# Ch-04
# Vectors
## Lect-06

# Today's Goal

Addition of Vectors in Cartesian Form

Addition of three or more Vectors

**Let** $$\vec{A} = Ax\hat{i} + Ay\hat{j}$$ $$\vec{B} = Bx\hat{i} + By\hat{j}$$

$$\vec{A} + \vec{B} = (Ax + Bx)\hat{i} + (Ay + By)\hat{j}$$

**$\vec{A}$ & $\vec{B}$ are directed as shown. Find $\vec{R}$ in Cartesian form, where $\vec{R} = \vec{A} + \vec{B}$**

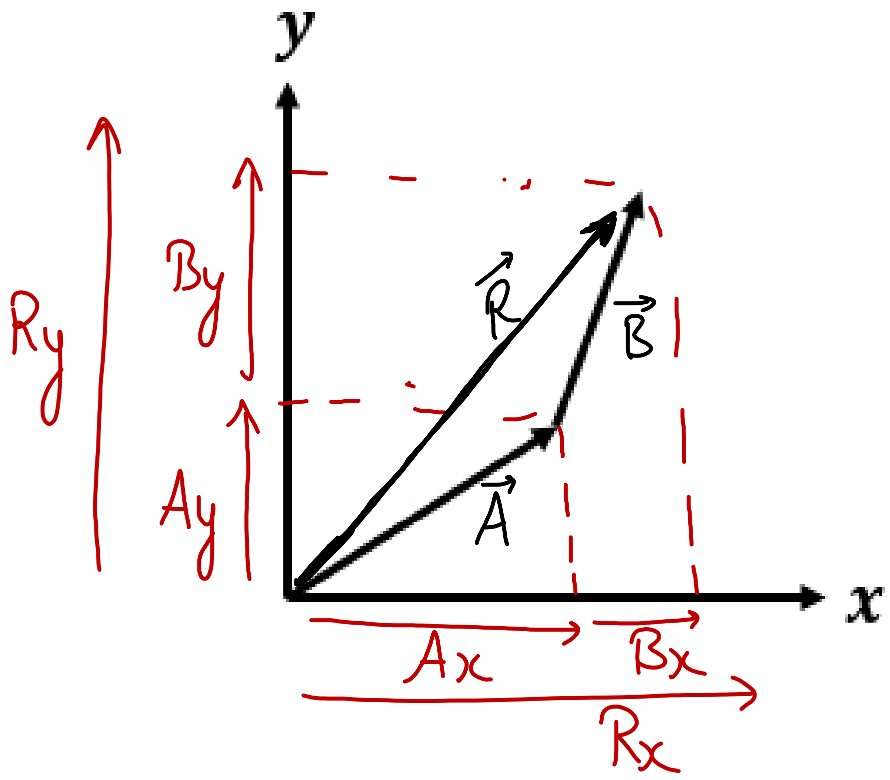
$$R_x = A_x + B_x$$

$$R_y = A_y + B_y$$

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

$$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$$



**Similarly if,**

$$\vec{A} = Ax\hat{\imath} + Ay\hat{\jmath} + Az\hat{k}$$
$$\vec{B} = Bx\hat{\imath} + By\hat{\jmath} + Bz\hat{k}$$

**Then** $\vec{R} = ?$

**where** $\vec{R} = \vec{A} + \vec{B}$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k}$$

Q)

$$\vec{A} = 8\hat{i} + 6\hat{j} + 4\hat{k}$$

$$\vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{A} + \vec{B} = 12\hat{i} + 9\hat{j} + 6\hat{k}$$

$$\vec{A} - \vec{B} = 4\hat{i} + 3\hat{j} + 2\hat{k}$$

## Q1) $\vec{A} = 2\hat{\imath} + \hat{\jmath}$ , $\vec{B} = 3\hat{\jmath} - \hat{k}$ and $\vec{C} = 6\hat{\imath} - 2\hat{k}$, Value of $\vec{A}$-2$\vec{B} + 3\vec{C}$ would be

a) $20\hat{i} + 5\hat{j} + 4\hat{k}$

b) $20\hat{i} - 5\hat{j} - 4\hat{k}$

c) $4\hat{i} + 5\hat{j} + 20\hat{k}$

d) $5\hat{i} + 4\hat{j} + 10\hat{k}$

$$\vec{A} - 2\vec{B} + 3\vec{C}$$

$$= (2i + \hat{j}) - (6\hat{j} - 2k) + (18\hat{i} - 6\hat{k})$$

$$= 2i + j - 6j + 2k + 18i - 6K$$

$$= 20\hat{i} - 5\hat{j} - 4\hat{k}$$

**Q2)Determine that vector which when added to the resultant of $\vec{A} = \widehat{3i} - 5\hat{j} + 7\hat{k}$ and $\vec{B} = 2\hat{i} + 4\hat{j} - 3\hat{k}$ gives unit vector along y-direction.**

