Ch-04
Vectors

Lect-08

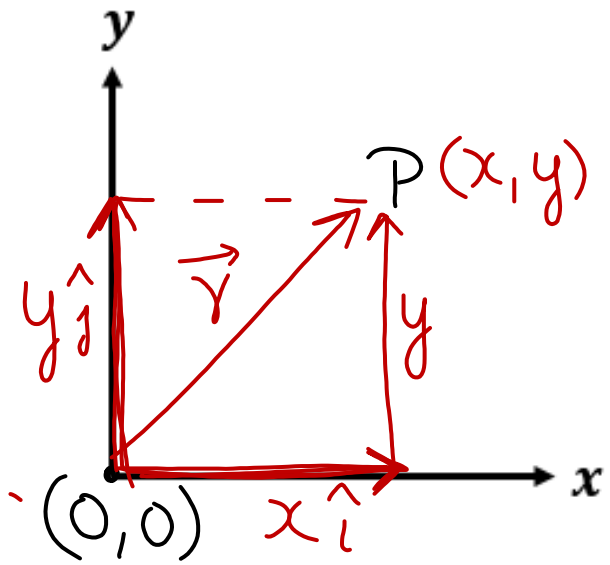
Today's Goal

✓

Position Vector & Displacement Vector

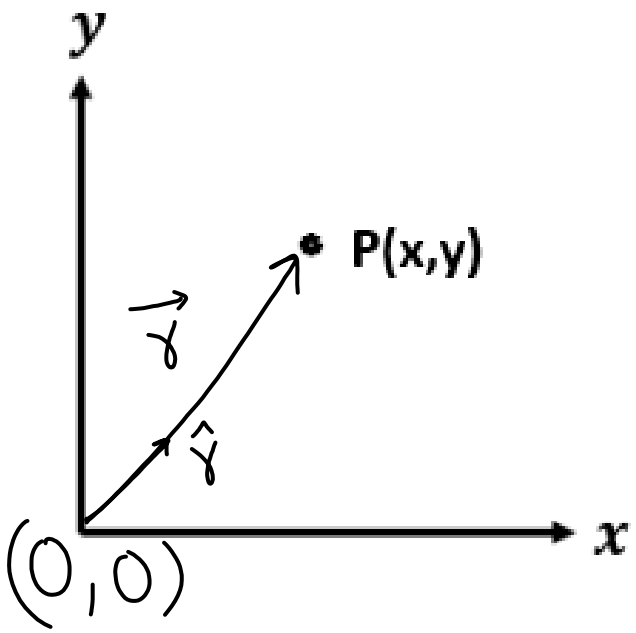
Position Vector

- ***Once you want to specify position of a point, you need to have a reference point. This reference point is called origin.***

- 

$$\vec{r} = x\hat{i} + y\hat{j}$$

- ***Position vector specifies the position of a particle in space with respect to an origin.***



$$\vec{r} = x\hat{i} + y\hat{j}$$

$$|r| = \sqrt{x^2 + y^2}$$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$$

Q1) Find $\overrightarrow{r_A}, \overrightarrow{r_B}, \overrightarrow{r_C}, \overrightarrow{r_D}$. Also find $\widehat{r_A}, \widehat{r_B}, \widehat{r_C}, \widehat{r_D}$

$$\overrightarrow{\gamma_A} = 2\hat{i} + 2\hat{j}$$

$$\overrightarrow{\gamma_B} = -2\hat{i} + 2\hat{j}$$

$$\overrightarrow{\gamma_C} = -2\hat{i} - 2\hat{j}$$

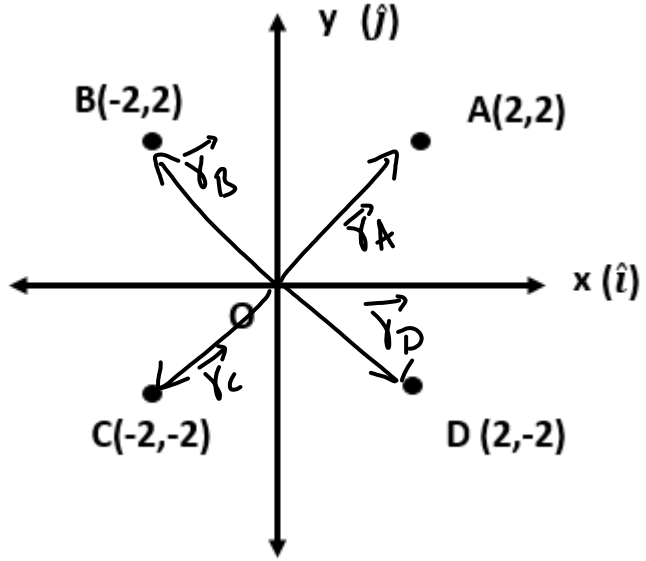
$$\overrightarrow{\gamma_D} = 2\hat{i} - 2\hat{j}$$

$$\hat{\gamma}_A = \frac{\overrightarrow{\gamma_A}}{|\gamma_A|} = \frac{2\hat{i}+2\hat{j}}{\sqrt{2^2+2^2}} = \frac{2\hat{i}+2\hat{j}}{\sqrt{8}}$$

