha_b - \alpha_c| \)  
(D) inversely proportional to \( |\alpha_b - \alpha_c| \)
4. A cube of coefficient of linear expansion $\alpha$, is floating in a bath containing a liquid of coefficient of volume expansion $\gamma$. When the temperature is raised by $\Delta T$, the depth upto which the cube is submerged in the liquid remains the same. Find the relation between $\alpha$ and $\gamma$, showing all the steps. [JEE 2004] 
Sol.

5. A 0.1 kg mass is suspended from a wire of negligible mass. The length of the wire is 1 m and its cross-sectional area is $4.9 \times 10^{-7}$ m$^2$. If the mass is pulled a little in the vertically downward direction and released, it performs simple harmonic motion of angular frequency 140 rad s$^{-1}$. If the Young’s modulus of the material of the wire is $E = 10^6$ N m$^{-2}$, the value of $n$ is [JEE 2010] 
Sol.
**Exercise - I**

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<table>
<thead>
<tr>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>37.</td>
<td>A,C,D</td>
<td>38.</td>
<td>A,C,D</td>
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<td></td>
</tr>
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**Objective Problems (JEE MAINS)**

1. (i) hollow sphere > solid sphere, (ii) hollow sphere = solid sphere
2. (a) 450       (b) 449       (c) 12:14:59   (d) (i) 1 (ii) 900/451 s (iii) \( \frac{1}{450 \times 10^{-8}} \)
3. (a) \( h' = h (1 + \alpha_x \Delta \theta) \), \( A' = A (1 + 2\alpha_x \Delta \theta) \), \( V' = V (1 + 3\alpha_x \Delta \theta) \)
   - volume of liquid \( V' = Ah(1 + \gamma \Delta \theta) \)
   - (b) \( h' = h (1 + \gamma \Delta \theta) \)
   - (c) (i) \( \gamma < 3\alpha_x \) (ii) \( \gamma > 3\alpha_x \) (iii) \( \gamma = 3\alpha_x \)
   - (d) \( \Delta V = Ah (\gamma_x - 3\alpha_x) \Delta \theta \)
   - (e) 3\( \alpha_x = h', \gamma \)
   - (f) \( h_x (1 - 3\alpha_x \Delta \theta) \), (g) \( h_x (1 + \gamma \Delta \theta) \). (h) (i) \( \gamma_x > 2\alpha_x \) (ii) \( \gamma_x < 2\alpha_x \) (iii) \( \gamma_x = 2\alpha_x \)

4. \( \gamma_x = \left( \frac{1}{g} + 27 \times 37 \times 10^{-8} \right) \)
5. (a) \( V_x d_2 \left[ \frac{1 + 3\alpha_x \Delta \theta}{1 - \gamma \Delta \theta} \right] \)
6. 5 sec slow    7. 10000 N    8. \( 4 \times 10^8 \) m/C    9. 5\( \alpha_x /3 \)
10. h/5R     11. 3V/20 12. 10 cm, 40 cm
13. (a) All lie (b) 50°X, 50°Y, 50°W. 14. -40°C or -40°F

**Exercise - II**

**JEE ADVANCED**

1. (i) hollow sphere > solid sphere, (ii) hollow sphere = solid sphere
2. (a) 450       (b) 449       (c) 12:14:59   (d) (i) 1 (ii) 900/451 s (iii) \( \frac{1}{450 \times 10^{-8}} \)
3. (a) \( h' = h (1 + \alpha_x \Delta \theta) \), \( A' = A (1 + 2\alpha_x \Delta \theta) \), \( V' = V (1 + 3\alpha_x \Delta \theta) \)
   - volume of liquid \( V' = Ah(1 + \gamma \Delta \theta) \)
   - (b) \( h' = h (1 + \gamma \Delta \theta) \)
   - (c) (i) \( \gamma < 3\alpha_x \) (ii) \( \gamma > 3\alpha_x \) (iii) \( \gamma = 3\alpha_x \)
   - (d) \( \Delta V = Ah (\gamma_x - 3\alpha_x) \Delta \theta \)
   - (e) 3\( \alpha_x = h', \gamma \)
   - (f) \( h_x (1 - 3\alpha_x \Delta \theta) \), (g) \( h_x (1 + \gamma \Delta \theta) \). (h) (i) \( \gamma_x > 2\alpha_x \) (ii) \( \gamma_x < 2\alpha_x \) (iii) \( \gamma_x = 2\alpha_x \)