$\vec{A} + \vec{B} = \vec{R}$

$\vec{R} = \vec{A} + \vec{B} = 5\hat{i} - 1\hat{j} + 4\hat{k}$

Let the required vector be $\vec{C}$

$\vec{C} + \vec{R} =$ Unit vector along y-direction

$\vec{C} + \vec{R} = \hat{j} \Rightarrow \vec{C} = \hat{j} - \vec{R}$

$\vec{C} = \hat{j} - \vec{R}$

$= \hat{j} - (5\hat{i} - \hat{j} + 4\hat{k})$

$= \hat{j} - 5\hat{i} + \hat{j} - 4\hat{k}$

$\boxed{\vec{C} = -5\hat{i} + 2\hat{j} - 4\hat{k}}$

Answer

**Q3)If vectors $\vec{A} = \hat{\imath} + 2\hat{\jmath} + 4\hat{k}$ and $\vec{B} = 5\hat{\imath}$ represent the two sides of a triangle, then the third side of the triangle can have length equal to**

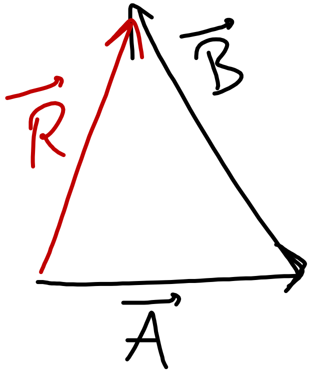
length of $\vec{R} = |R|$

a) 6

b) $\sqrt{56}$

c) Both of the above

d) None of the above




$$\vec{R} = \vec{A} + \vec{B}$$

$$\vec{R} = 6\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|R| = \sqrt{(6)^2 + (2)^2 + (4)^2} = \sqrt{36 + 4 + 16} = \sqrt{56}$$

Q4) Vector $\vec{A}$ is of length 2 cm and is 60° above the x-axis in the first quadrant. Vector $\vec{B}$ is of length 2 cm and 60° below the x-axis in the fourth quadrant . The sum of $\vec{A} + \vec{B}$ is a vector of magnitude

