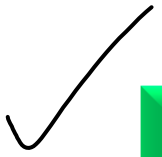
Ch-04
Vectors

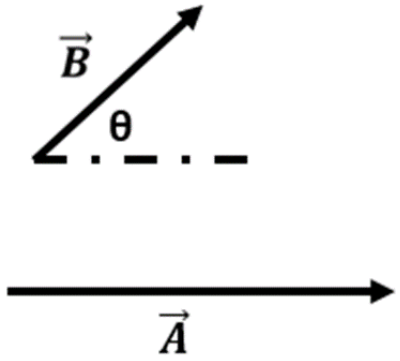
Lect-04

Today's Goal

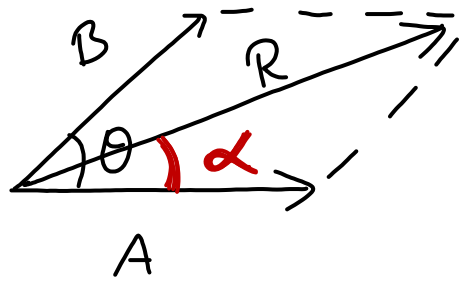


Subtraction of Vectors

We have Studied



$$\vec{R} = \vec{A} + \vec{B}$$



$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

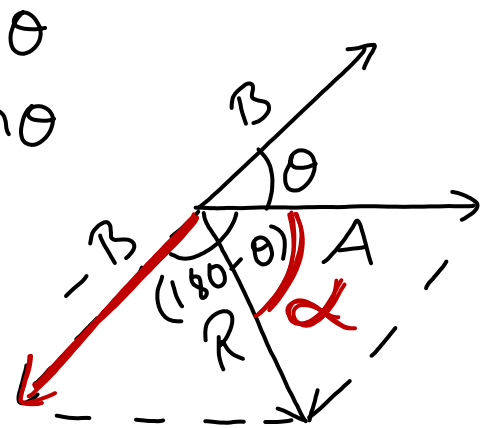
$$|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

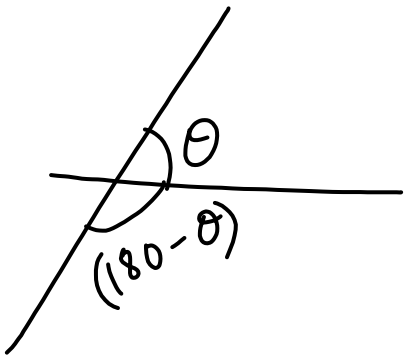
$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

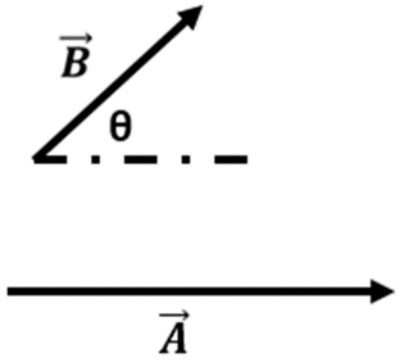
Now $\vec{A} - \vec{B}$

$\cos(180-\theta) = -\cos\theta$

$\sin(180-\theta) = +\sin\theta$







$|R| = |\vec{A} - \vec{B}|$

$\vec{R} = \vec{A} - \vec{B}$

$\vec{R} = \vec{A} + (-\vec{B})$

$(180-\theta)$

Ratla

$R^2 = A^2 + B^2 + 2AB\cos(180-\theta)$

$R^2 = A^2 + B^2 - 2AB\cos\theta$

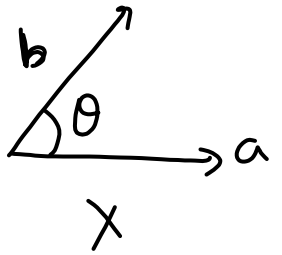
$|R| = \sqrt{A^2 + B^2 - 2AB\cos\theta}$ } $\theta \rightarrow$ Angle between $\vec{A}$ & $\vec{B}$