4. \( \gamma_x = \left( \frac{1}{g} + 27 \times 37 \times 10^{-8} \right) \)
5. (a) \( V_x d_2 \left[ \frac{1 + 3\alpha_x \Delta \theta}{1 - \gamma \Delta \theta} \right] \)
6. 5 sec slow    7. 10000 N    8. \( 4 \times 10^8 \) m/C    9. 5\( \alpha_x /3 \)
10. h/5R     11. 3V/20 12. 10 cm, 40 cm
13. (a) All lie (b) 50°X, 50°Y, 50°W. 14. -40°C or -40°F

**Exercise - III**

**JEE MAINS (Previous Ques.)**

7. A 8. D

**Exercise - IV**

**JEE PROBLEMS**

1. \( 2 \times 10^{-3} \)C 2. B,D 3. A 4. \( \gamma = 2\alpha_x \) 5. 4
1. C.
   Given \( \ell = 1 \text{ mm}, \Delta \ell = 6 \times 10^{-5} \text{ mm} \)
   \( \alpha = 12 \times 10^{-6} \text{ k}^{-1} \)
   then
   \[ \Delta \ell = \ell \alpha \Delta T \]
   \[ 6 \times 10^{-5} \text{ mm} = (1 \text{ mm}) (12 \times 10^{-6}) \Delta T \]
   \( \Delta T = 5^\circ \text{C} \)

2. A
   Given
   \( \ell = 25 \text{ cm}, \ A = 0.8 \times 10^{-4} \text{ cm}^2 \)
   \( \Delta T = 10^\circ \text{C}, \ \alpha = 10^3 \text{ \circ C}^{-1}, \ Y = 2 \times 10^{11} \text{ N/m} \)
   then
   \[ \frac{\Delta \ell}{\ell} = \alpha \Delta T = \frac{F}{A Y} \]
   \( F = \alpha A \gamma \Delta T \)
   \[ = (10^{-4})(0.8 \times 10^{-4}) \times (2 \times 10^{11}) \times 10 \]
   \[ = 160 \text{ N} \]

3. C.
   \( L_1 = L + L_0 \alpha \Delta T \)
   \( L_2 = L + L_0 \alpha \Delta T \)
   Stress, \( \frac{Y \Delta L_1}{L} \) \( Y \Delta L_2 \)
   \[ I = \frac{2 Y_1}{3 Y_2} \Rightarrow \frac{Y_1}{Y_2} = \frac{3}{2} \]

4. C.
   \( I = \text{CMR}^2 \)
   \( dI = 2 \text{CMRdR} = 2 \text{CMR} [\frac{R \Delta T}{\rho}] \)
   \[ = 2 \mu I \Delta T \]

5. B
   \[ F = \frac{AY \Delta L}{L} = AY \alpha \Delta T \]
   \[ f = k \sqrt{\frac{E}{\mu \alpha}} = k \sqrt{\frac{AY \alpha \Delta T}{\rho \alpha}} \]
   \[ \Rightarrow f \alpha = \sqrt{\frac{Y_0}{\rho}} \]

6. B
   We know that
   \[ U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \]
   \[ = \frac{1}{2} \times Y (\alpha \Delta T)^2 \text{ volume} \]
   \[ U = \frac{1}{2} Y (\alpha \Delta T)^2 \ell \]
   \[ U = \ell (\alpha \Delta T)^2 \]

7. B
   \[ \frac{\Delta \ell}{\ell} = \alpha \Delta T = -\alpha \Delta T \]
   means read more so actual is less.

8. B
   Given \( L = 20 \text{ cm}, \ L_1 = 0.075 \text{ cm}, \ L_2 = 0.045 \text{ cm} \)
   \( \ell = L \alpha \Delta T \)
   \( 0.075 = 20 \alpha_1 (100) \)
   \( 0.045 = 20 \alpha_2 (100) \)
   Let for third rod \( L_3 \) and \( L_4 = 20 - L_1 \)
   So \( \Delta L_3 = \Delta L_4 + \Delta L_2 \)
   \[ 0.06 = L_2 \alpha_1 (100) + (20 - 20) \alpha_2 100 \]
   \[ L_1 = 10 \text{ cm} \]