a) 2 along + y axis

b) 2 along + x axis ✓

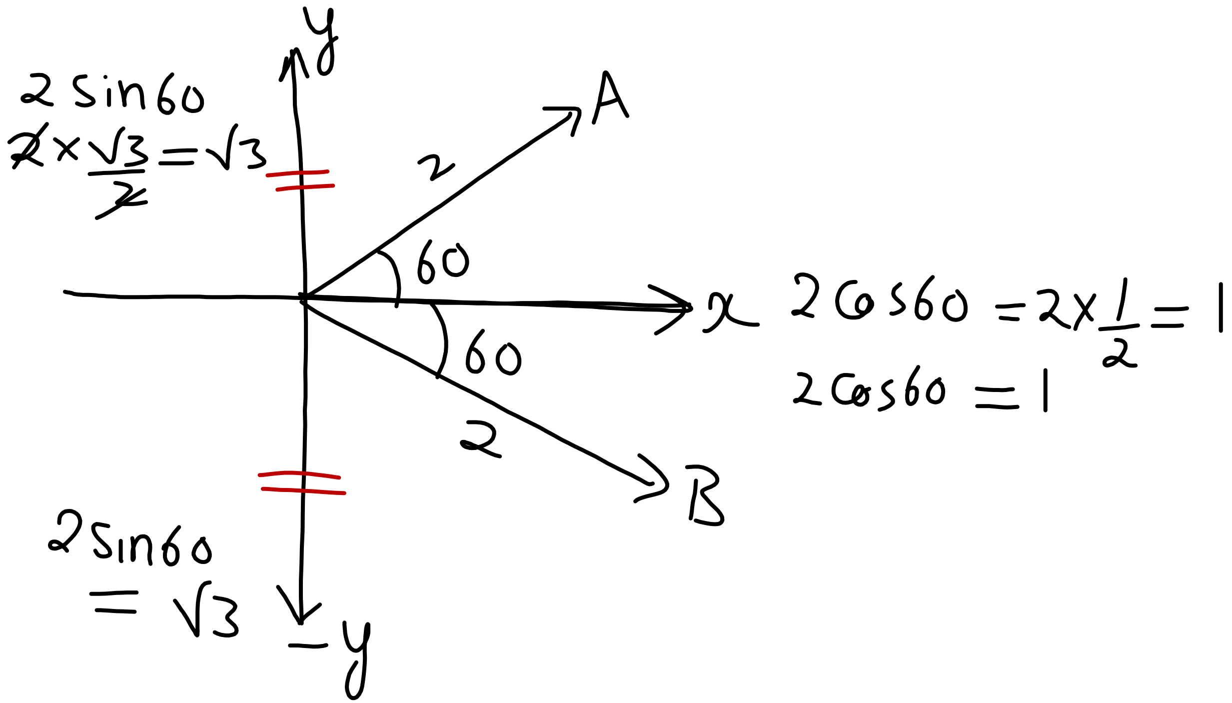
c) 1 along -x axis

d) 2 along -x axis

$$\vec{A} = 1\hat{i} + \sqrt{3}\hat{j}$$

$$\vec{B} = 1\hat{i} - \sqrt{3}\hat{j}$$

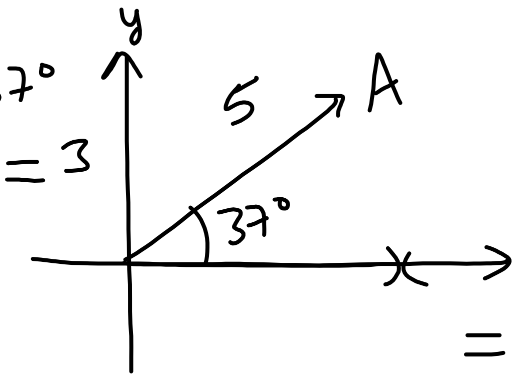
$$\boxed{\vec{A} + \vec{B} = 2\hat{i}}$$



**Q5) Find $\vec{A}+\vec{B}$ and $\vec{A}-\vec{B}$ . If $\vec{A}$ make angle 37° with positive x-axis and $\vec{B}$ make angle 53° with negative x-axis as shown and magnitude of $\vec{A}$ is 5 and of $\vec{B}$ is 10**

$5\sin 37^\circ$

$5\times\frac{3}{5}=3$



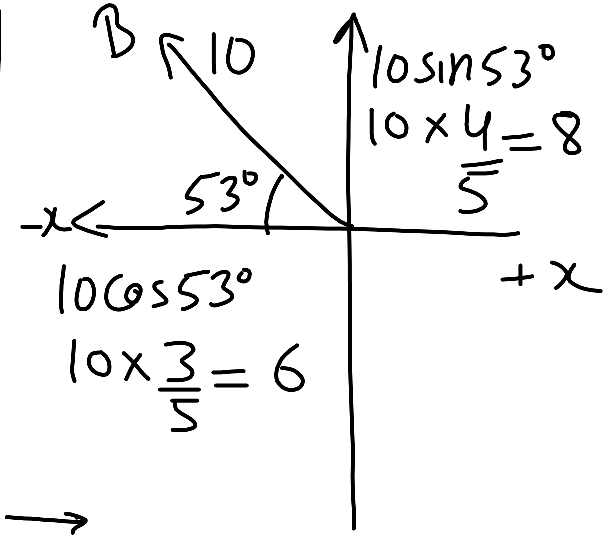

$5\cos 37^\circ$

$=5\times\frac{4}{5}=4$

$\vec{A}=4\hat{i}+3\hat{j}$

$\vec{A}+\vec{B}=-2\hat{i}+11\hat{j}$

$\vec{A}-\vec{B}=10\hat{i}-5\hat{j}$



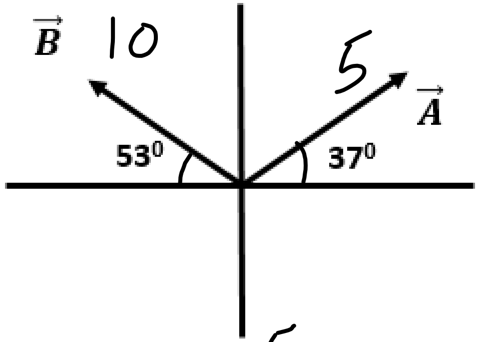

$10\sin 53^\circ$

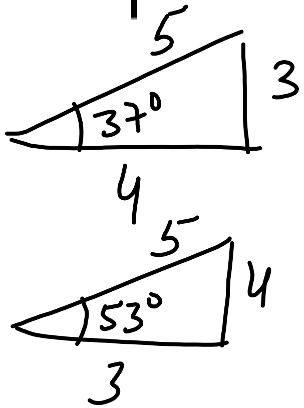
$10\times\frac{4}{5}=8$

$10\cos 53^\circ$

$10\times\frac{3}{5}=6$

$\vec{B}=-6\hat{i}+8\hat{j}$



## Q6) A body acted upon by 3 given forces is under equilibrium. If $|\widehat{F_1}| = 10$ N . $|\widehat{F}_2| = 6$ N. Find the values of $|\widehat{F_3}|$ & angle ( θ)

