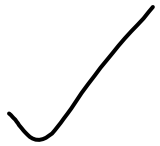
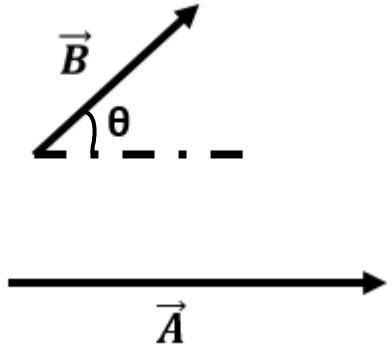
Ch-04
Vectors
Lect-03

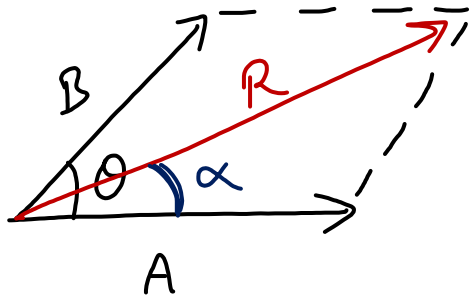
Today's Goal



Analytical Method Of Vector Addition

Parallelogram Law of Vector Addition



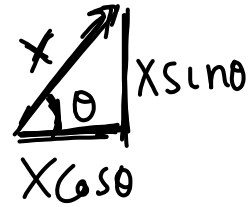


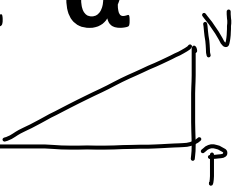
$\vec{A} + \vec{B} = \vec{R}$

→ Magnitude = length of this diagonal

→ direction → 'α' w.r.t $\vec{A}$

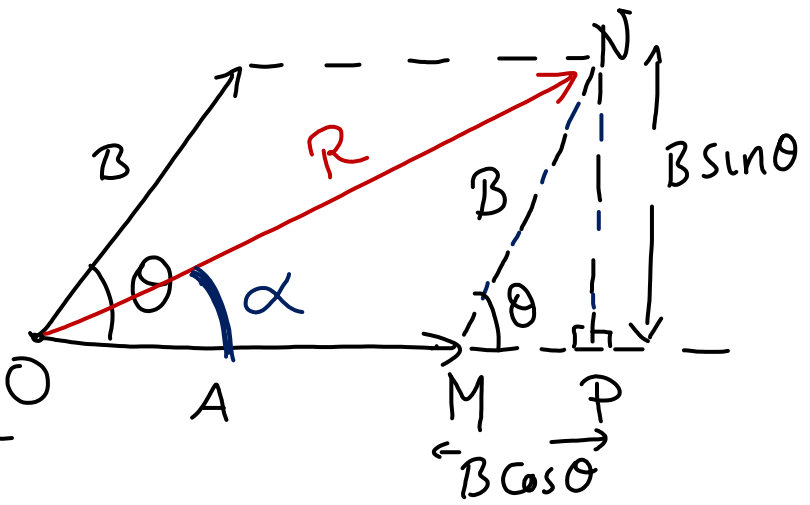
Magnitude of Resultant |R| & Direction of Resultant 'α'





In ΔONP,

$$(ON)^2 = (NP)^2 + (OP)^2$$

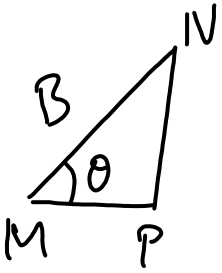


$$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$$

$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$

$$R^2 = A^2 + B^2(\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$$

$$\boxed{R^2 = A^2 + B^2 + 2AB\cos\theta}$$



$$\cos\theta = \frac{MP}{B}$$

$$MP = B\cos\theta$$

$$\sin\theta = \frac{NP}{B}$$

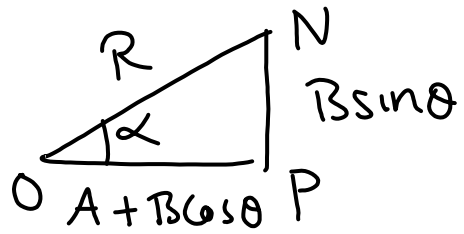
$$NP = B\sin\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta \quad \{0^\circ \le \theta \le \pi\}$$

