# THERMAL PROPERTIES OF MATTER

## HEAT AND TEMPERATURE:
- Heat and work both are ways of transfer of energy.
  - Non-mechanical Way
  - Mechanical Way
- Temperature is the measurement of hotness and coldness.
  - Reason: if & 1
    - Feel: Avg. kinetic energy of each molecule
    - (Energy per molecule)
- Heat is flown due to temp. difference, i.e., difference in vibrational kinetic energy.
- When two bodies at different temp. are connected the molecules with greater vibrational energy gives energy to molecules with lower energy and hence temp. of hotter body falls and temp. of colder body rises up to same value.

## THERMOMETRY:
- Measure of temperature.
  - For any scale... Reading - ice point = Constant
  - Steam point - ice point

<table>
<thead>
<tr>
<th>Boiling of water (Steam Point)</th>
<th>100°C</th>
<th>212 °F</th>
<th>373.15 K</th>
</tr>
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<tr>
<td>Reference Points</td>
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<tr>
<td>Melting of ice (Ice point)</td>
<td>0°C</td>
<td>32 °F</td>
<td>273.15 K</td>
</tr>
</tbody>
</table>
For fahrenheit and celsius...
\[
\frac{F - 32}{212 - 32} = \frac{C - 0}{100 - 0} \Rightarrow \frac{C}{5} = \frac{F - 32}{9}
\]

For kelvin and celsius...
\[
\frac{K - 273.15}{373.15 - 273.15} = \frac{C - 0}{100 - 0} \Rightarrow K = C + 273.15
\]

General form...
\[
\frac{F - 32}{212 - 32} = \frac{K - 273.15}{373.15 - 273.15} = \frac{C - 0}{100 - 0}
\]

Convert 27°C in K and F.
\[
\frac{27 - 0}{100 - 0} = \frac{K - 273.15}{373.15 - 273.15} = \frac{F - 32}{212 - 32}
\]

Find the value of F and K.

For Reumes's scale ice point is 0° and steam point is 80°. Find the value of 70°C in terms of °R.
\[
\frac{C - 0}{100 - 0} = \frac{R - 0}{80 - 0} \Rightarrow \frac{C}{100} = \frac{R}{80} \Rightarrow R = \frac{80}{100} \times 70 = 56°
\]

If temp. is changed from 30°C to 60°C. Find the value of ΔT in F and K.
\[
0°C \rightarrow 100°C \quad \{ \text{Change is same} \}
\]
\[
273.15 K \rightarrow 373.15 K \quad \Rightarrow \Delta T_C = \Delta T_K = 100°C
\]

0°C → 100°C?
\[
\begin{align*}
32°F & \rightarrow 212°F \\
\end{align*}
\]
\[
\frac{100 \Delta T_C}{\Delta T_F} = 1.8 \Rightarrow \Delta T_C = 1.8 \Delta T_F
\]
Now... \[ \Delta T_F = \frac{\Delta T_c}{1.8} \]

Job 'c me! 4 change ñi 4F ñi

\[ \Rightarrow \Delta T_F = 1.8 \times \Delta T_c = 1.8 \times 30 = 54 \, ^\circ F \]

\[ \Delta L = \Delta K = \Delta F \]

\[ \text{THERMAL EXPANSION :-} \]

When we give heat to a substance, its temp. rises and it expands.

Heat given \[ \Rightarrow \] Molecules vibrate

\[ \Rightarrow \] Expansion

\[ \text{Sawal Utha} \Rightarrow \text{Vibrate kaise} \text{ to door hi kyu jayenge... par bhi aar sakte hai.} \]

\[ \Rightarrow \text{Avg. position same} \]

\[ \Rightarrow \text{No expansion } \times \]

\[ \Rightarrow \text{Main reason of expansion :-} \text{ The well of energy-distance is not symmetrical and hence avg. distance is increases and hence substance expands.} \]
Linear expansion:

1. D Rod

\[ L \text{ (} L, T \text{)} \rightarrow \text{Heat} \rightarrow \text{L (} L+\Delta L \text{)} \]

\[ [\Delta T \text{ is not very large}] \]

It is observed that... \( \Delta L \propto \Delta T \) and \( \Delta L \propto L \)

\[ \Delta L = L \propto \Delta T \]

Co-efficient of linear expansion

\[ \alpha \]

- \( \alpha \) depends upon material
- \( \alpha \) depends upon temp. (It is a constant)
- Increase in length per unit length and per unit rise in temp.

\[
(\alpha = \frac{\Delta L}{L \Delta T}; \text{if } L=1 \text{ m } \Rightarrow \alpha = \Delta L)
\]

Volume expansion: (3-D)

- If \( \Delta T < 100^\circ C \) then ...

\[ \Delta V \propto V \]

\[ \Delta V = V \gamma \Delta T \]

Co-efficient of volume expansion

\[ \gamma \]

- \( \gamma \) depends on material
- \( \gamma \) depends on temp. largely
- Increase in volume per unit volume and per unit rise in temp...

\[
(\gamma = \frac{\Delta V}{V \Delta T}; V=1 \text{ m}^3 \Rightarrow \gamma = \Delta V)
\]
From graph we may conclude that...

i) At lower temp... $\gamma$ T with temp. ↑

ii) At higher temp... $\gamma$ is nearly constant

\[ \Rightarrow \text{Isotropic Solid} : -(\text{Amorphous}) \]

Some properties in all directions (Disordered)

\[ \Rightarrow \text{For these type of solids... } [\gamma = 3\alpha] \text{ (Experimentally verified)} \]

Proof:

\[ (V, l) \]

Rising temp. by $dT$...

\[ \frac{dV}{V \gamma} = \Delta T \]

\[ \Rightarrow 3l^2 \cdot dl = V \gamma \Delta T \]

\[ V = l^3 \Rightarrow \frac{dV}{V} = 3l^2 \cdot dl \]

\[ \Rightarrow 3l^2 (l \alpha \Delta T) = l^3 \gamma \Delta T \]

\[ \Rightarrow 3\alpha = \gamma. \text{ (Proved)} \]

\[ \Rightarrow \text{Area expansion} \quad \Delta A = AB \Delta T \]