9. A
   Given
   \[ f = \text{coefficient of cubical expansion} \]
   \[ \rho_{\text{bary}} = \rho' \]
   \[ \Rightarrow 266.5 \times 1.527 \]
   \[ 4 \left( \frac{7}{5} \right)^3 = 1 + 35f \]
   \[ 3 \pi \left( \frac{2}{2} \right) \]
   \[ f = 8.3 \times 10^{-4}/.c \]

10. B.
    Given volume at \( 0^\circ \text{C} = V_0 \)
    coefficient of Linear expansion = \( a_0 \)
    coefficient of cubical expansion = \( \gamma_m \)
    \[ h = \frac{V_0 - V' - V''}{V_0 (1 + \gamma_0 \Delta T) - V'' (1 + 3 a_0 \Delta T)} \]
    \[ = \frac{V_0 T (\gamma - 3 a_0)}{V_0 (1 + 2 a_0 \Delta T)} \]
    \[ A_0 \]
11. B

\[ F = \frac{A_y \Delta L}{L(1 + \alpha \Delta t)} \]

\[ F = \frac{A_{cm} \alpha t}{(1 - \alpha t)} \]

12. A.

At 0°C

\[ \rho_v, v, g = W_2 \] ...(1)

At t°C \( \rho_v, v, g = W \) ...(2)

\[ \rho_v, v, g = \frac{\rho_v}{(1 + \gamma t)} \] ...(3)

\[ W = W_2 + (\rho_v, (1 - \gamma t) v_1, (1 + \gamma t) - \rho_v v_2) g \]
\[ = W_2 [1 - (\gamma - \gamma t)] \]

13. C.

Initially \( P = \frac{V_r \rho_2}{A_r}, P' = \frac{V_r \rho_3}{A_r} \]

\[ P' = \frac{V_r (1 + 10^{-3})}{A_r (1 + 2 \times 10^{-3})} \]

\[ \frac{P'}{P} - 1 = \frac{1 - 10^{-2}}{1} \times 100 = -2\% \]

14 C.

\[ dx = \Delta dx \]

\[ \int \Delta dx = \int dx (3x + 2) \times 10^{-4} (20 - 0) \]

\[ \Delta L = (20 \times 10^{-4}) (\frac{3x^2 + 2x}{2}) \]

\[ \Delta L = (20 \times 10^{-4}) (\frac{3L^2}{2} + 2L) = 1.2 \text{ cm} \]

\( L_{new} = L + \Delta L \)

15. C.

Let eqn. temp = \( t \) then

\[ (m_2 s_2 t) = (m_2 s_2 (100 - t)) \] ...(1)

\[ d_4' = d_4 (1 + \alpha_4 t) \] ...(2)

\[ d_4' = d_4 (1 - \alpha_4 (100 - t)) \] ...(3)

Now \( d_4' = d_4' \) ...(4)

So. \( d_4 (1 + \alpha_4 t) = d_4 (1 - \alpha_4 (100 - t)) \)

\[ t = \frac{d_4 (1 - \alpha_4 (100 - d_4'))}{[d_4 - d_4]} \]

Put the above value of \( t \) in eq. 1.

\[ \frac{m_2 s_2 + 1}{m_2 s_2} \]

\( m_n = \frac{23}{54} \)

16. C.

\( \alpha_t, \alpha_p \) for \( \gamma - \gamma \) plane

\( \beta_{com} = 3 \times 10^{-3} \text{ per } ^\circ \text{C} \)

17. D.

\( \gamma_m = \gamma_{com} \Rightarrow D. \)

Volume increases but mass remains same.

18. C.

\( \gamma_m < \gamma_n \Rightarrow \rho_m > \rho_w \)

So completely Immersed

\( \Delta \rho < \Delta \rho_n \)

So \( W_j > W_i \) [Displaced mass of alcohol is less]

19. D.

Initially \( \rho, \rho, \text{ and } V \) density of sphere, density of liquid and volume.

\[ \frac{B_r - B_l}{B_r} \times 100 - \frac{V_r \rho_1 - V_1 \rho_1 g}{V_1 \rho_1 g} = \frac{[(1 + \gamma, M)(1 - \gamma, M) - 1]}{100} = -0.05 \text{ (decreases)} \]

20. B.

\[ \frac{\Delta L}{L} \times 100 = 1 = 100 \alpha \Delta t = 100 \alpha (T_2 - T_1) \]

\[ \frac{\Delta A}{A} \times 100 = 200 \alpha \Delta t = 2\% \]