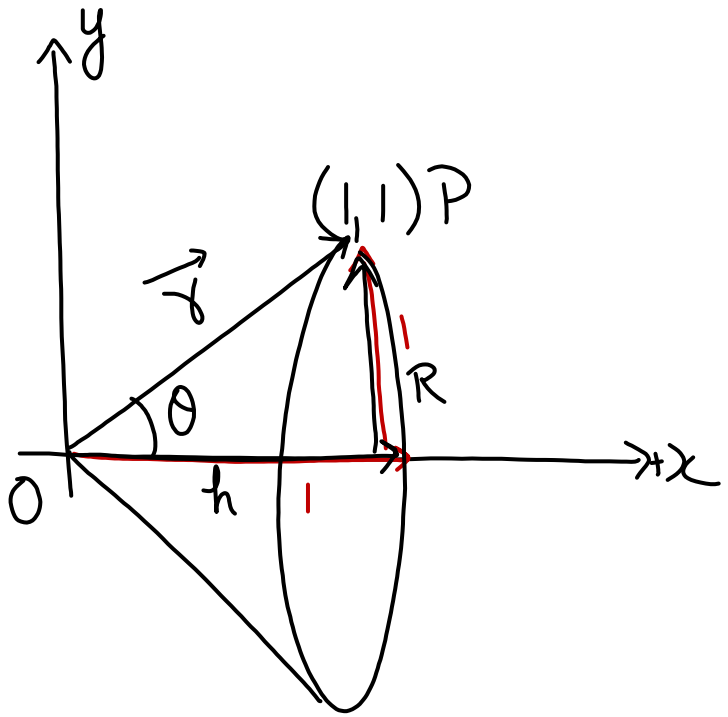
$$\hat{\gamma}_B = \frac{\overrightarrow{\gamma_B}}{|\gamma_B|} = \frac{-2\hat{i}+2\hat{j}}{\sqrt{8}}$$

$$\hat{\gamma}_C = \frac{\overrightarrow{\gamma_C}}{|\gamma_C|} = \frac{-2\hat{i}-2\hat{j}}{\sqrt{8}}$$

$$\hat{\gamma}_D = \frac{\overrightarrow{\gamma_D}}{|\gamma_D|} = \frac{2\hat{i}-2\hat{j}}{\sqrt{8}}$$



Q 2) The Co-ordinate of a point is (1,1) Find
(i) It's position vector ($\vec{r}$)
(ii) Angle made $\vec{r}$ by with +x axis.
(iii) The volume of a cone generated when the line segment representing $\vec{r}$ rotates about +x axis, one end fixed at origin.