Co-efficient of superficial/Area expansion.

\[ \beta = 2\alpha \quad \text{(Experimentally verified)} \]

Find the value of $\gamma$ for an ideal gas at constant pressure

\[ PV = nRT \Rightarrow P \frac{dV}{V} = nR \frac{dT}{T} \]

\[ \Rightarrow P(V \gamma dT) = nR \frac{dT}{T} \]

\[ \Rightarrow \gamma = \frac{nR}{PV} = \frac{nR}{nRT} = \frac{1}{T} \]
# ANOMALOUS EXPANSION OF WATER

- 4°C की ऊपर ★ contract ही नहीं है।
- 4°C की नीचे ★ expand ही नहीं है।

**Important Observations:**

1. In cold countries, water pipes are burst.

   As temp ↓↓ below 4°C...

   Water expands ★ Burst

2. At cold nights, crops are destroyed. Because ice start
   burst तो ज्यादा

3. Ice freezes only at lake surfaces and hence aquatic
   creatures cannot live below surface.

   As temp. ↓↑ above 4°C...

   Water contracts and become

   heavy and goes down.

   ★ Phir niche wala upon

   aake contract...

एक तम आयेगी तब साता पानी 4°C पर आयेगा।

Surface wallah 4°C से नीचे expand करेगा और

उपर से लेंगा।

⇒ Temp 0°C से तल्ले से surface wallah paani

ice ban jaayega.

⇒ Surface के niche paani mast 4°C पर

⇒ मृत्तिका Zinda hai.
**BIMETALLIC STRIP:**

If \( \alpha_2 > \alpha_1 \), then the 2nd rod will expand more and \( l_2 > l_1 \). Hence \( l_2 \) increases more than \( l_1 \) as shown in figure.

\[
\begin{align*}
( l_2 = (R+d)\theta ) \\
( l_1 = R\theta < l_2 )
\end{align*}
\]

- Used in fire alarm.
  - Temp. ↑↑ ⇒ Strip bent
  - Loose contact
  - Alarm sounds

\[
\frac{l_2}{l_1} = \frac{(R+d)\theta}{R\theta} = \frac{L\alpha_2 \Delta T}{L\alpha_1 \Delta T} \Rightarrow \frac{R+d}{R} = \frac{\alpha_2}{\alpha_1}
\]

\[
\Rightarrow \alpha_1 R + \alpha_2 d = \alpha_2 R
\]

\[
R = \frac{d\alpha_1}{\alpha_2 - \alpha_1}
\]

**FAULT IN METALLIC SCALES:**

- Rod \( l = 100 \text{ cm} \)
- Temp. ↑↑ \( l = 100 \text{ cm} \)
- Scale measures shorter length of the rod than actual.

Assuming length of rod doesn't change...
True Reading = 100 = \( \Box_{\text{M}} + 20 \)

\( \text{Measur} \)

\( \text{Length} \)

\( \text{Rise in length of scale} \)

Hence...

\( \text{True reading} = \text{M.L.} + \Delta L_{\text{scale}} \)

(Note that \( \Delta L_{\text{scale}} \) will be \(-ve\) if temp. is lowered.)

A steel scale calibrated for 20°C is used to measure the length of a rod at 30°C which comes out to be \( x \). Find the true length of the rod if \( \alpha_s \) is given for scale.

\( \text{Length of scale} = 100 \text{ cm} \)

\( \text{III} \rightarrow \text{Kai bhook stef ustali} \)

\( \text{Phir Se} \)

\( \text{Galti}!! \)

\( \text{L} = x + 1000x_s(30-20) \)

\( \Rightarrow L = x + 1000x_s \)

\( \Rightarrow L(1-10x_s) = x \)

\( \Rightarrow L = \frac{x}{1-10x_s} \)

\( \text{L} \times \text{Length of scale} \)

\( \text{L} = x + 1000x_s \)

(300 % correct sah \( \text{c} \))

\( \star \rightarrow \text{Steel Scale ke utne hi part ka expansion karo ki jitne ki actual rod ho.} \)

\( \Rightarrow \text{Pahle wallah hi sahi tha} \)

\( \Rightarrow L = \frac{x}{1-10x_s} \) (Ab 100 % sahi)

\( \text{1) Temp } \uparrow \Rightarrow \text{Readings stretched } \Rightarrow \text{Rod Gota} \)

\( \text{2) Temp } \downarrow \Rightarrow \text{Readings compressed } \Rightarrow \text{Rod Lamba} \)

\( \text{3) For same rise/fall... kitna lamba/gota = } \frac{x}{\Delta T} \)

(Rod ka Act Length)
If temp. is increased length of pendulum will also increase and hence time period will also increase \( (T \propto \sqrt{L}) \). 
\[ T = 2\pi \sqrt{\frac{L}{g}} \quad \text{and} \quad T' = 2\pi \sqrt{\frac{L(1 + \alpha \Delta \theta)}{g}} \quad (\Delta \theta \to \text{temp. change}) \]

Hence... \[ T' = T \sqrt{1 + \alpha \Delta \theta} = (1 + \alpha \Delta \theta)^{1/2} \]

As \( \alpha \) is very small... \[ \frac{T'}{T} = 1 + \frac{\alpha \Delta \theta}{2} \]

\[ \implies T' = T + \frac{T \alpha \Delta \theta}{2} \]

\( \text{Put} \ \alpha \Delta \theta \text{ if temp. is lowered.} \)