equilibrium $\vec{F}_{net} = 0$

Resultant Force = 0

$\theta = ?$ , $F_3 = ?$

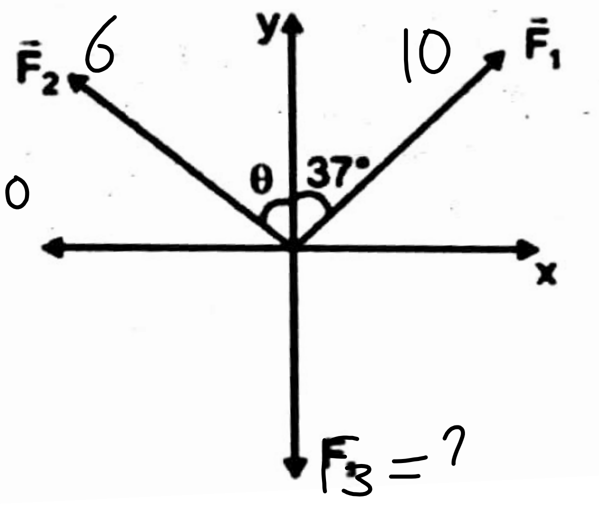
$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$

$\vec{R} = 0$

$R_x = 0$ $R_y = 0$

$R_x = F_{1x} + F_{2x} + F_{3x} = 0$

$R_y = F_{1y} + F_{2y} + F_{3y} = 0$



$R_x = 0$

$F_{1x} + F_{2x} + F_{3x} = 0$

$+10\sin 37^\circ - 6\sin\theta + 0 = 0$