$\vec{r} = \hat{i} + \hat{j}$

$\tan\theta = \frac{1}{1}$

$\theta = 45^\circ$

Cone

Volume $= \frac{1}{3}\pi R^2 h$

$h = 1$

$R = 1$

Volume $= \frac{1}{3}\pi (1)^2 (1)$

$= \frac{\pi}{3}(\text{unit})^3$

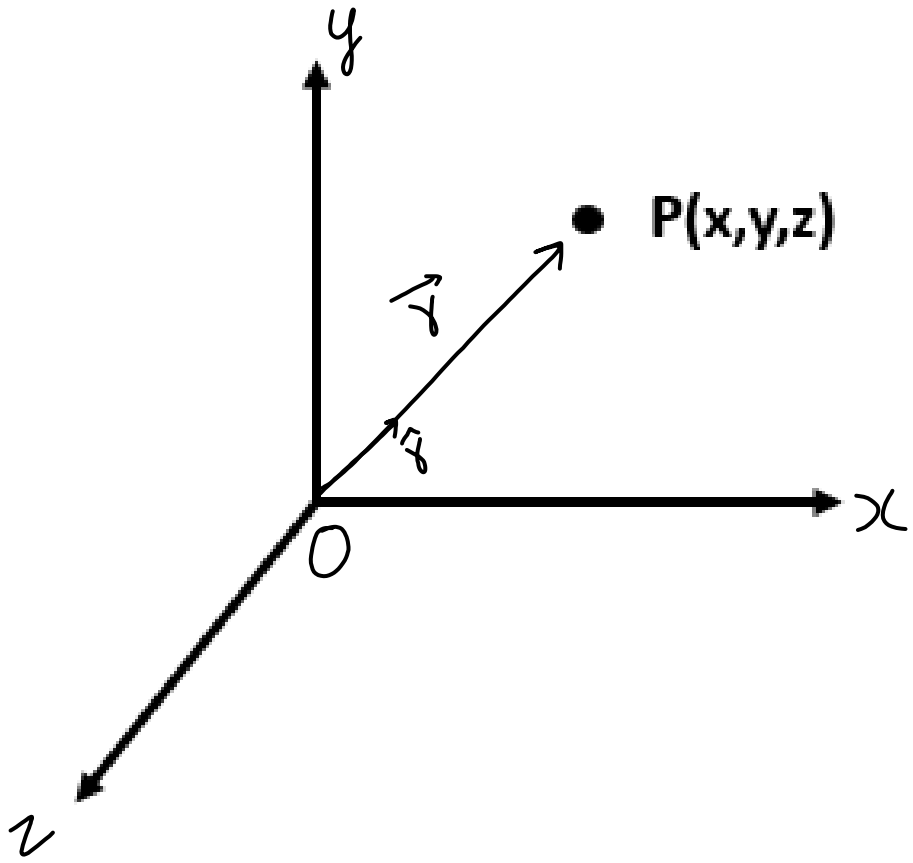
In 3-D

Position Vector

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

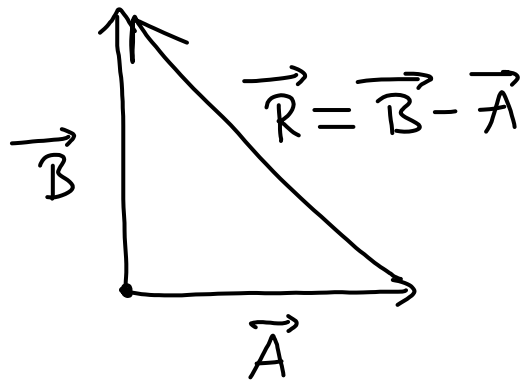
$$|r| = \sqrt{x^2 + y^2 + z^2}$$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2 + y^2 + z^2}}$$



Concept : Short trick

(1)

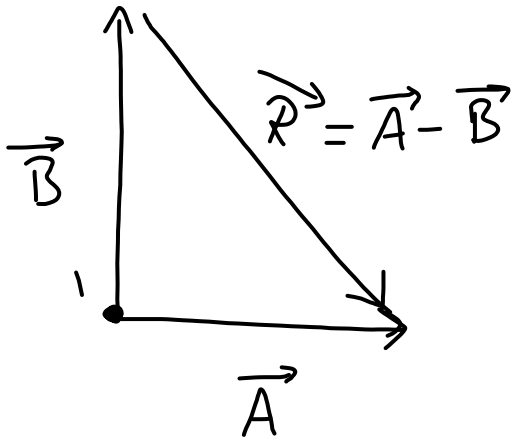


Tail से Tail जुड़ते हैं।

$$\vec{A} + \vec{R} = \vec{B}$$

$$\vec{R} = \vec{B} - \vec{A}$$

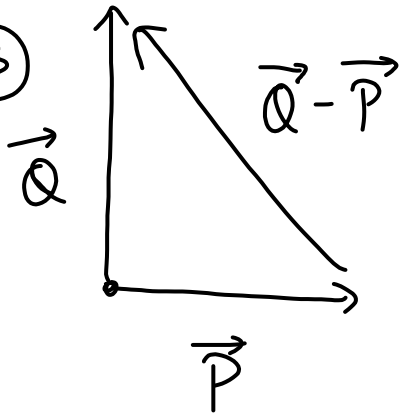
(2)



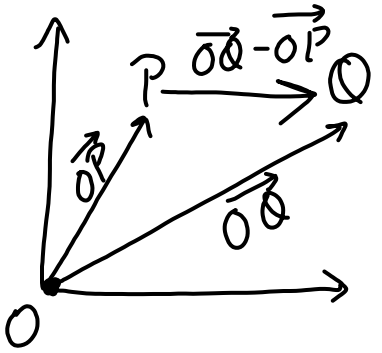
$$\vec{B} + \vec{R} = \vec{A}$$

$$\vec{R} = \vec{A} - \vec{B}$$

(3)

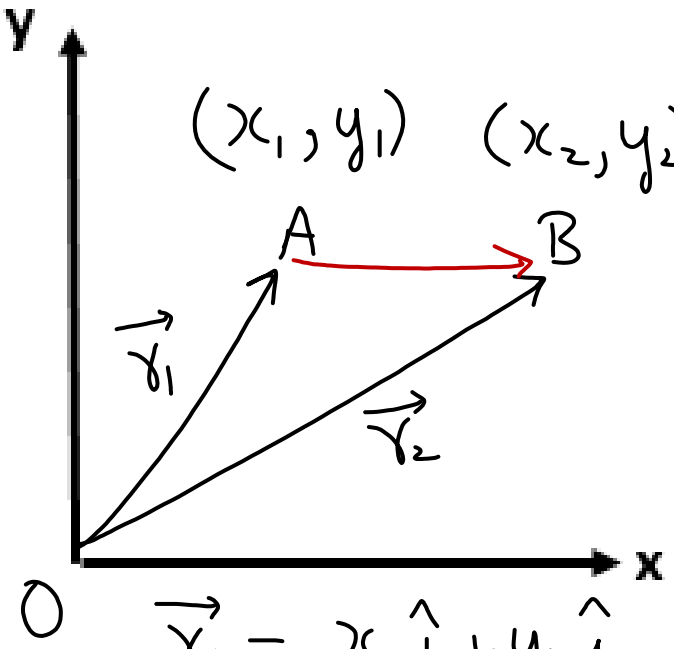


(4)



Displacement Vector

Displacement Vector is a vector joining initial position to final position.