Time lost/gain in measuring one time period... \( (\Delta T) = \frac{1}{2} T \alpha \Delta \theta \)

\[ \implies \text{Time lost/gain during } \Delta t \text{ sec} = \frac{T \alpha \Delta \theta}{2} \]

\[ \Delta t = \frac{T \alpha \Delta \theta}{2} = \frac{1 \times 24 \times 3600 \times 1.2 \times 10^{-5} \times (30 - 20)}{2} \]

\[ = 5.184 \text{ sec.} \]
# EFFECT ON DENSITY OF LIQUID:

\[(m, V, \rho) \xrightarrow{\Delta T} (m, V+\Delta V, \rho)\]

\[\rho = \frac{m}{V} \quad \rho' = \frac{m}{V+\Delta V}\]

Now \[\frac{\rho'}{\rho} = \frac{V}{V+\Delta V} = \frac{1}{1 + \gamma \Delta T}\]

\[\Rightarrow \rho' = \rho (1 + \gamma \Delta T)^{-1}\]

\[\Rightarrow \rho' = \rho (1 - \gamma \Delta T)\quad \text{(as } \gamma \text{ is very small)}\]

(Put \(\Delta T\) with signs)

# SPECIFIC HEAT CAPACITY := \(C\)

It is the amount of heat required to raise the temp.

of 1 g substance by 1°C.

\[\text{e.g. } C_{\text{water}} = 4.2 \text{ J/g} \cdot \text{°C (high)} \Rightarrow \text{Do not heat/cool rapidly}\]

\[C_{\text{copper}} = 0.4 \text{ J/g} \cdot \text{°C (low)} \Rightarrow \text{Heats and cools rapidly}\]

Higher SHC means more heat absorbs means less heat conducts and hence bad conductors.

High HSC \(\Rightarrow\) Bad conductors

Low HSC \(\Rightarrow\) Good Conductors

\[C = \frac{Q}{m \Delta T}\]

\[\Rightarrow \quad Q = mC \Delta T\]

(Use when temp. is changing)

Per unit mass

and per unit temp. rise/fall
1. Find the amount of heat required to heat 2 g water from 10°C to 40°C.

\[ Q = mc \Delta T = 2 \times 4.2 \times 30 = 252 \text{ J} \]

2. Find the amount of heat released when 2 g water is cooled from 70°C to 10°C.

\[ Q = mc \Delta T = 2 \times 4.2 \times 60 = 506 \text{ J} \]

3. A heater gives heat at a rate of 100 W. It is used to heat 50 g water from 20°C to 30°C. Find the time taken to do this.

Heat given by heater = Heat absorbed by water.

\[ 100t = mc \Delta T \]
\[ 100t = 50 \times 4.2 \times 10 \]
\[ t = 21 \text{ sec.} \]

4. A refrigerator extracts energy at a rate of 100 W. How much time will it take to cool a lemon squash from 30°C to 5°C. \((C_{\text{lemon}} = 4.2 \text{ g}^{-1} \text{°C}^{-1} \text{ J})\) (Mass = 100 g)

\[ 100t = mc \Delta T \Rightarrow 100t = 100 \times 4.2 \times 25 \Rightarrow t = 105 \text{ sec.} \]

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**PRINCIPLE OF CALORIMETRY / METHOD OF MIXTURES** -

Heat loss by \(\text{hot body}\) = Heat gained by \(\text{cold body}\)

(Based on law of conservation of energy)
10 g water at 30°C is mixed with 50 g sugar at 10°C. Find the temp. of mixture. \((c_{water} = 4.2 \text{ J/g} \cdot ^\circ C; \ c_{sugar} = 0.4 \text{ J/g} \cdot ^\circ C)\)

Heat lost by water = Heat gain by copper

\[ \Rightarrow 10 \times 4.2 \left(30 - \theta \right) = 50 \times 0.4 \left(\theta - 10\right) \]

\[ \Rightarrow \theta = \ldots \]

20 g of solid at 20°C is mixed with 100 g of cold water at 5°C. If final temp. of mixture is 10°C. Find SHC of solid.

Heat loss by solid = Heat gained by water

\[ \Rightarrow 20 \times c \times (20 - 10) = 100 \times 4.2 \times (10 - 5) \]

\[ \Rightarrow c = \ldots \]

In a copper vessel, we pour 20 g of water at 80°C. Now 100 g of cold water at 10°C is added to vessel. Find the final temp. of mixture if vessel weighs 50 g. \((c_{copper} = 0.4 \text{ J/g} \cdot ^\circ C)\)

Temp. of vessel = Temp. of content.

\[ \Rightarrow \text{Hot bodies are both hot water and copper vessel.} \]

\[ \Rightarrow \text{Heat lost by hot water} + \text{Heat lost by copper} = \text{Heat gained by cold water}. \]

\[ \Rightarrow 20 \times 4.2 \left(80 - \theta \right) + 50 \times 0.4 \left(80 - \theta \right) = 100 \times 4.2 \left(\theta - 10\right) \]

\[ \Rightarrow \theta = \ldots \]

The temp. of 600 g of cold water rises by 15°C when 300 g of hot water is added. Find initial temp. of cold water. \((\text{Temp. of hot water} = 50°C)\)
\[ Q_{\text{hot water}} = Q_{\text{cold water}} \]
\[ \Rightarrow 300 \times C \times (50 - (x + 15)) = 600 \times C \times (x + 15 - x) \]
\[ \Rightarrow x = 5^\circ C. \]

\[ (x \rightarrow \text{Initial temp. of cold water}) \]
\[ (x + 15) \rightarrow \text{E}q^{\text{m}} \text{ temp.} \]

**HEAT CAPACITY:**

- Amount of heat required to raise the temp. of whole body by 1°C. (Bada C)
  \[ C = mc \]

- Heat capacity of 20 g water = 20 \times 4.2 = 84 \text{ J}.

- She does not depend on mass but heat capacity depends on the given mass of substance.

- As \( Q = mc \Delta T \Rightarrow Q = C \Delta T \)

**LATENT HEAT:**

\[ \Rightarrow \text{Gopi Hui (Jo dikhai na de)} \]

- Latent heat is a hidden heat used when a substance changes its state.