21. C.

\[ \alpha_t = \frac{\Delta L}{L}, \Delta L = 3L \eta, \Delta L = L x \Delta t = (2L) (2\alpha) \Delta t \]

\[ \alpha_{cm} = \frac{\alpha + 4\alpha}{3} = \frac{5\alpha}{3} \]

22. B.

Given \( \beta = 1.4 \times 10^{-3} \frac{P_a \alpha}{\Delta V/V} \cdot \Delta T = 30^\circ C - 20^\circ C = 10^\circ C \)

\[ \beta = -\frac{\Delta P}{\Delta V/V} \Rightarrow \Delta P = -\beta \frac{\Delta V}{V} \]
\[
\Delta P = \beta (3 \alpha \Delta T) = 1.4 \times 10^2 \times 3 \times 1.7 \times 10^4 \times 10 = 7.14 \times 10^7 \text{ Pa.}
\]

23. A.
At 40°C
1 Unit will be = \(1(1 + \alpha \Delta T)\) units
\(= 1(1 + 12 \times 10^{-4} \times 40)\) Units
So 100 Unit will be = \(100(1 + 12 \times 10^{-4} \times 40) = 100 (1 + 40 \times 10^{-4}) = 100\) units
Actual
\(100 (1 + 40 \times 10^{-4}) = l_1 (1 + (2 \times 10^{-4}) = 100 [1 + 400 \times 10^{-4}] > 100\)mm.

24. C
\(V_m\) denote volume of mercury
\[V_m = V_{max} - V_m = V'_{max} - V'_{m}\]
\[V_{max} \times 300 = V_{max} [1 + 3 \times (9 \times 10^{-4}) \Delta t] - 300 [1 + 8 \times 10^{-4} \Delta t]
\]
\[V_{max} = \frac{(300 \times 1.8 \times 10^{-4}) \Delta t = 2000 \text{ cm}^3}{27 \times 10^{-8} \Delta t}
\]

25. B.
Because floating
\[\rho_s V_g = \rho \left(\frac{V}{2}\right) g\]
\[2 \rho_s = \rho.
\]

26. A.
if \(\gamma_i > \gamma_s\) then submerged more else come out of liquid respectively
and \(\gamma_i > \gamma_s\) (always)

27. A.
\[\gamma' = \gamma [1 + \gamma \Delta t]\]
\[\rho' = \rho [1 - \gamma \Delta t]\]
\[\rho \left(\frac{V}{2}\right) g = \rho' \left(\frac{V'}{2}\right) g\]
\[\rho \left(\frac{V}{2}\right) g = \rho [1 - \gamma \Delta t] \left(\frac{V}{2}\right) (1 + \gamma_i \Delta t)g\]
\[(1 - \gamma \Delta t) (1 + \gamma_i \Delta t) = 1\]
\[(1 - \gamma \Delta t) (1 + 3 \gamma_i \Delta t) = 1\]
\[3 \alpha_i - \gamma = 0\]

28. A.
Initially \(\rho_s (A_i h) g = (\rho_i A_i h) g\) \(...(1)\)
Now \(\rho' (A_i' h) g = (\rho_i A_i' h) g\) \(...(2)\)
\[\rho_i (1 - \gamma \Delta t) h = \rho_i (1 - 3 \alpha_i \Delta t) h \quad (1 + \alpha_i \Delta t)
\]
\[h = 2 \alpha_i
\]

29. A.
\[\rho' < \rho_i \quad \text{or} \quad \rho_i = \frac{\rho}{2} > \frac{\rho_i}{2}\]
\[\frac{1 + \gamma_i \Delta t}{\gamma_i \Delta t} > \frac{2}{\gamma_i} \quad 1 + \gamma_i \Delta t > 2 \Delta t > \frac{1}{\gamma_i}\]
\[T_f - T > \frac{1}{\gamma_i} = T_f > T + \frac{1}{\gamma_i}
\]

30. C.
Given \(\gamma_i - \gamma_s = c\)
and \(\gamma_i - \gamma_s = s\)
\[\gamma_i = \frac{c + \gamma_s - s = 3 \alpha_i}{3}
\]