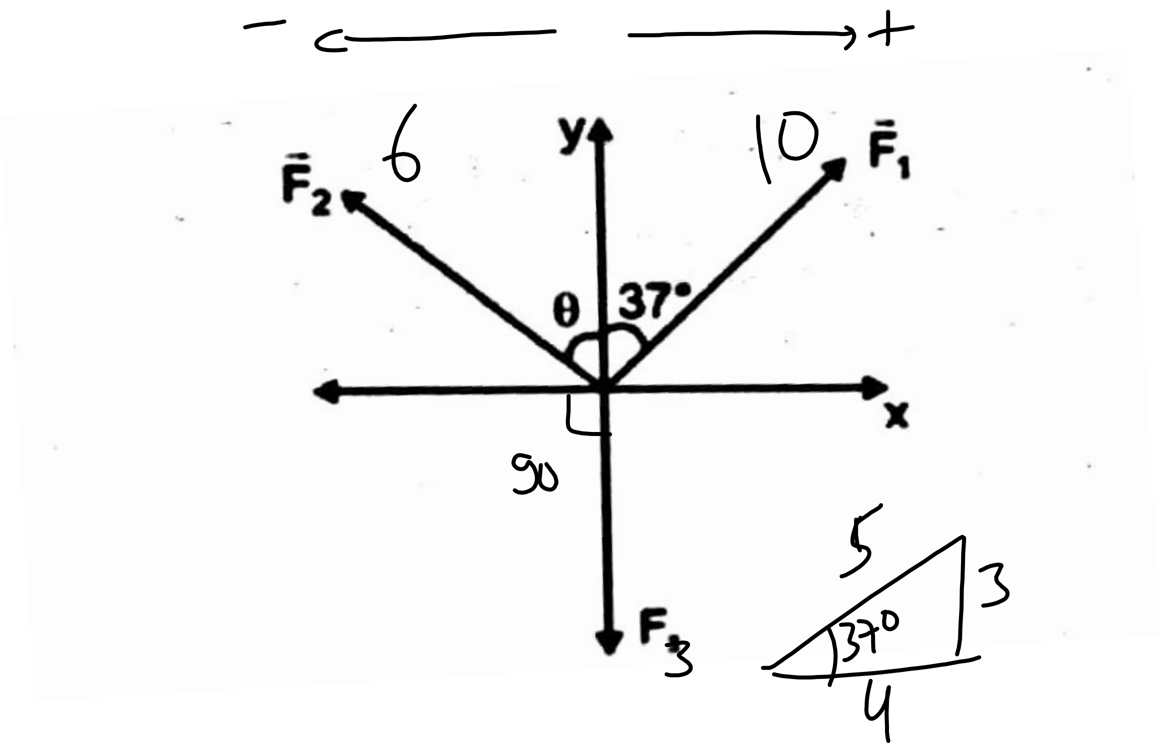
$10\sin 37^\circ = 6\sin\theta$

$10 \times \frac{3}{5} = 6\sin\theta$

$\cancel{6} = \cancel{6}\sin\theta$

$1 = \sin\theta$

$\boxed{\theta = 90}$



$$R_y = 0$$

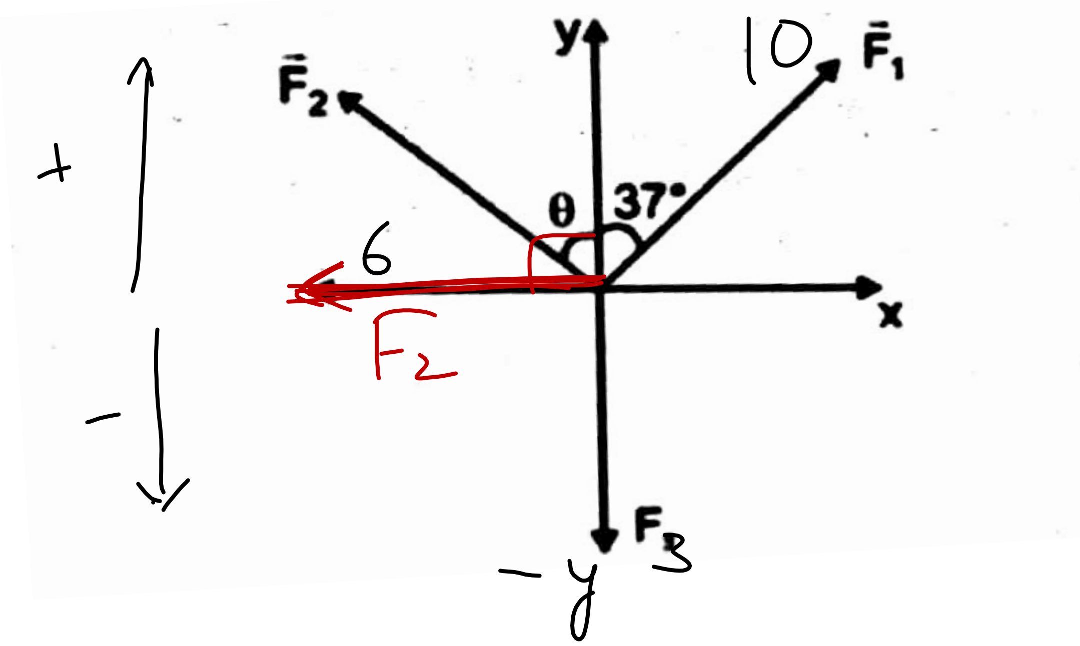
$$F_{1y} + F_{2y} + F_{3y} = 0$$

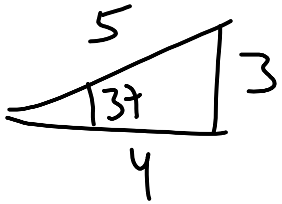
$$+10\cos 37^\circ + 6\cos 90 - F_3 = 0$$