- When we give heat during phase change, the temp. doesn't change. This heat is used to break/weaken the bonds for changing the phase.

\[ \text{Ice (0}^\circ \text{C)} \xrightarrow{\text{Heat}} \text{Water (0}^\circ \text{C)} \]
Specific latent heat: To change the phase of 1 g substance.

Specific latent heat of fusion of ice:

\[ \text{Ice} \xrightarrow{\text{Heat} 336 \text{ J}} \text{Water (0°C)} \]

\[ L_f = 336 \text{ J/g} = 80 \text{ cal/g} \]

Fusion ← Melting

Specific latent heat of vaporisation of water:

\[ \text{Water} \xrightarrow{\text{Heat}} \text{Water vapours (100°C)} \]

\[ L_v = 2268 \text{ J/g} = 540 \text{ cal/g} \]

Condensation ← Vaporisation/Boiling

As specific latent heat is the heat required per unit mass...

\[ L = \frac{Q}{m} \Rightarrow Q = mL \]

Specific latent heat (Use for phase change)

Find the amount of heat required to change 10 g of ice at -5°C to water at 20°C. \((\text{SHC}_{\text{water}} = 4.2 \text{ J/g°C})\)

\((\text{SHC}_{\text{ice}} = 2.1 \text{ J}^{-1}\text{°C}^{-1}\cdot\text{g})\) \((\text{SLH}_\text{fusion} = 336 \text{ J/g})\).

\[ \text{Ice} \xrightarrow{\text{Heat}} \text{Ice} \xrightarrow{\text{Heat}} \text{Water} \xrightarrow{\text{Heat}} \text{Water} \]

\[ (10 \text{ g}) \xrightarrow{\text{Heat}} (10 \text{ g}) \xrightarrow{\text{Heat}} (10 \text{ g}) \xrightarrow{\text{Heat}} (10 \text{ g}) \]
Total heat required
= $Q_1 + Q_2 + Q_3$
= $(10)(2.1)(5) + 10(336) + 10(4.2)(20)$

Q. Find the amount of heat released when 1 kg of water vapours is cooled to water at 20°C.
($S_{HC\_water} = 4.2 \text{ J/}^\circ\text{C}$) ($S_{LH\_vap} = 2260 \text{ J/}^\circ\text{C}$).

\[
\begin{align*}
\text{Water Vapours} & \xrightarrow{\theta_1} \text{Water} \\
(100^\circ\text{C}) & \xrightarrow{\theta_2} \text{Water} \\
1 \text{ kg} & \xrightarrow{\theta_3} 8 \text{ kg} \\
& \xrightarrow{\theta_4} 1 \text{ kg}
\end{align*}
\]

Total heat released
= $Q_1 + Q_2 + Q_3$
= $(1000 \times 2260) + (1000)(4.2)(80)$

Q. A piece of ice of mass 40 g is added to 200 g of water at 50°C. Calculate the final temp. when all ice is melted.
($S_{HC\_water} = 4.2 \text{ J/}^\circ\text{C} \cdot \text{K}$) ($S_{LH\_ice} = 336 \text{ J/}^\circ\text{C}$).

\[
\begin{align*}
(50^\circ\text{C}) & \xrightarrow{\text{mcAT}} (0^\circ\text{C}) \\
(200 \text{ g}) & \xrightarrow{\text{mcAT}} (0^\circ\text{C}) \\
& \xrightarrow{\text{mcAT}} (40 \text{ g}) \\
& \xrightarrow{\text{mcAT}} (0^\circ\text{C}) \\
\end{align*}
\]

Heat released by water = Heat gained by ice
\[
\Rightarrow \text{mcAT} = \text{mcAT} + \text{mcAT}
\Rightarrow (200)(4.2)(50-\theta) = (40)(336) + (40)(4.2)(\theta-0)
\Rightarrow \theta = \ldots
\]

Q. Result of mixing 10 g ice at -10°C with 10 g water at 10°C.
($C_w = 4.2 \text{ J/}^\circ\text{C} \cdot \text{K}$, $C_i = 2.1 \text{ J/}^\circ\text{C}$, $L_f = 336 \text{ J/}^\circ\text{C}$).
Water \( \rightarrow \) Water (0°C) \( \rightarrow \) Water (10°C) \( \rightarrow \) Ice (0°C) \( \rightarrow \) Ice (-10°C)

\[ 10 \times 4.2 \times (10 - 0) = 10 \times 4.2 \times 0 + 10 \times 336 + 10 \times 2.1 \times 10 \]

\[ 420 - 0 = 420 \]

\[ \Theta = -ve \quad \text{X} \quad (Kw6 \text{ to gadhad hai... Daya !!!)} \]

Max. heat released by water when it come to 0°C
\[ = m \times c \times \Delta T = 10 \times 4.2 \times 10 = 420 \text{ J} \]

Heat acquired by ice to reach zero
\[ = m \times c \times \Delta T = 10 \times 2.1 \times 10 = 210 \text{ J} \]

Heat remaining = 420 - 210 = 210 J

If all ice melts then heat required = mL = 10 \times 336 = 3360 J

\[ \Rightarrow \text{All ice will not melt} \]

Max. mass of ice that can be melted \( (m) = \frac{210}{336} = 0.025 \text{g} \)

\[ \Rightarrow \text{Some ice is still remaining} \Rightarrow T_f = \Theta = 0 \text{°C}. \]

# CONDUCTION :-

(Generally in solids)

- It is a way of heat transfer by molecular collisions.
- No actual flow of matter.
- \( T_1 \rightarrow T_2 \)

Molecules vibrate and transfers energy to neighbouring...
molecules by just vibrating collisions.

\[ \text{Variable State:} \]

- Heating and from one end... has cross-section of molecule. Krig heat absorb karke temp. aaste aur krig heat pass kar dengi.