$\tan\alpha = \dfrac{B\sin(180-\theta)}{A + B\cos(180-\theta)} = \dfrac{B\sin\theta}{A - B\cos\theta}$

Q1) Two vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$. What is the angle between $\vec{a}$ and $\vec{b}$?

a) 0°
b) 90°
c) 60°
d) 180°

$$|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$$



$$\sqrt{a^2+b^2+2ab\cos\theta} = \sqrt{a^2+b^2-2ab\cos\theta}$$

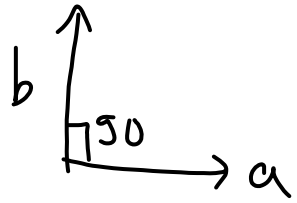
$$a^2+b^2+2ab\cos\theta = a^2+b^2-2ab\cos\theta$$

$$4ab\cos\theta = 0$$

$a \neq 0$
$b \neq 0$

$$\cos\theta = 0$$

$$\theta = 90$$



Q2) A particle is moving with velocity 5m/s towards the east and its velocity changes to 5m/s north in 10 sec. Find the acceleration.

a) $\sqrt{2}$ m/s2 N-W

b) $\frac{1}{\sqrt{2}}$ m/s2 N-W

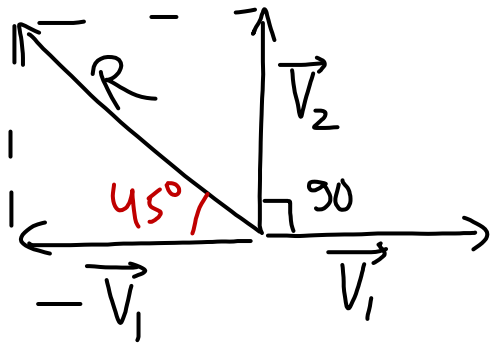
c) $\frac{1}{\sqrt{2}}$ m/s2 N-E

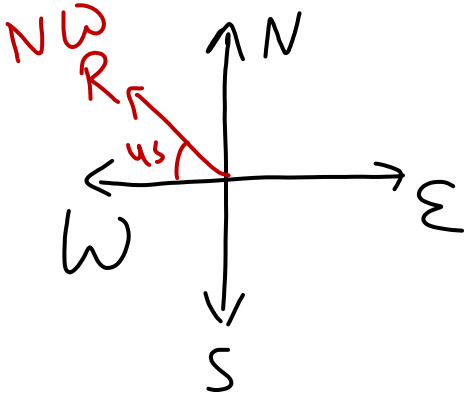
d) $\sqrt{2}$ m/s2 N-E

$\vec{V_1} = 5\,m/s$ E

$t = 10\,s$

$\vec{V_2} = 5\,m/s$ N





$\vec{a} = \frac{\text{Change in velocity}}{\text{time}}$

$\vec{a} = \frac{\vec{V_2} - \vec{V_1}}{t} = \frac{\vec{R}}{t} = \frac{5\sqrt{2}\,m/s}{10\,s} = \frac{\sqrt{2}}{2\sqrt{2}}$