$\vec{r_1} = x_1\hat{i} + y_1\hat{j}$

$\vec{r_2} = x_2\hat{i} + y_2\hat{j}$

displacement vector $\overrightarrow{AB}$

$\overrightarrow{AB} = \vec{r_2} - \vec{r_1}$

$= (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$

$$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\hat{AB} = \frac{\overrightarrow{AB}}{|AB|} = \frac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Imp

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA}$

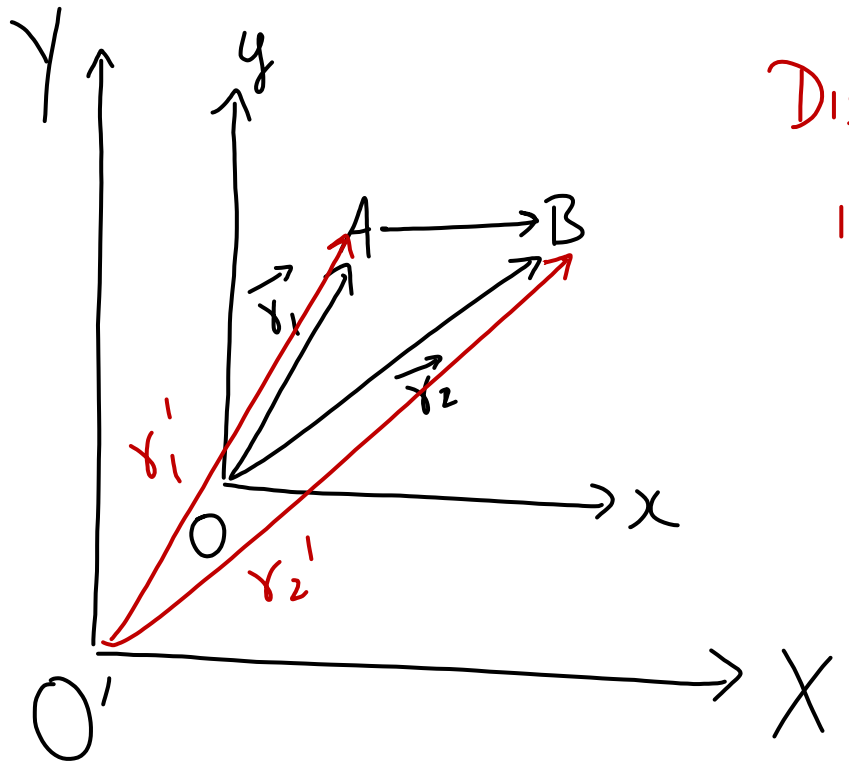
$\overrightarrow{RS} = \overrightarrow{OS} - \overrightarrow{OR}$

$\overrightarrow{RQ} = \overrightarrow{OQ} - \overrightarrow{OR}$

Q3) Which is independent of choice of origin.

a) **Position vector**
b) **Displacement vector**
c) **Both of the above**
d) **None of the above**

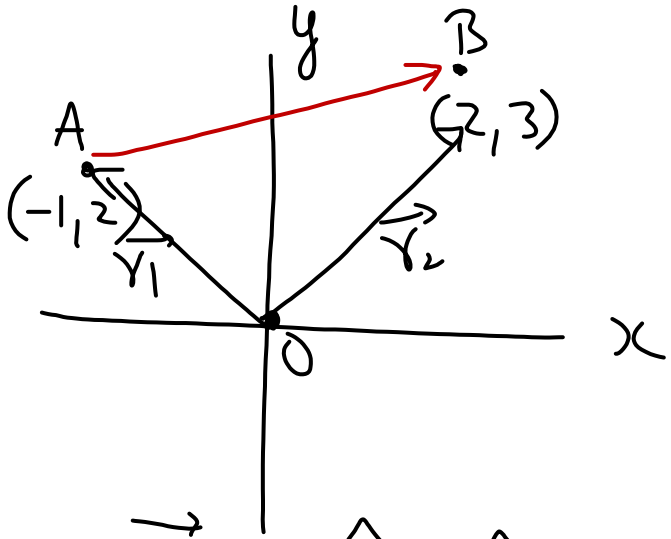
Position Vector depends on origin



Displacement Vector is INDEPENDENT of origin

Q 4) A particle moves from (-1,2) to (2,3). Find the displacement vector

Feel + Concept.