- Temp. of each cross-section samy ke sath badalta hai.

\[ \text{Steady State:} \text{ Temp. of each point coi will be same.} \]

- Temp of each cross-section remains constant w.r.t time.

- Koi bhi cross-section heat absorb ni hain.

\[ \text{Heat passing through each cross-section is same.} \]

- Steady state me heat flow hoti hai.

- During steady state... \( \frac{dT}{dx} \neq 0 \) but \( \frac{dT}{dt} \) = 0.

\[ \text{Maths:} \]

\[ (T_1 - T_2) \]

\[ \phi \rightarrow \text{Amount of heat that flows in time} \ 't'. \]

\[ \phi \rightarrow \text{Amount of heat that flows in time} \ 't'. \text{ (So that the heat does not lose by radiation)} \]

\[ \phi \]
It is experimentally observed that...

\[ \Phi \propto A \]
\[ \Phi \propto (T_1 - T_2) \]
\[ \Phi \propto \frac{1}{l} \]

\[ \Phi = \frac{kA(T_1 - T_2)}{l} \]

Thermal conductivity (constant for a material)

SI unit: \( \frac{W}{m \cdot ^\circ C} \)

Common unit: \( \text{cm} \cdot \text{s} \cdot ^\circ C \)

\[ \text{Heat Current (H)} = \text{Rate of flow of heat} \]

\( \text{(Heat per second)} \)

\[ H = \frac{\Phi}{l} \Rightarrow H = \frac{kA\Delta T}{l} \]

(Differential from \( H = \frac{d\Phi}{dt} \))

A copper cube (a = 10 cm)
Temp. of two ends (100°C, 0°C)

\( k_{\text{copper}} = 400 \text{ W/m} \cdot ^\circ C \)

What is the rate of heat flow?

\[ H = \frac{kA\Delta T}{l} = \frac{400 \times (0.1)^2 \times 100}{0.1} = 4000 \text{ W} \]

Water \( (100^\circ C) \) \( \rightarrow \) Ice \( (0^\circ C) \) \( \rightarrow \) Water \( (100^\circ C) \) \( \rightarrow \) Ice \( (0^\circ C) \)

\[ \frac{m_1}{m_2} \rightarrow \text{Ratio of the ice melted} \rightarrow ? \] (Provided 't' same)

\[ \Phi_1 = H_1 \cdot t = \frac{k(A) \Delta T}{x} = 2 \]

\[ \Phi_2 = H_2 \cdot t = \frac{2k(A) \Delta T}{4x} \]

\[ \implies \frac{m_1}{m_2} = 2 \Rightarrow \frac{m_1}{m_2} = 2 \]
Temperature gradient: It is assumed that temp. varies linearly with length at steady state.

\[ \text{Temp. gradient} = \frac{dT}{dl} = \frac{\Delta T}{l} \]

\( \downarrow \) Constant as per assumption

Analogy: 

<table>
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<tr>
<th>Heat</th>
<th>Electricity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat current ( H )</td>
<td>Electrical current ( i )</td>
</tr>
<tr>
<td>Why? ( \rightarrow \Delta T )</td>
<td>Why? ( \rightarrow \Delta V )</td>
</tr>
</tbody>
</table>

\[ H = \frac{KADT}{l} = \frac{\Delta T}{l/kA} \]

\[ i = \frac{\Delta V}{R_e} \]

\[ R_{th} = \frac{l}{KA} \]

\[ R_e = \frac{S}{A} \]

Thermal Resistance

Summary:

1) \( \Phi = \frac{KADT}{l} \)

2) \( H = \frac{\Phi}{l} = \frac{KADT}{l} = \frac{d\Phi}{dt} \)

3) \( \Delta T = \frac{HR_{th}}{V} = \frac{iR}{V} \)

4) \( R_{th} = \frac{l}{KA} \)
**Series Combination** (Steady state me sab me heat same)

\[ H = \frac{\Delta T}{R_1} \]
\[ H = \frac{\Delta T}{R_2} \]
\[ H = \frac{\Delta T}{R_e} \]

\[ T_1 - \theta = HR_1 \quad \text{(1)} \]
\[ \theta - T_2 = HR_2 \quad \text{(2)} \]
\[ T_1 - T_2 = HR_e \quad \text{(3)} \]

\[ T_1 - \theta + \theta - T_2 = HR_1 + HR_2 \]
\[ T_1 - T_2 = HR_e = H(R_1 + R_2) \]

\[ R_e = R_1 + R_2 \]

\[ A \text{... } R = \frac{l}{KA} \]
\[ l_1 + l_2 = \frac{l_1}{K_1 A} + \frac{l_2}{K_2 A} \]
\[ \Rightarrow l_1 + l_2 = \frac{l_1}{K_1} + \frac{l_2}{K_2} \]

\[ K_e = \frac{l_1 + l_2}{\frac{l_1}{K_1} + \frac{l_2}{K_2}} \]

Find:
1. \( R_e \)
2. \( K_e \)
3. \( \theta \)

\[ \theta = 100^\circ C \]

\[ H \xrightarrow{K} \theta \xrightarrow{2K} 0^\circ C \]

\[ (ii) \quad R_e = R_1 + R_2 \Rightarrow \frac{4l}{K_e A} = \frac{l}{K_1 A} + \frac{3l}{K_2 A} \]
\[ 4 = \frac{l}{K_e} + \frac{3}{2K_e} \Rightarrow K_e = \frac{8K}{5} \]

(ii) \[ R_e = \frac{4l}{K_eA} = \frac{4l}{\frac{8K}{5}A} = \frac{5 \cdot l}{2KA} \]

(iii) \[ H_1 = H_2 \Rightarrow \frac{\Delta T}{R_1} = \frac{\Delta T}{R_2} \Rightarrow \frac{100 - \Theta}{l/KA} = \frac{\Theta - 0}{3l/2KA} \]
\[ \Rightarrow 300 - 3\Theta = 2\Theta \]
\[ \Rightarrow 5\Theta = 300 \Rightarrow \Theta = 60^\circ C \]