Magnitude

$$|R| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$$

In $\triangle ONP$



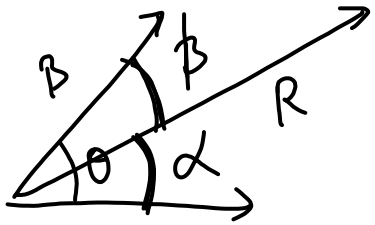
$$\tan\alpha = \frac{NP}{OP}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\alpha = \tan^{-1}\left(\frac{B\sin\theta}{A + B\cos\theta}\right)$$

α w.r.t $\vec{A}$

β w.r.t $\vec{B}$



$$\tan\beta = \frac{A\sin\theta}{B + A\cos\theta}$$

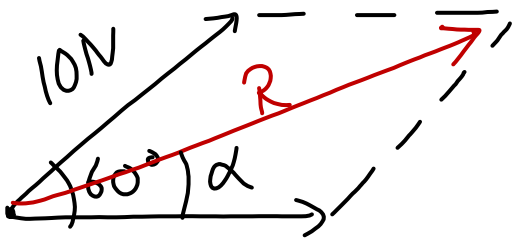
Q1) Two Forces each of magnitude 10 N act at a point as shown. Find the Resultant.

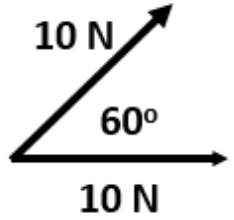
a) $10\sqrt{3}$, 30o with horizontal ✓

b) $10\sqrt{3}$, 60o with horizontal

c) $10\sqrt{2}$, 30o with horizontal ✗

d) $10\sqrt{2}$, 60o with horizontal ✗





$R^2 = A^2 + B^2 + 2AB\cos\theta$

$R^2 = 10^2 + 10^2 + 2\times10\times10\times\cos60°$

$R^2 = 10^2 + 10^2 + 2\times10^2\times\frac{1}{2}$

$R^2 = 3\times10^2$

$R = \sqrt{3\times10^2}$

$R = 10\sqrt{3}$

$\tan\alpha = \frac{B\sin\theta}{A+B\cos\theta}$

$= \frac{10\sin60}{10+10\cos60}$

$= \frac{10\times\frac{\sqrt{3}}{2}}{10+10\times\frac{1}{2}}$

$$\tan\alpha = \frac{10\frac{\sqrt{3}}{2}}{10 + 10 \times \frac{1}{2}}$$

$$= \frac{5\sqrt{3}}{10+5} = \frac{8\sqrt{3}}{15\,3}$$

$$\tan\alpha = \frac{\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\alpha = 30^\circ$$

Q2) Two forces whose magnitude are in the ratio 3:5 give a resultant of 28 N. If the angle of their inclination is 60° , find the magnitude of each force.

a) **16 N, 24 N**

b) **12 N, 16 N**

c) **12 N, 20 N**

d) **16 N, 20 N**

$R = 28$

$\theta = 60°$

$F_1 = 3x$

$F_2 = 5x$

$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$(28)^2 = (3x)^2 + (5x)^2 + 2 \times 3x \times 5x \cos 60$

$(28)^2 = 9x^2 + 25x^2 + 2 \times 3x \times 5x \times \frac{1}{2}$

$(28)^2 = 9x^2 + 25x^2 + 15x^2$

$(28)^2 = 49x^2$

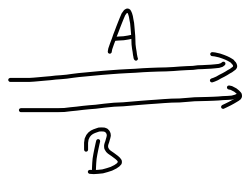
$28 = 7x$

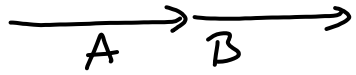
$x = \frac{28}{7} = 4$

$F_1 = 3x = 3 \times 4 = 12\ N$

$F_2 = 5x = 5 \times 4 = 20\ N$

Special Cases





$\theta = 0$ $\quad \cos 0^\circ = 1$

Case I: θ= 0°

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 + 2AB}$$

$$R = \sqrt{(A+B)^2} \implies \boxed{R = A + B} \quad \text{Max } R$$

$$\boxed{|R| = |A| + |B|} \quad \text{Max}$$

Case II: θ= 180°

$\cos 180^\circ = -1$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \sqrt{A^2 + B^2 + 2AB\cos 180} = \sqrt{A^2 + B^2 - 2AB}$$