31. B.
\[
\Delta Q_{ca} = 100 \times 4 \times 60 = 24000 \text{ cal. for } 0°C \quad \text{water}
\]
\[
\Delta Q_i = (100 \times 0.2 \times 20) + (200 \times 0.5 \times 20)
\]
\[+ (200 \times 80) = 18400 \text{ cal. for } 25.5°C
\]
So let temp is t then.
\[
24000 - 18400 = (200 \times 1 + 100 \times 0.2)t
\]
\[t = 25.5°C
\]

32. C.
\[
\rho_{25} h_i g = \rho_{25} h_2 g
\]
\[
\rho_2 (120) = \rho_2 (1 - \gamma 30) (124)
\]
\[
\gamma = \left(1 - \frac{120}{124}\right) \frac{1}{30} = 11 \times 10^{-4} /°C
\]

33. A.
\[
(212 - 37)°F 
\frac{(100 - 0)°C}{25°C = 45°F}
\]

34. D.
at 0°C
\[V_{ho} = 20A\]
\[V_{ho} = 30A\]
Now at time T y read 120°C
\[\text{So, } V_{sy} = A (120) = 30A (1 + \gamma_m T)\]
\[\text{and } V_{sy} = Ah = 20A (1 + \gamma_m T)\]

Dividing \[
\frac{120}{h} = 30 \Rightarrow h = 80.\]
**Multiple Choice Question**

35. C, D
   
   for Adiabatic
   
   \[ PV = \text{const.} \]
   
   \[ P \times \frac{1}{V} \]
   
   \[ PV = nRT \]

36. B, C
   
   Strain \rightarrow \text{Same}
   
   Stress = \frac{F}{A} = \text{constant}
   
   \[ F \times A \rightarrow F \times r^2 \]
   
   Energy = \frac{1}{2} \text{stress} \times \text{strain} \times \text{volume}
   
   \[ \times \text{Area} \]
   
   \[ \times r^2 \]

37. A, C, D
   
   Gravitational Potential Energy \( U_g = Mgl \)
   
   Elastic Potential Energy \( U_e = \)
   
   \[ \frac{1}{2} \text{stress} \times \text{strain} \times \text{volume} \]
   
   \[ = \frac{1}{2} \frac{F}{2A/r^2} \times V \left( F - mg \right) \]
   
   \[ V = A/r_0 \]
   
   \[ = \frac{1}{2} mgl \]

   Heat Produced = \( U_r = \frac{1}{2} Mgl \)

38. A, C, D
   
   (A) % rise in area = \( \beta \Delta T \)
   
   \[ = 2(\alpha \Delta T) \]
   
   \[ = 2 \times 0.2 = 0.4\% \]
   
   (C) % rise in volume = \( 3 \alpha \Delta T \)
   
   \[ = 3 \times 0.2 = 0.6\% \]
   
   (D) \( \alpha = \frac{0.2}{80 \times 100} = 0.25 \times 10^{-4}/\degree C \)
Exercise - II

1. (i) \( V_{aw} = V_{material} \)
   So Hollow feel more pressure from inside and increase more due to air pressure

2. (a) \( \frac{15 \times 60}{2} = 450 \) oscillation
   (b) \( T = k \sqrt{r} \)
   \[ T' = k \sqrt{r + \Delta r} - k \sqrt{r} \left( 1 - \frac{\Delta r}{r} \right)^{3/2} \]
   \[ = k \sqrt{r} \left( 1 + \frac{\Delta r}{2r} \right) \]
   \[ T' = T \left( 1 + \frac{\Delta r}{2r} \right) \]
   So \( T' \) time = 1 oscillation

3. (a) \( V_i' = Ah \left( 1 + 3 \alpha \Delta t \right) \)
   (b) \( h_i' = Ah \left( 1 + \gamma \Delta t \right) \)
   (c) (i) \( \gamma_i < 3 \alpha \)
   (ii) \( \gamma_i > 3 \alpha \)
   (iii) \( \gamma_i = 3 \alpha \)
   (d) \( \Delta V = V_i' - V_o' = Ah \left( 1 + \gamma_i \Delta t \right) - Ah \left( 1 + 3 \alpha \Delta t \right) \)
   \[ = Ah \left( \gamma_i - 3 \alpha \right) \Delta t \]