$\vec{r}_1 = -\hat{i} + 2\hat{j}$

$\vec{r}_2 = 2\hat{i} + 3\hat{j}$

disp Vector $\vec{AB}$

$\vec{AB} = \vec{r}_2 - \vec{r}_1$

$= (2\hat{i} + 3\hat{j}) - (-\hat{i} + 2\hat{j})$

$= 3\hat{i} + \hat{j}$

Formula

$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$\vec{AB} = 3\hat{i} + \hat{j}$

Q 5) A particle whose speed is 50 m/s moves along the line from A(2,1) to B (9,25). Find its velocity vector in the form of $a\hat{\imath} + b\hat{\jmath}$

a) $(7\hat{\imath} + 24\hat{\jmath})\,\mathrm{m/s}$

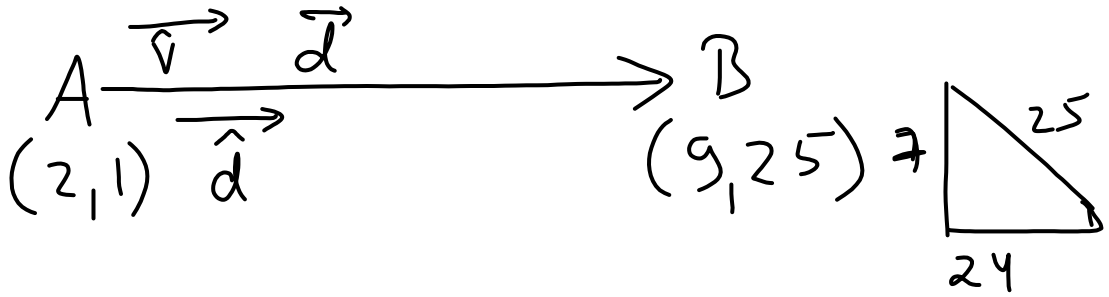
b) $2(7\hat{\imath} + 24\hat{\jmath})\,\mathrm{m/s}$

c) $4(7\hat{\imath} + 24\hat{\jmath})\,\mathrm{m/s}$

d) $5(7\hat{\imath} + 24\hat{\jmath})\,\mathrm{m/s}$

$\hat{v} = \hat{d}$

Velocity is in direction of displacement



$\vec{d} = (9-2)\hat{\imath} + (25-1)\hat{\jmath}$

$\vec{d} = 7\hat{\imath} + 24\hat{\jmath}$

$\hat{d} = \dfrac{\vec{d}}{|d|} = \dfrac{7\hat{\imath}+24\hat{\jmath}}{\sqrt{7^2+24^2}} = \dfrac{7\hat{\imath}+24\hat{\jmath}}{25}$

$\vec{V} = |V|\,\hat{v}$

$\vec{V} = 50\,\hat{v} = 50\left(\dfrac{7\hat{\imath}+24\hat{\jmath}}{25}\right)$

$\vec{V} = 2(7\hat{\imath}+24\hat{\jmath})\ \mathrm{m/s}$

In 3-D

Displacement Vector

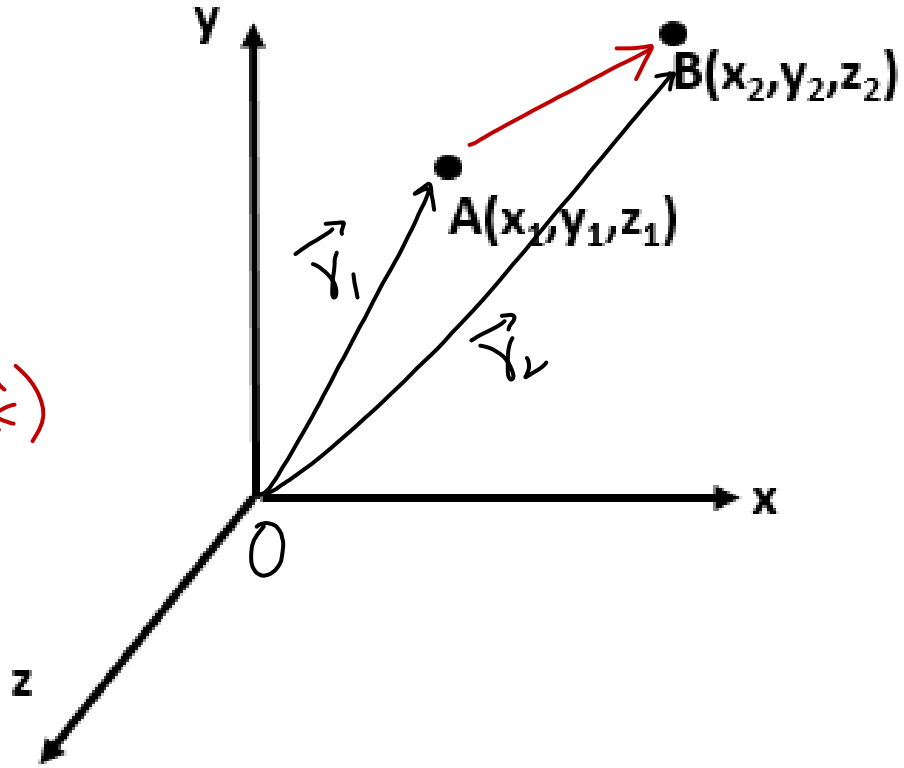
$$\overrightarrow{AB} = \vec{r}_2 - \vec{r}_1$$

$$\overrightarrow{AB} = (x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) - (x_1\hat{i} + y_1\hat{j} + z_1\hat{k})$$

$$\overrightarrow{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$\hat{AB} = \frac{\overrightarrow{AB}}{|AB|} = - - - -$$