$$R = \sqrt{(A-B)^2} = A - B$$

$$\boxed{|R| = A \sim B} \quad \text{Min}$$

$$\boxed{|R|_{Min} = |A - B|}$$

$$|A \sim B| \leq |R| \leq A + B$$

The Resultant of $\vec{A} = 3$ units, $\vec{B} = 4$ units can never be

a) 2 units ✓

b) 3 units ✓

c) 6 units ✓

d) 8 units ✗

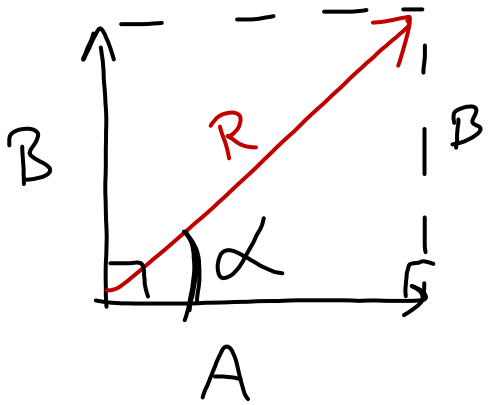
$$1 \leq |R| \leq 7$$

$\cos 90 \rightarrow 0$

Case III: θ= 90°

$$R = \sqrt{A^2 + B^2 + 2AB\cos 90}$$

$$R = \sqrt{A^2 + B^2}$$



$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta} = \frac{B\sin 90}{A + B\cos 90} = \frac{B}{A}$$

$$\tan\alpha = \frac{B}{A}$$

$$R = \sqrt{A^2 + B^2}$$

Q3) The $|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$, then angle between $\vec{A}$ and $\vec{B}$ will be

a) 90°
b) 120°
c) 0°
d) 60°

$$\vec{A}+\vec{B}=\vec{R}$$

$$|\vec{A}+\vec{B}|=|\vec{R}|$$

$$|\vec{A}+\vec{B}|=\left||\vec{A}|-|\vec{B}|\right|$$

$$|\vec{R}|=|A-B|$$

$$R=A\sim B$$

$\theta=180^\circ$

$$|\vec{A}+\vec{B}|=|\vec{A}|+|\vec{B}|$$

$$|\vec{R}|=|\vec{A}|+|\vec{B}|$$

$$R=A+B$$

Max

$\theta=0^\circ$

Q4) Which pair of the following forces will never give resultant force of 2N ?

a) **2N and 2 N** ✓ $0 \leq R \leq 4$

b) **1 N and 1 N** ✓ $0 \leq R \leq 2$

c) **1 N and 3 N** ✓ $2 \leq R \leq 4$

d) **1 N and 4 N** ✗ $3 \leq R \leq 5$

Q5) The maximum and minimum magnitude of the resultant of two given vectors are 17 units and 7 units , respectively. If these two vectors are at right angles to each other, the magnitude of their resultant is