3. (e) \( A(h - h_i) = V_i' - V_o' \)
   \[ Ah - Ah_i = Ah \left( 1 + \gamma_i \Delta t \right) - Ah \left( 1 + 3 \alpha \Delta t \right) \]
   \[ 0 = h\left(\gamma_i - 3 \alpha \right) \Delta t \]
   (f) (i) \( Ah_i = Ah \)
   \[ h = \frac{Ah_i}{A_i} = \frac{h_i}{\left( 1 + 2 \alpha \Delta t \right)} \]
   Now \( h = h_i \left( 1 + \alpha \Delta t \right) \)
   So \( h_i' = h_i \left( 1 + 3 \alpha \Delta t \right) \)

(f) (ii) \( Ah_i = V_o \)
   \[ V_i = V_o \left( 1 + \gamma_i \Delta t \right) \]
   \[ Ah_i' = V_i \left( 1 + \gamma_i \Delta t \right) \]
   \[ h_i' = h_i \left( 1 + \gamma_i \Delta t \right) \]

(f) (iii)
   (1) \( \gamma_i > 3 \alpha \)
   (2) \( \gamma_i < 3 \alpha \)
   (3) \( \gamma_i = 3 \alpha \)

4. \( \rho = \frac{156.25 - 56.25}{V_i} \) at 15°C
   \[ \rho_i' = \rho \left( 1 - \gamma \Delta t \right) \] at 52°C
   \[ V_i' = V_i \left( 1 + \gamma \Delta t \right) \]
   \[ \frac{156.25 - 66.25}{V_i \left( 1 + \gamma \Delta t \right)} = \frac{156.25 - 56.25}{V_i} \left( 1 - \gamma \Delta t \right) \]
   \[ \Rightarrow \frac{90}{1 + 3 \times 9 \times 10^{-3}} = 100 \left( 1 - \gamma_i \times 37 \right) \]
   \[ \gamma_i = \]
\[
\frac{1}{3700} \left[ 100 - \frac{90}{1 - 37 \times 9 \times 3 \times 10^{-3}} \right] \\
\Rightarrow \gamma = 2.72 \times 10^{-1} \degree C
\]

5 (a) 
initially \( \rho_0 = \rho_n = d_a \) 
\[ F_{inert} = \rho_n^i (V_o^i)g \]
\[ = d_a (1 - \gamma, \Delta \theta) V_o (1 + 3a, \Delta \theta) g \]
(b) 
(i) \( 3a_r > \gamma_L \) 
(ii) \( 3a_r < \gamma_L \) 
(iii) \( 3a_r = \gamma_L \)

6. \( T = k \sqrt{\gamma} \)

\[ \frac{dT}{T} = \frac{k}{2} \frac{d\sqrt{\gamma}}{\sqrt{\gamma}} = \frac{k}{2} \sqrt{\gamma} \Delta \theta \]

In T + dT lag by \( = dT \)

In 10^4 slow by \( \frac{dT}{T + dT} \times 10^6 \text{sec} \)

\[ = \frac{dT}{1 - \frac{dT}{T}} \times 10^6 \text{Sec} \]

\[ = \frac{1}{2} \sqrt{\gamma} \Delta \theta \times 10^4 \text{sec} \]

\[ = \left( \frac{1}{2} \sqrt{\gamma} \right) \left( 1 - \frac{1}{2} \sqrt{\gamma} \right) \times 10^4 \]

\[ = \frac{1}{2} \sqrt{\gamma} \Delta \theta \times 10^4 \]

\[ = \frac{1}{2} \times 10^4 \times (30 - 20) \times 10^6 \]

\[ = 5 \text{ sec slow.} \]

7. 
\[ F = Ay \frac{\Delta \theta}{\gamma} \]

\[ = Ay \Delta \theta \]

\[ = 10^{-1} \times 10^{11} \times 10^{-4} (100 - 0^\circ) \]

\[ = 10000 \text{ N} \]

8. C.M \( = \gamma = \frac{h}{3} \text{ from 0} \)

\[ dy = \frac{1}{3} dh \]

\[ dy = \frac{1}{3} (h \Delta \theta) \]

\[ \frac{dy}{d\theta} = \frac{1}{3} h \Delta \theta = \frac{2}{3} (\cos 30^\circ) \alpha \]

\[ = \left[ \frac{1}{3} \left( 2 \times \sqrt{3} \times 4.3 \times 10^{-4} \right) \right] \text{m/}^\circ \text{C} \]

\[ = 4 \times 10^{-4} \text{ m/}^\circ \text{C} \]