$$10 \times \frac{4}{5} + 0 - F_3 = 0$$

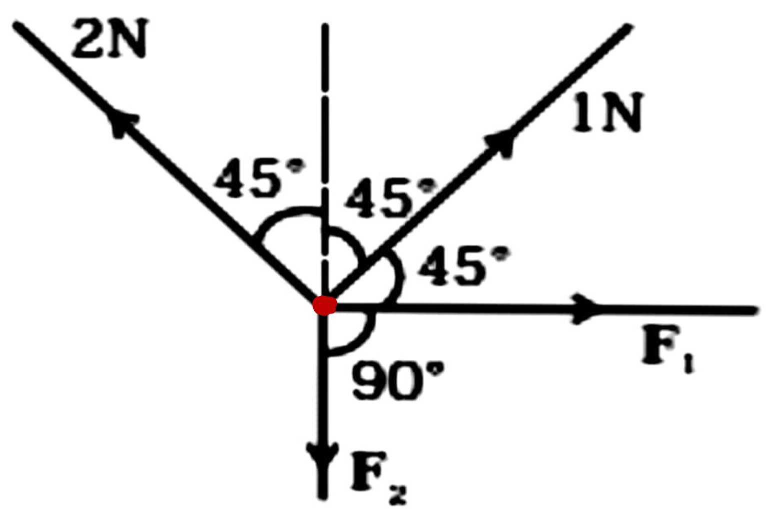
$$10 \times \frac{4}{5} = F_3$$

$$8 = F_3 \Rightarrow \boxed{F_3 = 8N}$$



**Q7) A Particle is in equilibrium under the presence of four forces shown . Find $F_1$ and $F_2$**




equilibrium $F_{net} = 0$

Resultant Force = 0

$\vec{R} = \vec{F_1} + \vec{F_2} + \vec{F_3} + \vec{F_4} = 0$

$\vec{R} = 0$

$R_x = 0$ $R_y$

$R_x = 0$

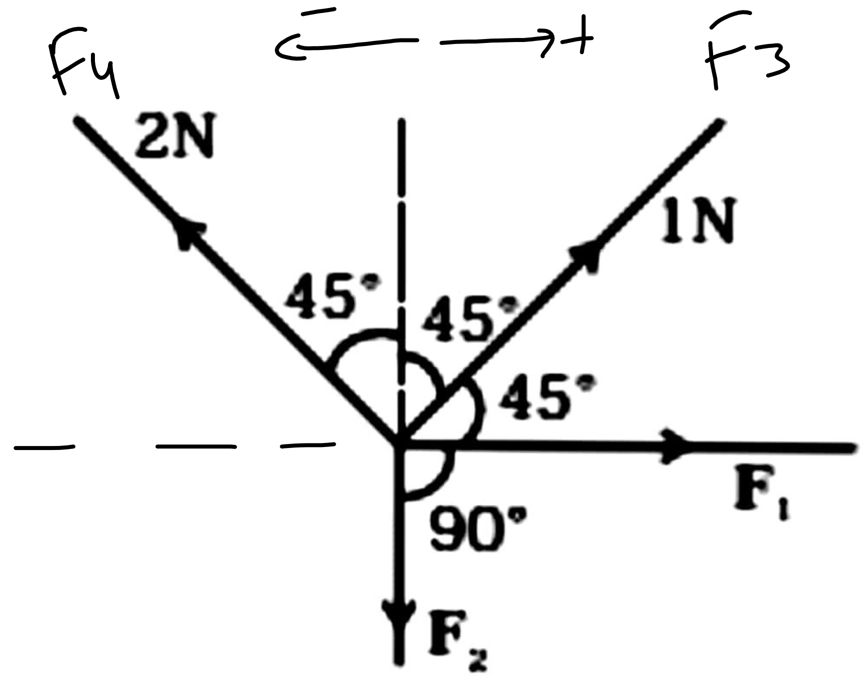
$F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0$