![Diagram of temperature difference](image)

\[ k_1 = \frac{l}{k_1A} \quad k_2 = \frac{l}{k_2A} \]
\[ \Rightarrow \frac{k_1}{k_2} = \frac{7}{3} \]
\[ \Rightarrow 10\Theta = 300 \Rightarrow \Theta = 30^\circ C \]

27 Parallel Combination (Temp difference same)

\[ H = \frac{T_1 - T_2}{R_e}; \quad H_1 = \frac{T_1 - T_2}{R_1} \quad \text{and} \quad H_2 = \frac{T_1 - T_2}{R_2} \]

Also... \[ H = H_1 + H_2 \]
\[
\frac{T_1 - T_2}{R_e} = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} \Rightarrow \frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2}
\]

\[
A_s \quad R = \frac{l}{KA} \quad \Rightarrow \quad K_e (A_1 + A_2) = \frac{K_i A_1}{l} + \frac{K_i A_2}{l}
\]

\[
\Rightarrow K_e = \frac{K_i A_1 + K_i A_2}{K_i + K_2}
\]

\[
R_1 = \frac{l}{6kA} \quad R_2 = \frac{l}{4kA} \Rightarrow \frac{l}{R_2} = \frac{6kA}{l} + \frac{4kA}{l}
\]

\[
\Rightarrow K_e (2A) = \frac{6kA}{l} + \frac{4kA}{l}
\]

\[
\Rightarrow K_{eq} = 5k \quad \text{and} \quad R_e = \frac{l}{5K(2A)} = \frac{l}{10kA}
\]

\[
A_{Al} = 1 \text{cm}^2 \quad K_{Al} = 200 \quad \text{W/m}^\circ\text{C}
\]

\[
A_{Cu} = 1 \text{cm}^2 \quad K_{Cu} = 400 \quad \text{W/m}^\circ\text{C}
\]

\[
\theta = 60^\circ \text{C} \quad \text{Find the rate of heat flow through copper rod.}
\]

\[
H_{Cu} = \frac{\Delta T}{R_{Cu}} = \frac{(60-40) \times 400 \times 10^{-4}}{1} = 1.6 \text{ W}
\]

\[
H_2 + H_2 = H \quad \Rightarrow \quad 90 - \theta + 30 - \theta = \theta - \theta
\]

\[
\Rightarrow 180 - 2\theta = \theta
\]

\[
K_i, l, A \rightarrow \text{Same.} \quad \Rightarrow R \text{ same} \quad \Rightarrow \theta = 60^\circ \text{C} \quad H \text{ same}
\]
K_{Cu} = 0.192 \text{ cal/cm}^2 \text{ cm}^2 \text{s} \\
K_{\text{Brass}} = 0.26 \text{ cal/cm}^2 \text{ cm}^2 \text{s} \\
K_{\text{Steel}} = 0.12 \text{ cal/cm}^2 \text{ cm}^2 \text{s} \\
Find \ the \ rate \ of \ heat \ flow \ through \ copper \ rod \ in \ cal/s.?\\n\[ H = H_1 + H_2 \] \\
\[ \implies \frac{100 - \Theta}{R_{Cu}} = \frac{\Theta - 0}{R_{Brass}} + \frac{\Theta - 0}{R_{Steel}} \] \\
\[ \Rightarrow (100 - \Theta)0.192A = \Theta \cdot 0.26A + \Theta \cdot 0.12A \] \\
\[ \Rightarrow 200 - 2\Theta = 2\Theta + \Theta \] \\
\[ \Rightarrow \Theta = 40^\circ C \] \\
Find \ K_2. \\
\[ K_2 = \frac{K_1A_1 + K_2A_2}{A_1 + A_2} \] \\
\[ = \frac{K_1\pi R_1^2 + K_2\pi(2R)^2 - R^2}{\pi R_1^2 + \pi(2R)^2 - R^2} \] \\
\[ = \frac{k_1 + 5k_2}{4} \] \\
In 1 sec, certain amount of heat flows through 1st combination. Find the time in which same amount of
Heat flows through II\textsuperscript{nd} combination.

\[ R_1 = \frac{l}{kA} = 2R \quad \text{and} \quad R_2 = \frac{l}{2kA} = R \quad \text{(Say)} \]

\[(R_\text{e})_I = R_1 + R_2 = 3R \quad \text{and} \quad (R_\text{e})_\text{II} = \frac{(R)(2R)}{R + 2R} = \frac{2R}{3}\]

\[ Q = H \frac{\Delta T}{3} \quad \text{and} \quad Q = \frac{H \Delta T}{2R/3} \]

As \( Q \) is same...

\[ \frac{\Delta T \times 3}{R} = \frac{3\Delta T \times x}{2R} \Rightarrow x = 2 \text{ sec.} \]

# RADIATION:

- It is a way of heat transfer which requires no medium as it is an electromagnetic waves.

\[ \Rightarrow \text{Pâveost's theory of exchange:} \]

1. All body emits radiation at all temp.
2. All body absorbs heat radiations falling on it at all temp.
3. Amount of emission/absorption depends on temp. and material of body.

\[ T_{\text{surr}} > T_{\text{body}} \Rightarrow \text{Absorption} \rightarrow \text{Emission} \Rightarrow \text{Body temp.} \uparrow \uparrow \]

\[ T_{\text{surr}} < T_{\text{body}} \Rightarrow \text{Absorption} < \text{Emission} \Rightarrow \text{Body temp.} \downarrow \downarrow \]

\[ \Rightarrow T_{\text{surr}} = T_{\text{body}} \Rightarrow \text{Emission} = \text{Absorption} \]
Emissive Power (E) :-

Heat emitted by a body... \( \varphi \propto A \) ? \[ \text{E = EAT} \]

Emissive Power Unit: \( \frac{J}{s \cdot m^2 \cdot W} \)

Spectral emissive power

- Alag alag wavelength ke liye
  - Alag alag emit karne ki tendency.