$R^2 = V_1^2 + V_2^2 - 2V_1V_2\cos 90$

$R^2 = 5^2 + 5^2 - 0$

$R^2 = 2 \times 5^2$

$|R| = 5\sqrt{2}\,m/s$

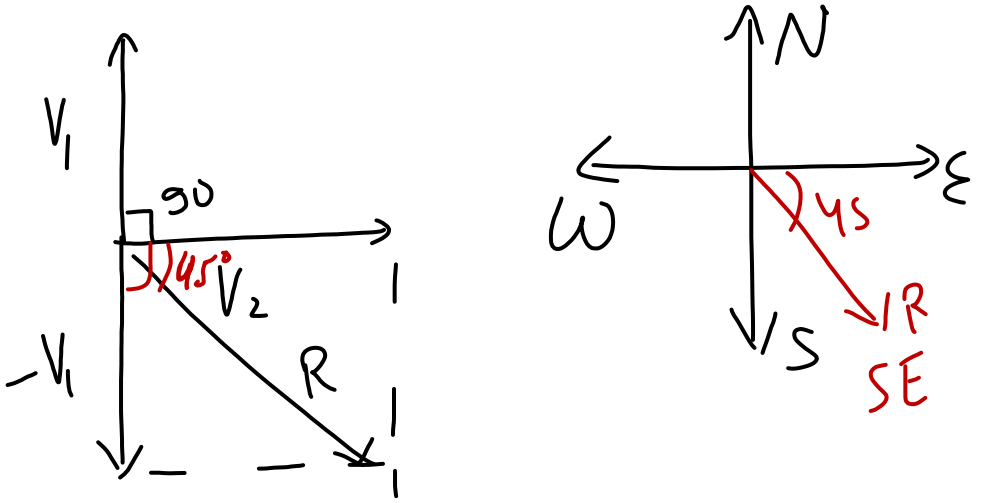
Q3) A car going due North at $10\sqrt{2}$ ms^{-1} turns right through an angle of 90° without changing speed. The change in velocity of the car is

a) $20\ m/s$ in South-East direction

b) $20\sqrt{2}\ m/s$ in South- East direction

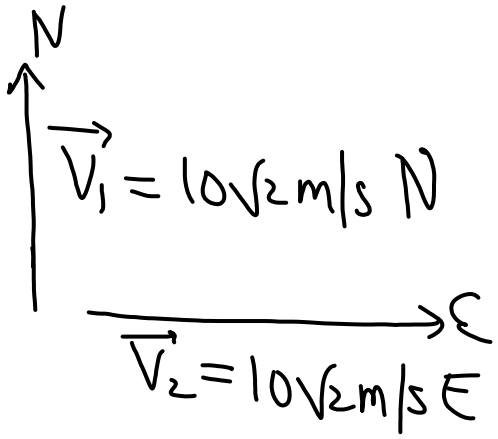
c) 20 m/s North-East direction

d) 20 m/s in North-West direction



Change in velocity

$\vec{R} = \vec{V_2} - \vec{V_1}$



$R^2 = V_1^2 + V_2^2 - 2V_1V_2\cos 90$

$R^2 = (10\sqrt{2})^2 + (10\sqrt{2})^2 - 0$

$R^2 = 100\times2 + 100\times2 = 400$

$|R| = 20$

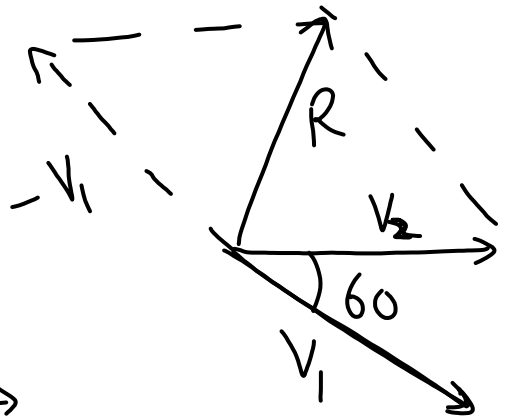
Q4) A particle slides with a speed of 5m/s at P. When it reaches Q, it still moves at 5 m/s after describing an angle of 60° at O as shown in the figure. Find the change in the velocity of the particle between P and Q. Assume that the path followed by the particle is circular from P to Q