$+F_1 + 0 + 1\cos 45^\circ - 2\sin 45 = 0$

$F_1 + 1 \times \frac{1}{\sqrt{2}} - 2 \times \frac{1}{\sqrt{2}} = 0$

$F_1 + \left(\frac{-1}{\sqrt{2}}\right) = 0$

$$\boxed{F_1 = \frac{1}{\sqrt{2}} N}$$



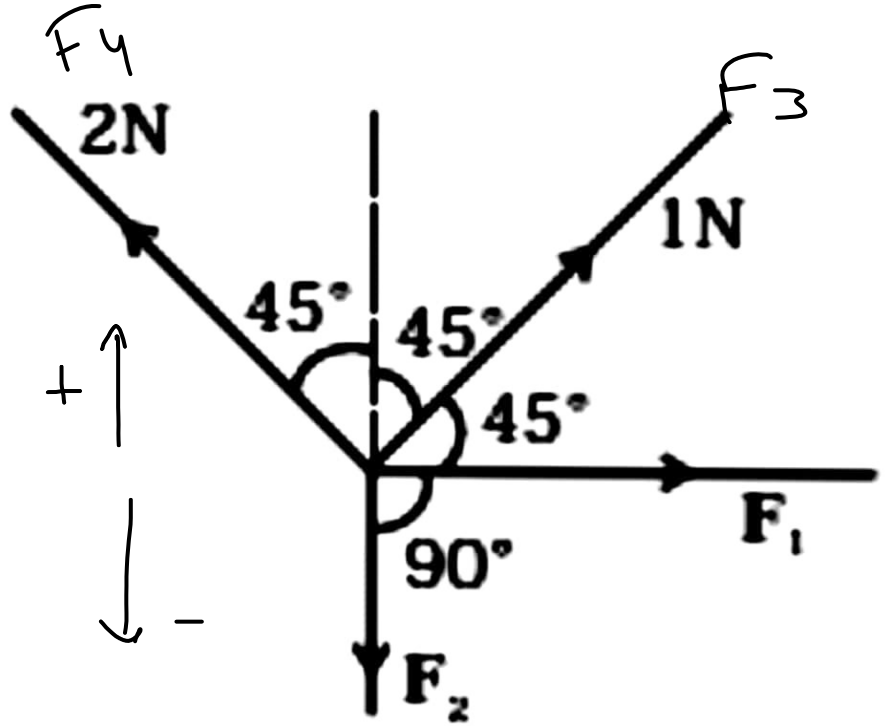
$R_y = 0$

$F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$

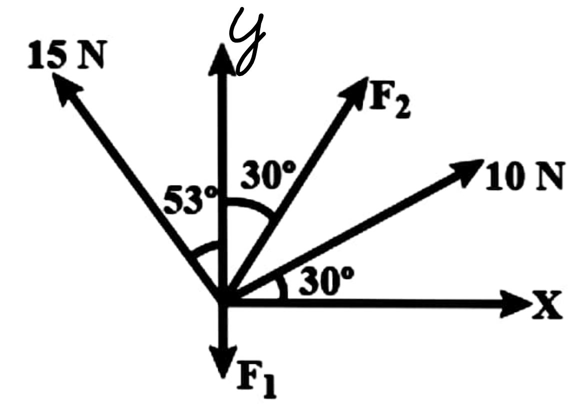
$0 + -F_2 + 1\sin 45 + 2\cos 45 = 0$

$-F_2 + 1 \times \frac{1}{\sqrt{2}} + 2 \times \frac{1}{\sqrt{2}} = 0$

$$\boxed{\frac{3}{\sqrt{2}} N = F_2}$$



**Q8)** A particle is in equilibrium in the presence of four forces as shown in the figure. (take, $\sqrt{3} = 1.7$)




This question has multiple correct options

A. $F_1 = 19.95N$

B. $F_2 = 7N$

C. $F_1 = 7N$

D. $F_2 = 19.95N$

equilibrium → Resultant Force

$\vec{R} = 0$

$R_x = 0 \quad R_y = 0$

$$R_x = 0$$

$$F_{1x} + F_{2x} + F_{3x} + F_{4x} = 0$$

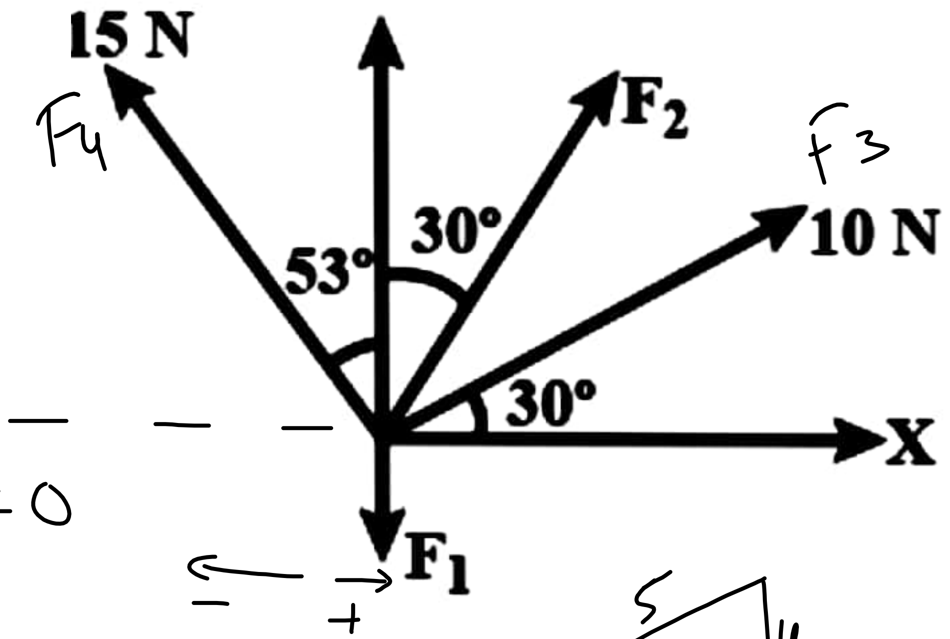
$$0 + F_2 \sin 30 + 10 \cos 30 - 15 \sin 53^\circ = 0$$