Absorbtive power (\( \alpha \)) :- Percentage of heat absorbed out of total heat incident.

\[ \alpha = \frac{\text{Heat absorbed by body}}{\text{Heat incident on body}} \] (Unitless)

Heat absorbed \[ \varphi = \alpha (\text{Heat incident}) \]

\( 0 \leq \alpha \leq 1 \)

- Ideal Reflector
- Perfect reflector

\( \alpha = \frac{50}{100} = 0.5 \) 80 Radiations

\( \Rightarrow (i) \) Material
\( \Rightarrow (ii) \) Wavelength

\( \Rightarrow \) Spectral absorbive power

Substance jis colour ka hoga uss cuwavelength ke liye emissive power jyada aur baki sab ke liye absorbive power jyada.
BLACK BODY:-(Sun/Stars)

An ideal black body absorbs all radiations falling on it completely at any temp and at any incident angle.

- $\alpha = 1$
- Zero reflection
- Zero transmittance
- Zero emittance

Black color ki body $\rightarrow$ Can be black body, but... black body $\rightarrow$ Need not be necessarily black. No reflection $\rightarrow$ But emission $\rightarrow$ Temp ki wajah se khud ki waves de sakta hai. Black $\rightarrow$ Dusre ke colors ko raapas nahi bhejta.

Secondary $\rightarrow$ Black body emits maximum radiations at a given temp. as compared to other bodies.

Both are in thermal eqn $\Rightarrow$ Jitni absorb utni emit

Absorbs less ($\alpha < 1$) $\rightarrow$ Will emit less

Absorbs more ($\alpha = 1$) $\rightarrow$ Will emit more

Secondary definition explained.
Practical black bodies:

1) Lamp black (1st reflection)
2) Graphite (3rd reflection)
3) Platinum/Gold black
4) An Isothermal enclosure

Kirchhoff's Law:

Good absorbers are good emitters.

\[ \text{Absorbs} \uparrow \Rightarrow a \uparrow \]
\[ \text{Emits} \uparrow \Rightarrow E \uparrow \]

For any body...

\[ E \propto a \]

Also... \[ E_{\lambda} \propto a_{\lambda} \]

"Lek shram jisme absorb karega temp. badhne pas utna hi emit karega."

Proof:

All have same dimensions.

Heat absorbed = \( a \) (Heat incident)

Heat emitted = \( E \lambda T \)

\[ Q_{\text{absorbed}} = Q_{\text{emitted}} \]

\[ q_1 = A \lambda \]
\[ q_2 = A \lambda \]
\[ 1 \times Q = E_b \lambda T \]

\[ \frac{q_1}{E_1} = \frac{q_2}{E_2} = \frac{1}{E_b} \]

\[ q_1 = A \lambda \]
\[ q_2 = A \lambda \]
\[ 1 = A \lambda \]

Hence...

\[ \frac{E_1}{a_{11}} = \frac{E_2}{a_{12}} = E_b \Rightarrow \frac{E_{\lambda_1}}{a_{\lambda_1}} = \frac{E_{\lambda_2}}{a_{\lambda_2}} \]

\[ \frac{E_{\lambda_1}}{a_{\lambda_1}} = \frac{E_{\lambda_2}}{a_{\lambda_2}} \Rightarrow \frac{E_{\lambda_1}}{E_{\lambda_2}} = \text{constant} \Rightarrow E_{\lambda_1} \propto (\lambda \text{ at a given temp.}) \]
Temp. of α is falling rapidly.
⇒ Emits heat faster
⇒ $E_x > E_y$
⇒ $a_x > a_y$ (⇒ $E < a$)

# WIEE's DISPLACEMENT LAW :- (Ideally followed by black body)

- Intensity of Radiation ($\frac{Q}{A t}$)
- $(T_3 > T_2 > T_1)$
- $(\lambda_1 > \lambda_2 > \lambda_3)$
- $\lambda_{max} \rightarrow$ No. of radiation of this $\lambda$ is maximum.

- $T_3 = 4000$ K
- $T_2 = 3000$ K
- $T_1 = 2000$ K
- $\lambda_3$ $\lambda_2$ $\lambda_1$
- Wavelength

- $\lambda_{max}$
- Temp.
- $2000$ K $1.5 \times 10^{-6}$ m
- $3000$ K $1 \times 10^{-6}$ m
- $4000$ K $0.75 \times 10^{-6}$ m

- As temp. increases, $\lambda_{max}$ displace towards shorter wavelength.

- Here we can see that... $\lambda_1 T_1 = \lambda_2 T_2 = \lambda_3 T_3$

- $\frac{High\ Wavelength}{\downarrow}\Rightarrow \lambda_{max}. T = constant = b$ \Rightarrow WIEE's constant

- $\frac{Low\ temp.}{\downarrow}\Rightarrow T = \frac{b}{\lambda_{max}}$

- $b = 2.898 \times 10^{-8}$ m-k

Used to measure temp. of distant stars.
Area under graph gives total emission of intensity from body at that temperature.

\[ T_3 > T_2 > T_1 \Rightarrow I_3 > I_2 > I_1 \]

(Higher temp. \(\rightarrow\) More Radiations)

In all cases... it is not necessary that peak of graph goes upward with rise in temp. but area will surely increase.

\[ \lambda_{\text{max}} \rightarrow \lambda_3 > \lambda_2 > \lambda_1 \]

\[ T_3 < T_2 < T_1 \]

(By Wien's law)

\[ \text{Find } \frac{T_{\text{sun}}}{T_{\text{north star}}} \]

\[ \lambda_{\max} = 510 \text{nm} \]
\[ \lambda_{\max} = 350 \text{nm} \]

\[ \frac{T_{\text{sun}}}{T_{\text{north star}}} = \frac{350}{500} = 0.69 \]

# STEFAN'S - BOLTZMANN LAW :-

For a perfectly black body...