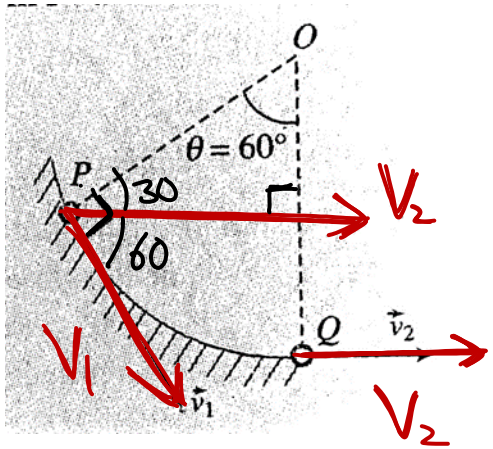
a) 5 m/s

b) 3 m/s

c) 6 m/s

d) 10 m/s





Change in velocity $= \vec{V_2} - \vec{V_1}$

$\vec{R} = \vec{V_2} - \vec{V_1}$

$|R| = ?$

$|R| = 5\ m/s$

$R^2 = V_1^2 + V_2^2 - 2V_1V_2\cos 60$

$R^2 = 5^2 + 5^2 - 2 \times 5^2 \times \frac{1}{2}$

$R^2 = 5^2$

Q5) A body is moving along a circular path with constant speed v, what is change in velocity and when the angle is described at centre of the circle is θ

a) v sinθ

b) 2v sinθ

c) v sin(θ/2)