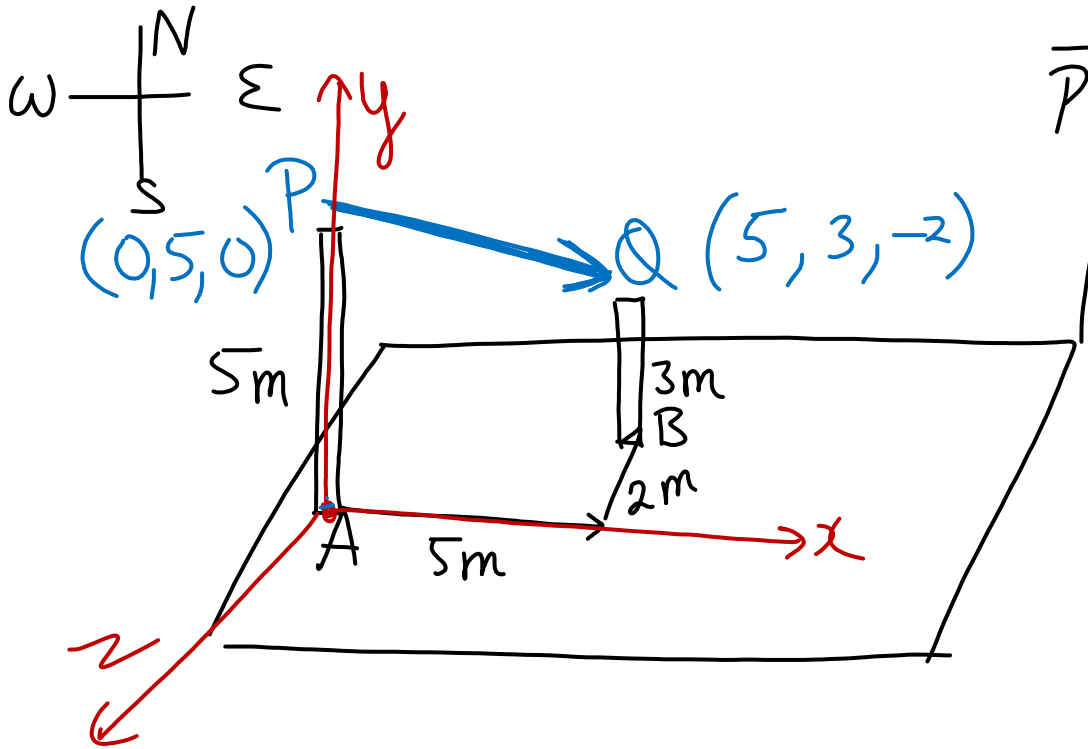
Q 6) On a horizontal flat ground, a person is standing at a point A . At this point, he installs a 5 m long pole vertically. Now, he moves 5 m towards east and then 2 m towards north and reaches at a point B. There he installs another 3 m long vertical pole. A bird flies from the top of the first pole to the top of the second pole. Find the magnitude of the displacement of the bird.