Intensity of radiation \(\propto T^4\)

\[ \Rightarrow \frac{Q}{A \times t} \propto T^4 \]

\[ \Rightarrow \frac{Q}{A \times t} = \sigma T^4 \]

\[ \sigma \rightarrow \text{Universal Constant} \]
\[ 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 \]
\[ 6 \times 10^{-8} \text{ W/m}^2 \text{ K}^4 \]
\[ \Theta = \sigma AT^4 \]

\[ \text{Radiated} \]

\[ P = \sigma e AT^4 \]

(For perfectly black body)

(For a given material)

\[ (0 \leq e \leq 1) \]

Kai bhi body black body se jyada heat emit aatega, aaiti bhi

(∴ Absorb aatega aur emit aatega)

Heaters: \( P_{\text{radiated}} = 6000 \text{ W}, \) coil area = 0.1 \( \text{m}^2 \). Find the temp of coil assuming it as a black body.

\( (\sigma = 6 \times 10^{-8} \text{ W/m}^2\text{K}^4) \)

\[ P = \sigma AT^4 \Rightarrow 6000 = 6 \times 10^{-8} \times 0.1 \times T^4 \]

\[ T = 1000 \text{ K} \]

When surrounding temp. is given:

\[ T_0 \]

Powers radiated = \( e\sigma AT^4 \)

Powers absorbed = \( e\sigma AT_0^4 \)

Net power = \( e\sigma A(T^4 - T_0^4) \) Radiator

\[ \frac{\text{Star - A}}{\text{Star - B}} \]

\[ R_A = 400R_B \quad \text{and} \quad P_A = 10^4 P_B \]

Find \( (\frac{\lambda_A}{\lambda_B})^4 \). (\( \lambda \to \text{Max. probable wavelength} \)
\[ P_A = P_B \times 10^4 \Rightarrow R_A^2 \times T_A^4 = R_B^2 \times T_B^4 \times 10^4 \]
\[ \Rightarrow 16 \times 10^4 \times T_A^4 = T_B^4 \times 10^4 \]
\[ \Rightarrow \frac{T_B}{T_A} = 2 = \frac{\lambda_A}{\lambda_B} \]

# Newton's Law of Cooling:

\[ \text{Rate of cooling} \propto \Delta T \quad (\text{Temp. difference body and surrounding}) \]
\[ \Rightarrow \text{Fall in temp.} \propto \Delta T \]
\[ \Rightarrow \frac{dT}{dt} \propto T_b - T_s \]

Q. Hot water in the container:

\[ 90^\circ C \rightarrow 65^\circ C ; 65^\circ C \rightarrow 60^\circ C ; 60^\circ C \rightarrow 55^\circ C \]

\( (t_1) \quad (t_2) \quad (t_3) \)

As rate of cooling decreases with time \( t_1 < t_2 < t_3 \).

A piece of metal is heated to temp. \( \Theta \) and kept in surrounding of temp. \( \Theta_0 \). The nearly correct graph for temp. of metal (\( T \)) v/s time (\( t \)) is:

\[ \frac{dT}{dt} \text{ should be decreasing and final temp. } \Theta_0. \]
Problem solving technique:- (Approx method) Not 100%

\[ T_i \quad \Delta t \quad T_b \]

Variable all the time

\[ \text{Rate of cooling} = \frac{\text{Fall in temp.}}{\text{time taken}} = \frac{T_i - T_b}{\Delta t} \ll \frac{T_b - T_s}{\Delta t} \]

\[ \Rightarrow \frac{T_i - T_b}{\Delta t} = k \left( \frac{T_i + T_b - T_s}{\Delta t} \right) \]

\[ 70 - 60 = k (65 - 30) \quad \text{and} \quad 80 - 50 = k (55 - 30) \]

\[ \Rightarrow 2 = 35k \quad \text{(1)} \quad \text{and} \quad \frac{10}{k} = 25k \quad \text{(2)} \]

\[ \frac{1}{2} \ldots \quad t = 7 \text{ min} \]

A liquid cools from 70°C to 60°C in 5 minutes. Calculate the time in which it cools from 60°C to 50°C if temp of surrounding is 30°C.

\[ 80 - 60 = k (70 - 30) \quad \text{and} \quad \frac{60 - 50}{4} = k (55 - 30) \]

\[ \Rightarrow \frac{20}{3} = 70 - 30 \quad \Rightarrow \quad 30^\circ C. \]
Derivation of Newton’s Law of Cooling:

Rate of heat loss = $e_0A(T^4 - T_0^4)$

(Power radiated)

$\Rightarrow \frac{dQ}{dt} = e_0A(T^4 - T_0^4)$

$\Rightarrow mc \frac{dT}{dt} = e_0A(T^4 - T_0^4)$

$\Rightarrow \frac{dT}{dt} = \frac{e_0A}{mc} (T^4 - T_0^4)$

$\Rightarrow \frac{dT}{dt} = k \left( T^4 - T_0^4 \right)$ \hspace{1cm} \left( k = \frac{e_0A}{mc} = \text{constant} \right)$

$\Rightarrow \frac{dT}{dt} = k \left( T_0 + \Delta T \right)^4 - T_0^4 \right)$

$\Rightarrow \frac{dT}{dt} = kT_0^4 \left( \left( \frac{T_0 + \Delta T}{T_0} \right)^4 - 1 \right)$

$\Rightarrow \frac{dT}{dt} = kT_0^4 \left( 1 + 4 \frac{\Delta T}{T_0} - 1 \right)$ \hspace{1cm} \left[ \text{Assuming } \Delta T \ll T_0 \right]$

$\Rightarrow \frac{dT}{dt} = 4KT_0^3 \cdot \Delta T$

$\Rightarrow \frac{dT}{dt} = k' \Delta T \Rightarrow \frac{dT}{dt} \propto \Delta T \hspace{1cm} \text{Proved}$