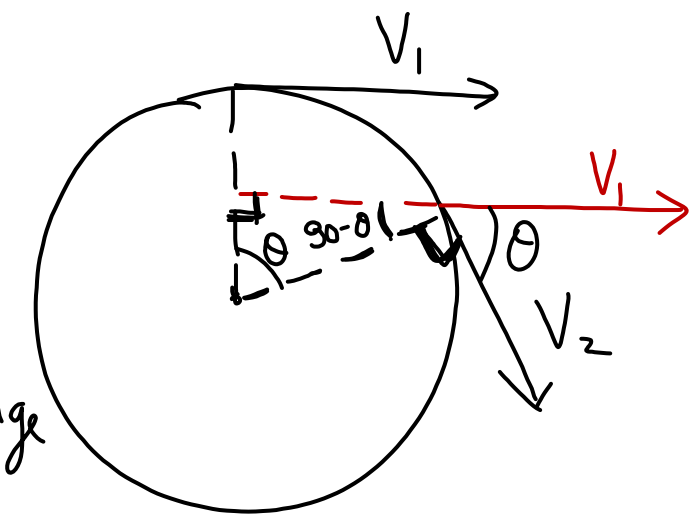
d) ~~2v sin(θ/2)~~

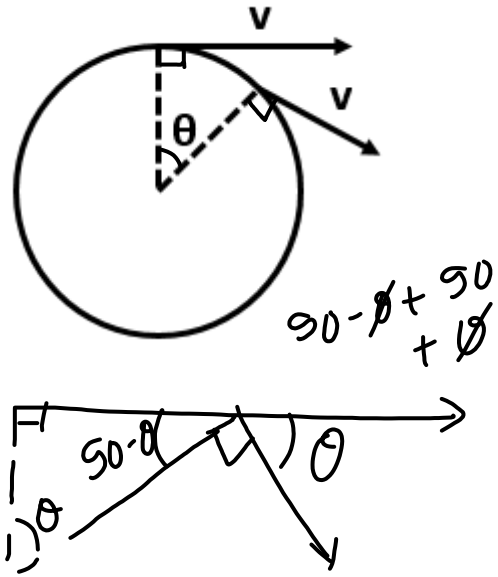
Uniform Circular Motion

↓

Speed → same

Velocity → change





Change in velocity

$\vec{R} = \vec{V_2} - \vec{V_1}$

$$\vec{R} = \vec{V_2} - \vec{V_1}$$

$$|R| = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$$

$$|R| = \sqrt{V^2 + V^2 - 2V^2\cos\theta}$$

$$|R| = \sqrt{2V^2 - 2V^2\cos\theta}$$

$$|R| = \sqrt{2V^2(1-\cos\theta)}$$

$$|R| = \sqrt{2V^2\, 2\sin^2\frac{\theta}{2}}$$

$$|R| = 2V\sin\frac{\theta}{2}$$

$$|R| = 2V\sin\frac{\theta}{2}$$

$$|\vec{V_2} - \vec{V_1}| = 2V\sin\frac{\theta}{2}$$

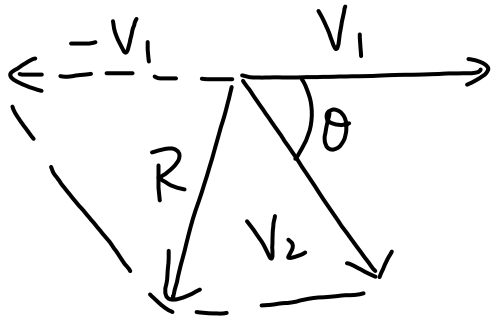
check

$$|\vec{V_2} - \vec{V_1}| = 2\times 5\times\sin\left(\frac{60}{2}\right)$$

$$= 2\times 5\times\sin 30$$

$$= 2\times 5\times\frac{1}{2} = 5$$

$1-\cos 120 = 2\sin^2 60$

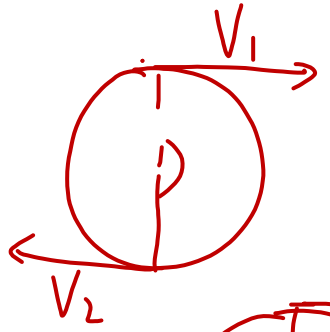


$1-\cos 2\theta = 2\sin^2\theta$

$1-\cos\theta = 2\sin^2\frac{\theta}{2}$

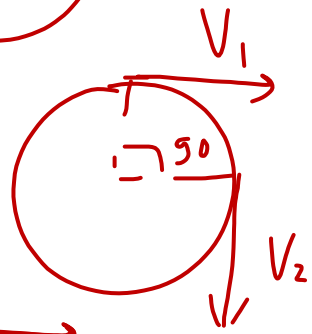
Q) A particle Moving in a circle with constant speed of 100m/s Find change in velocity after

i) $\frac{1}{2}$ revolution ⟶



$= 2V\sin\left(\frac{180}{2}\right) = 2\times100\times\sin 90 = 200\ m/s$

ii) $\frac{1}{4}$ revolution ⟶



$= 2V\sin 45 = 2\times100\times\frac{1}{\sqrt{2}}$

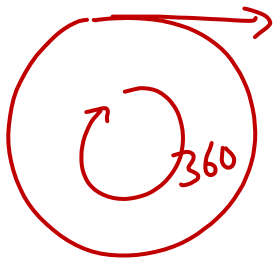
iii) $\frac{3}{4}$ revolution = ⟶

$= $ ⟶

$\frac{3}{4} \rightarrow 75$

iv) $\frac{5}{6}$ revolution

iv) $\frac{5}{6}$ of revolution $\longrightarrow$



Complete revolution $\longrightarrow$ 360°

θ

1 rev $\longrightarrow$ 360°

$\frac{5}{6}$ rev $\longrightarrow$ $360 \times \frac{5}{6}$

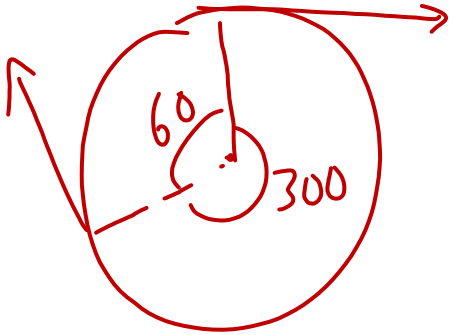
$= 300$

$= 2v \sin\frac{\theta}{2}$

$= 2v \sin\frac{60}{2}$

$= 2 \times 100 \times \sin 30$

$= \cancel{2} \times 100 \times \frac{1}{\cancel{2}}$



Q6) The resultant of $\vec{P}$ and $\vec{Q}$ is $\vec{R}$. If $\vec{Q}$ is doubled, $\vec{R}$ is doubled ; when $\vec{Q}$ is reversed, $\vec{R}$ is again doubled. Find P:Q:R

HW

a) $\sqrt{3}:\sqrt{2}:\sqrt{5}$

b) $\sqrt{5}:\sqrt{2}:\sqrt{3}$

Thank You

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