9. \( \Delta L = L_1 + L_2 \)

\[ = (3L) \alpha_2 \Delta \theta + 2L \alpha_2 \Delta \theta \]

\[ = \frac{1}{3} (\alpha + 42) = \frac{5\alpha}{3} \]

10. \( g_{en} = g_n \left( 1 - \frac{2h}{R} \right) \)

\[ T = k \sqrt{\frac{\ell}{g_{en}}} \text{ at 20}^\circ \text{C and height h} \]

\[ T' = k \sqrt{\frac{\ell (1 + \alpha (30 - 20))}{g_{en}}} \text{ at 30}^\circ \text{ at height h} \]

\[ T'' = k \sqrt{\frac{\ell (1 + \alpha (30 - 20))}{g_n}} \]

\[ T = T'' \]

\[ \sqrt{\frac{1}{\frac{2h}{R}}} = \sqrt{1 + \alpha (30 - 20)} \]
\[ \alpha = \left[ \frac{2h}{R - 2h} \right] \frac{1}{10} R \gg \gg h \]

So \(\alpha = \frac{h}{5R}\)

11. \(\gamma_s = 20 \alpha_s\)
\[ V - V_o' = V' - V_o' \]
\[ V - V_o = V(1 + 3 \alpha_s \Delta \theta) - V_o(1 + \gamma_m \Delta \theta) \]
\[ 0 = 3\alpha_s V - \gamma_m V_o \]
\[ 0 = 3\alpha_s V - 20 \alpha_s V_o \Rightarrow V_o = \frac{3V}{20} \]

12. \(\frac{\Delta r}{r} = \alpha_n \Delta \theta \Rightarrow \frac{0.05}{25} = \alpha_n (100)\)

and \(0.04 = \alpha_n (100)\)

\(\Delta r = \Delta_r \approx \Delta \theta\)
\[ 0.03 = (\alpha_n (50) + (50 - \alpha_n) \alpha_n) 50 \]
\[ \alpha_n = \frac{0.03 - 2500\alpha_n}{(\alpha_n - \alpha_n)50} \]
\[ = 10 \text{cm} \]

13. (a) \(R_s = 90 - (-20) = 90^\circ\)
\(R_s = (90 - 0) = 90^\circ\) same
\(R_s = (120 - 30) = 90^\circ\)

(b) \(50^\circ W < 50^\circ y < 50^\circ x\)

14. \(\frac{212 - 32}{100 - 0} \times T^\circ C = T^\circ F - 32\)
\[ T = -40 \]
1. **D**

Elastic energy stored in the wire is

\[ U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \]

\[ = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta L}{L} \times \frac{A \times L}{2} \]

\[ = \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1J \]

2. **D**

Work done in stretching the wire = potential energy stored

\[ = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \]

\[ = \frac{1}{2} \times \frac{F}{A} \times \frac{1}{L} \times \frac{A \times L}{2} = \frac{1}{2} \frac{F \times L}{2} \]

3. **B**

Given \( S = \text{Stress} \)

\( Y = \text{Young's modulus} \)

We know that \( U = \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume} \)

Energy per unit volume

\[ \frac{U}{\text{Volume}} = \frac{1}{2} \times \frac{\text{stress}}{Y} \]

(strain = \( \frac{\text{stress}}{Y} \))

\[ \frac{U}{V} = \frac{1}{2} \frac{S}{Y} \]

\[ \frac{U}{V} = 2 \frac{S'}{Y} \]

4. **A**

Let us consider the length of wire as \( L \) and cross-sectional area \( A \), the material of wire has Young's modulus as \( Y \).

**Case 1**

**Case 2**

Then for 1st case

\[ Y = \frac{w/A}{1/L} \]

For 2nd case,

\[ Y = \frac{w/A}{2F/L} \]

\[ \Rightarrow f = \frac{1}{2} \]

So, total elongation of both sides = \( 2f = 1 \)

5. **D**

\( A_i l_i = A_j l_j \)

\[ \Rightarrow l_2 = \frac{A_j A_i}{A_i} = \frac{A \times l_1}{3A} = \frac{l_1}{3} \]

\[ \Rightarrow \frac{l_1}{l_2} = 3 \]

\( \Delta x_1 = \frac{F_1}{A_1} \times l_1 \) \( \ldots (i) \)