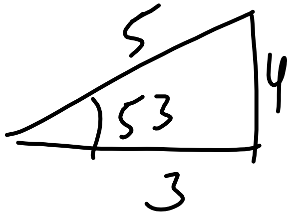
$$F_2 \times \frac{1}{2} + \overset{5}{\cancel{10}} \times \frac{\sqrt{3}}{\cancel{2}} - \overset{3}{\cancel{15}} \times \frac{4}{\cancel{5}} = 0$$

$$\frac{F_2}{2} + 5\sqrt{3} - 12 = 0$$

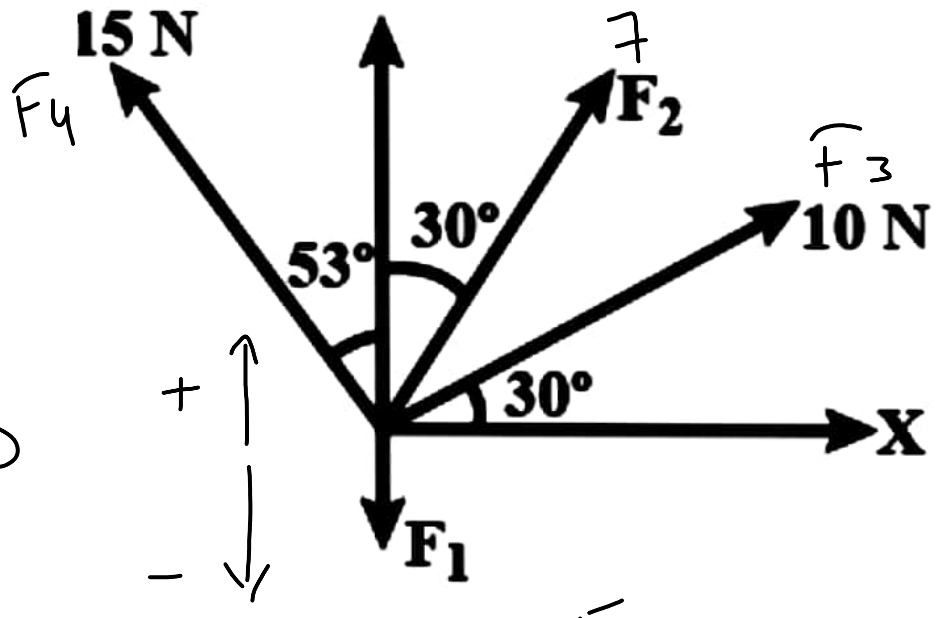
$$\frac{F_2}{2} = 12 - 5\sqrt{3} \Rightarrow F_2 = 24 - 10\sqrt{3} = 24 - 10 \times 17$$

$$\boxed{F_2 = 24 - 17 = 7}$$



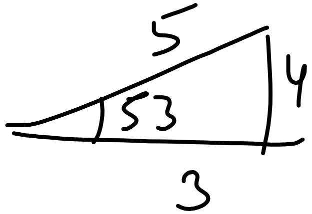
$$R_y = 0$$




$$F_{1y} + F_{2y} + F_{3y} + F_{4y} = 0$$

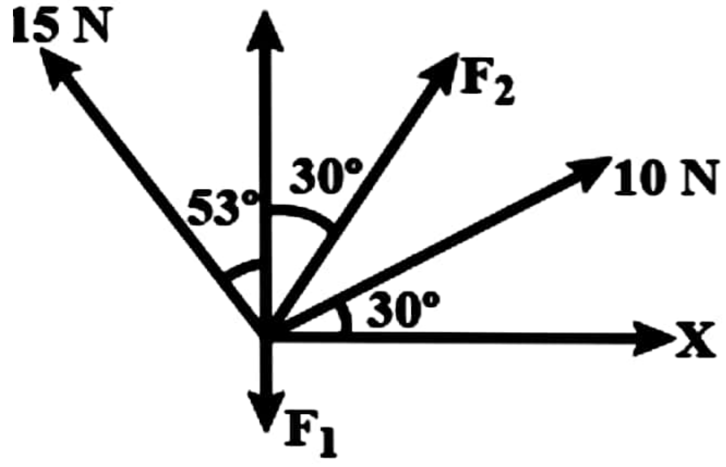
$$-F_1 + 7\cos 30 + 10\sin 30 + 15\cos 53 = 0$$

$$-F_1 + 7\times\frac{\sqrt{3}}{2} + \overset{5}{\cancel{10}}\times\frac{1}{\cancel{2}} + \overset{3}{\cancel{15}}\times\frac{3}{\cancel{5}} = 0$$




$$-F_1 + \frac{7\times 17}{2} + 14 = 0$$

$$\boxed{F_1 = 14 + \frac{7\times 17}{2} = 14 + \frac{\overset{5.95}{\cancel{11.9}}}{\underset{1}{\cancel{2}}} = 19.95\,N}$$


15 N
$F_2$
30°
10 N
53°
30°
X
$F_1$

# Thank You

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