a) 14
b) 16
c) 18
d) 13

$\vec{A}$, $\vec{B}$ $(|A| > |B|)$

$R_{Max} = A + B = 17$

$R_{Min} = A - B = 7$

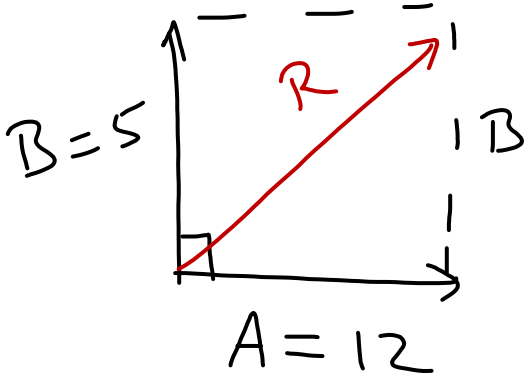
$A + B = 17$

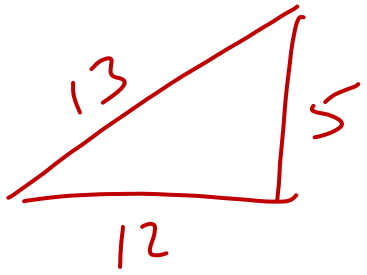
$\oplus$ $A - B = 7$

$2A = 24$

$\boxed{A = 12}$

$\boxed{B = 5}$





$$R = \sqrt{A^2 + B^2 + 2AB\cos 90^\circ} = \sqrt{A^2 + B^2} = \sqrt{12^2 + 5^2} = 13$$

Q6) The sum of the magnitudes of two forces acting at a point is 18 and the magnitude of their resultant is 12. If the resultant is at 90° with the force of smaller magnitude, what are the magnitudes of forces?