a) $\sqrt{33}$

b) $\sqrt{35}$

c) $\sqrt{37}$

d) $\sqrt{39}$

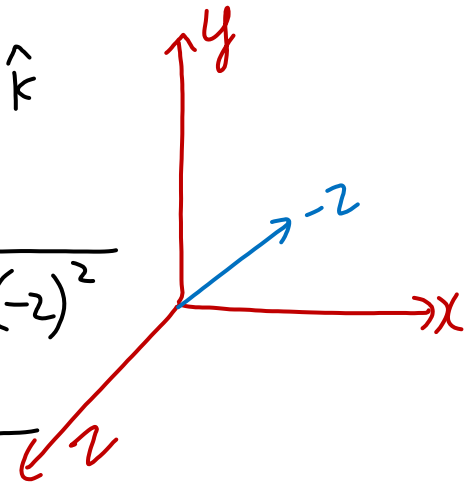


$\vec{PQ} = 5\hat{i} - 2\hat{j} - 2\hat{k}$

$|\vec{PQ}| = \sqrt{5^2 + (-2)^2 + (-2)^2}$

$= \sqrt{25 + 4 + 4}$

$= \sqrt{33}$ m



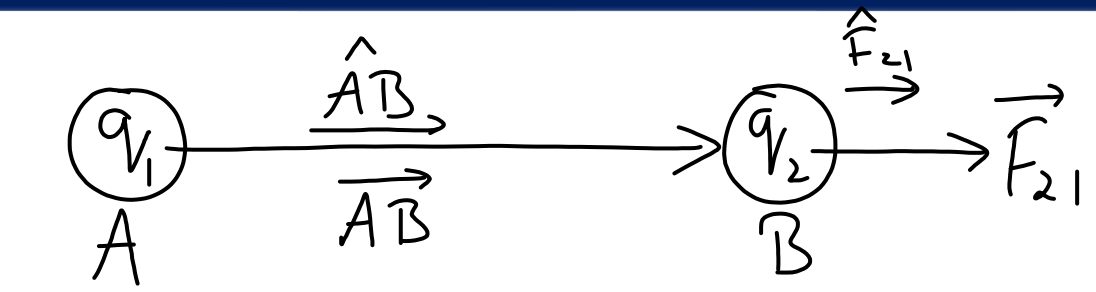
Q 8) Two point charges q_1 and q_2 are placed at (0,0,0) and (1,2,2) m respectively. They repel each other with a force of 3 N. The force on q_2 due to q_1 is $F_{21} = (x\hat{i} + y\hat{j} + z\hat{k})$ N. Find the value of $x + y + z$

a) 3

b) ~~5~~

c) 7

d) 9



$\hat{AB} = \hat{F}_{21}$

$\vec{F}_{21} = x\hat{i} + y\hat{j} + z\hat{k}$

$x + y + z$

Ans $\vec{F}_{21} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\vec{F}_{21} = |\vec{F}_{21}|\ \hat{F}_{21}$

$\vec{F}_{21} = 3\ \hat{F}_{21}$

$\vec{F}_{21} = 3 \times \dfrac{(\hat{i} + 2\hat{j} + 2\hat{k})}{3}$

$\overrightarrow{AB} = \hat{i} + 2\hat{j} + 2\hat{k}$

$\hat{AB} = \dfrac{\overrightarrow{AB}}{|AB|} = \dfrac{\hat{i} + 2\hat{j} + 2\hat{k}}{\sqrt{1^2 + 2^2 + 2^2}} = \dfrac{\hat{i} + 2\hat{j} + 2\hat{k}}{3}$

$x + y + z$

$1 + 2 + 2$

$= 5$

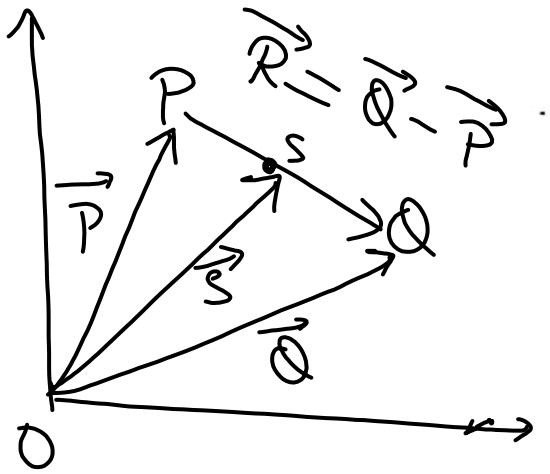
Q 9) Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is b | $\vec{R}$ | . The general relation among vector $\vec{P}$, $\vec{Q}$ and $\vec{S}$ is: [JEE Advanced 2017]

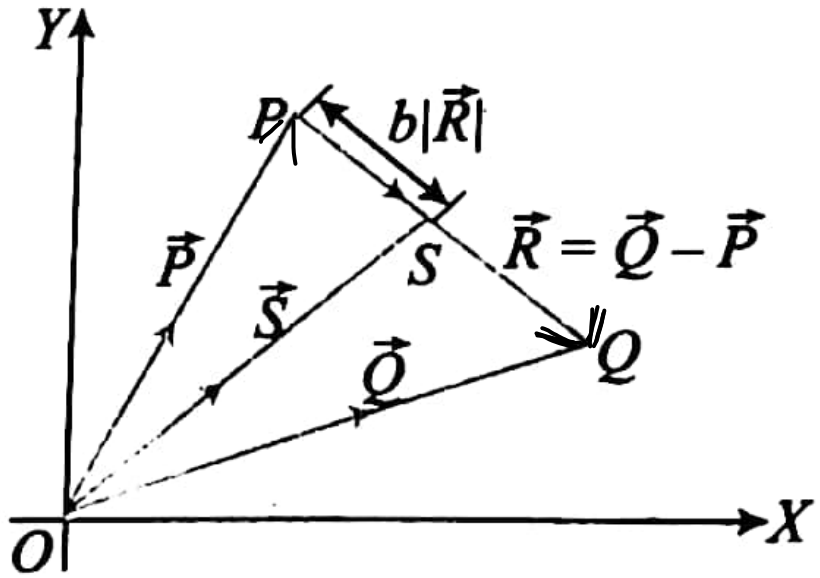
a) $\vec{S} = (1 - b)\vec{P} + b2\vec{Q}$