\( \Delta x_2 = \frac{F_2}{3A_2} \times l_2 \) \( \ldots (ii) \)

Here

\( \Delta x_1 = \Delta x_2 \)

\[ \frac{F_1}{A_1} \times l_1 = \frac{F_2}{3A_2} \times l_2 \]

\[ F_2 = 3F_1 \times \frac{l_1}{l_2} = 3F_1 \times 3 = 9F \]

6. **C**

\[ \Delta L = \alpha L \Delta T = \frac{FL}{AY} \]

\[ \Rightarrow \text{Stress} = \frac{F}{A} = \gamma a \Delta T \]

7. **B**

Given \( d = 20 \text{ cm, } t = 0^\circ \text{C to } t = 100^\circ \text{C} \)

\( V = V_0 (1 + \gamma t) \)

\( V = V_0 (1 + 3a \Delta t) \) \( (\gamma = 3a) \)

change in volume = \( V - V_0 \)

\[ = 3V_0 \alpha \Delta t \]

\[ = 3 \times 4 \pi \frac{(d/2)^3}{3} \times 23 \times 10^4 \times 100 \]

\[ = 3 \times 4 \pi \left( \frac{0.2}{2} \right)^3 \times 23 \times 10^4 \times 100 \]

\[ = 28.9 \text{ cc} \quad (1 \text{ cc} = 10^{-6} \text{m}^3) \]

8. **A**

Increase in length

\[ \frac{\Delta L}{L} = \alpha \Delta T \]

\[ \frac{\Delta L}{L} = \alpha \Delta T \]

Then thermal stress developed is

\[ \frac{T}{S} = \frac{\Delta L}{L} = \gamma a \Delta T \]

\[ T = SY \gamma a \Delta T \]

From FBD of one part of the wheel

\( F = 2T \)

Where \( F \) is force that one part of wheel is applies on the other part.

\( F = 2SY \gamma a \Delta T \)
1. \( \rho_1 = \frac{\rho_2}{1 + \gamma t} \)
\[ \rho_1 + h_k \rho_2 g - h \rho_{ns} g = \rho_2 + h_k \rho_2 g - h \rho_{ns} g \]
\[ \rho_{ns} = \frac{h_k}{h_k - h} = \frac{\rho_2}{1 + 95T} = \frac{\rho_2}{h_k + h} \]
\[ \Rightarrow r = 2 \times 10^{-4} \text{g/cm}^3, \quad \alpha = \frac{r}{g} = 6.7 \times 10^{-1} \text{g} \]

2. \( B, D \)
\[ f_0 (1 + \alpha_n \Delta T) = (R + d) \theta \]
\[ f_0 (1 + \alpha_c \Delta T) = R \theta \]
\[ R - d = \frac{1 + \alpha_n \Delta T}{1 + \alpha_c \Delta T} = 1, \quad d = \frac{1 + \alpha_n \Delta T}{1 + \alpha_c \Delta T} \]
\[ \text{[Binomial Expansion]} \]
\[ R = \frac{d}{(1 + \alpha_n \Delta T) + \alpha_n \Delta T} = \frac{1}{\Delta T} \quad \text{and} \quad R = \frac{1}{\alpha_n - \alpha_c} \]

3. \( A \)
\[ f_1 u_1 = f_2 u_2 \]
\[ \Rightarrow f_1 = \frac{u_1}{f_2} = \frac{u_1}{f_2 + f_1} = \frac{u_1}{u_2} \]
\[ \Rightarrow \gamma = 2a_1 \]

4. \( T \uparrow V_{gig} \uparrow p \uparrow \)
Depth submerged in liquid remains same
Upthrust = Weight
\[ v_i \rho_i g = v_i' \rho_i' g \]
\[ (Ah_i) \rho_i g = A(1 + 2a_1 \Delta T) h_i \left( \frac{\rho_i}{1 + \gamma \Delta T} \right) g \]
\[ \Rightarrow \gamma = 2a_1 \]

5. \( \omega = \frac{k}{m} k = 0.1 (140)^2 \)
\[ Y = \frac{A}{L}, \quad Y = \frac{0.1 \times 140 \times 140 \times 1 \times 10}{4.9 \times 10^{-7} \times 10} \]
\[ = 4 \times 10^9 \text{Nm}^{-2} \quad N = 4 \]