$\frac{1}{0} = \infty$ undefined

a) 12,6

b) 15,3

~~c) 13,5~~

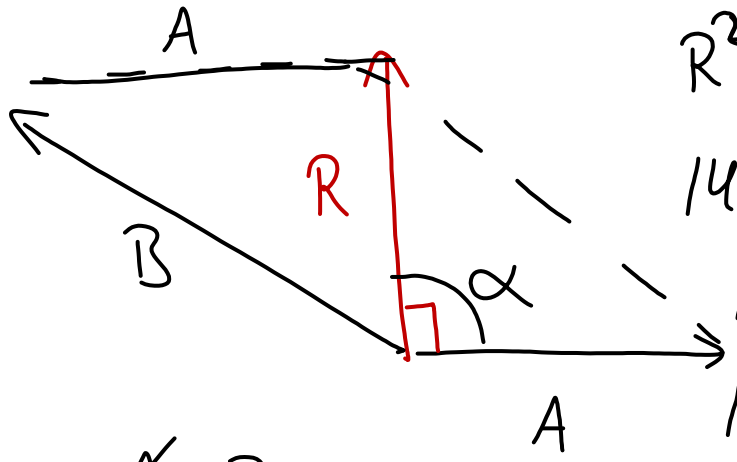
d) 10,8

$\vec{A} + \vec{B} = \vec{R}$

$A + B = 18$

$R = 12$

Assume $\longrightarrow$ A is smaller $(A < B)$



$R^2 = A^2 + B^2 + 2AB\cos\theta$

$144 = A^2 + B^2 + 2AB\cos\theta$

$144 = A^2 + B^2 + 2A(-A)$

$144 = A^2 + B^2 - 2A^2$

$144 = B^2 - A^2$

$144 = (B+A)(B-A)$

$144 = 18 \times B - A$

$8 = B - A$

$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$

$\tan 90 = \frac{B\sin\theta}{A + B\cos\theta} \Rightarrow A + B\cos\theta = 0$

undefined

$B\cos\theta = -A$

$A + B = 18$

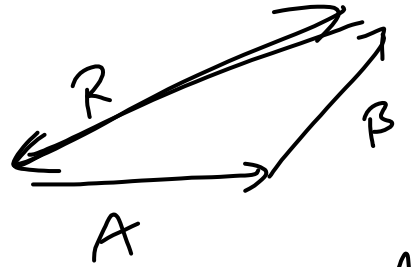
$\oplus \quad B - A = 8$

$2B = 26$

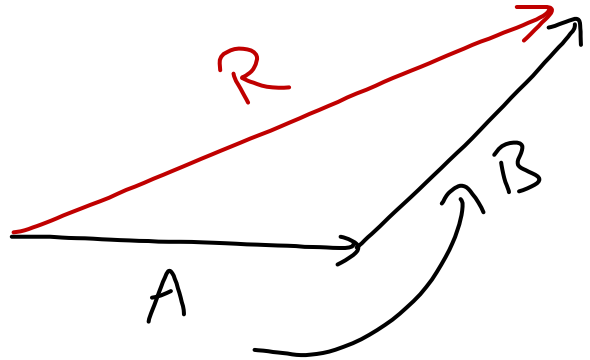
$B = 13$

Triangle Law

Method 2

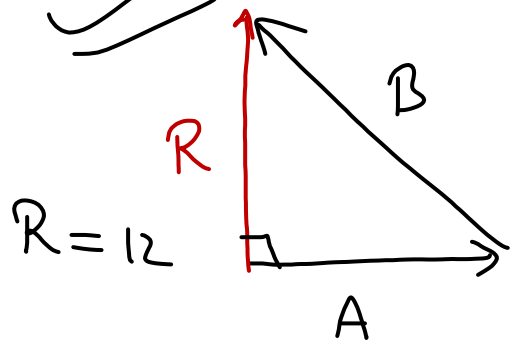


$A + B = 18$



$\Rightarrow$

100 |



$A + B = 18$

$+ \quad B - A = 8$

$2B = 26$

$\boxed{B = 13}$

$\boxed{A = 5}$

$R^2 = B^2 - A^2$

$R^2 = (B - A)(B + A)$

$144 = (B - A) \times 18$

$8 = B - A$

Q7) The resultant of two vectors $\vec{A}$ and $\vec{B}$ is perpendicular to the vector $\vec{A}$ and its magnitude is equal to half of the magnitude of the vector $\vec{B}$. Find out the angle between $\vec{A}$ & $\vec{B}$ ⟶ when $\vec{A}$ & $\vec{B}$ joined Tail to Tail

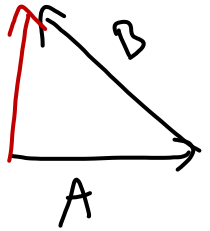
a) θ=30°

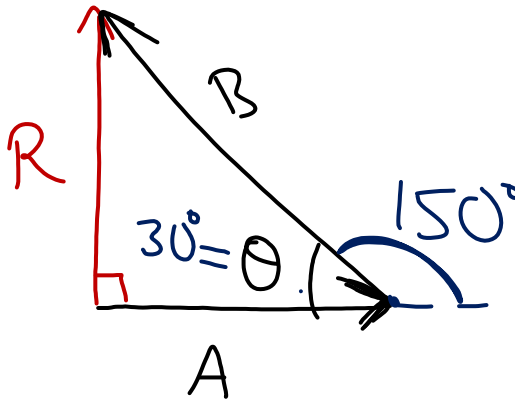
b) θ=150°

c) θ=60°

d) θ=120°

Triangle law





$R = \frac{B}{2}$

$\sin\theta = \frac{R}{B}$

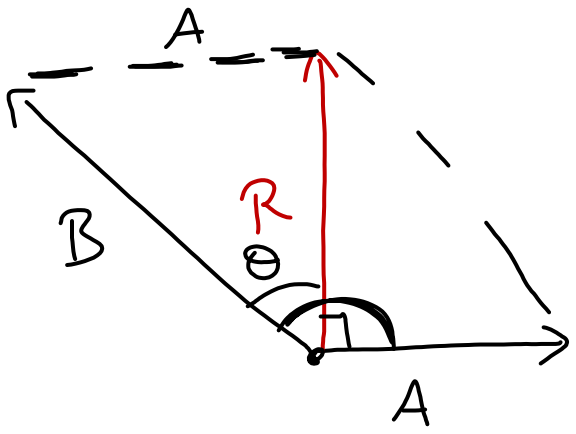
$= \frac{B/2}{B}$

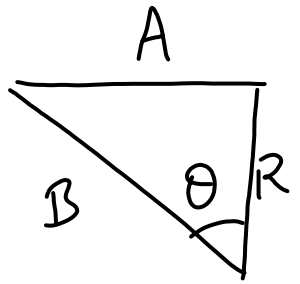
$\theta = 30°$

$\sin\theta = \frac{1}{2}$

Method 2

Ilgm Law





$$\cos\theta = \frac{R}{B} = \frac{B/2}{B}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$

$$R = \frac{B}{2}$$

Tail se Tail Jude

$$= 90 + \theta$$

$$= 90 + 60 = 150$$

Thank You

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