b) $\vec{S} = (b - 1)\vec{P} + b\vec{Q}$

c) ✓ $\vec{S} = (1 - b)\vec{P} + b\vec{Q}$

d) $\vec{S} = (1 - b2)\vec{P} + b\vec{Q}$





$|PS| = b|R|$

$$\overrightarrow{PS} = \vec{S} - \vec{P}$$

$$b|R|\hat{R} = \vec{S} - \vec{P}$$

$$b|R|\frac{\vec{R}}{|R|} = \vec{S} - \vec{P}$$

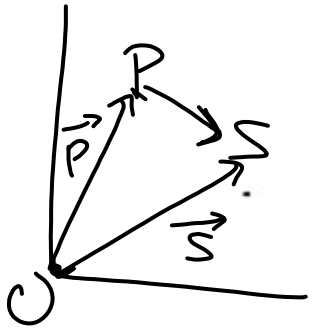
$$b\vec{R} = \vec{S} - \vec{P}$$

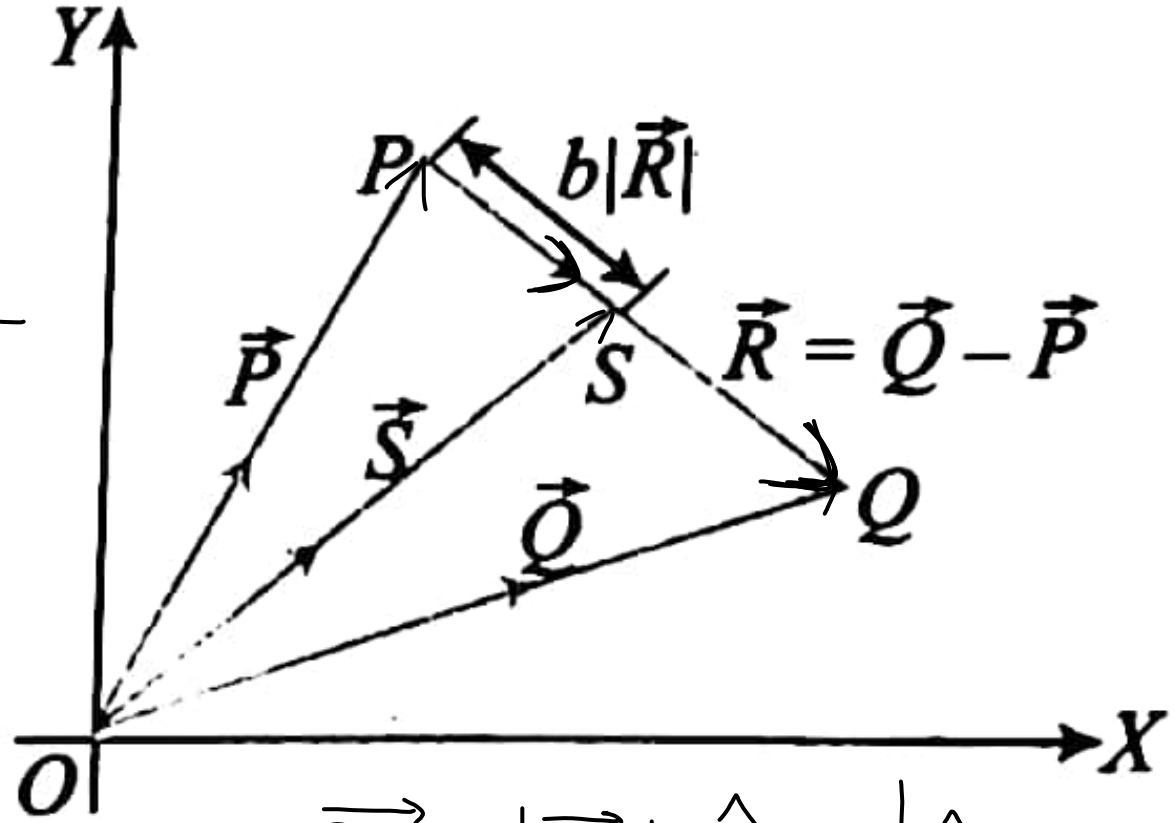
$$b(\vec{Q} - \vec{P}) = \vec{S} - \vec{P}$$

$$b(\vec{Q} - \vec{P}) + \vec{P} = \vec{S}$$

$$\vec{S} = b\vec{Q} - b\vec{P} + \vec{P}$$

$$\boxed{\vec{S} = b\vec{Q} + \vec{P}(1-b)}$$





$$\overrightarrow{PS} = |\overrightarrow{PS}|\,\hat{PS}$$

$$\overrightarrow{PS} = b|R|\,\hat{R}$$

$$\hat{R} = \frac{\vec{R}}{|R|}$$

Thank You

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