Ch-04
Vectors
Lect-05

Today's Goal

- Resolution of Vector
- Unit Vector
- Vector in Cartesian Form

Resolution of Vector

In ΔABC

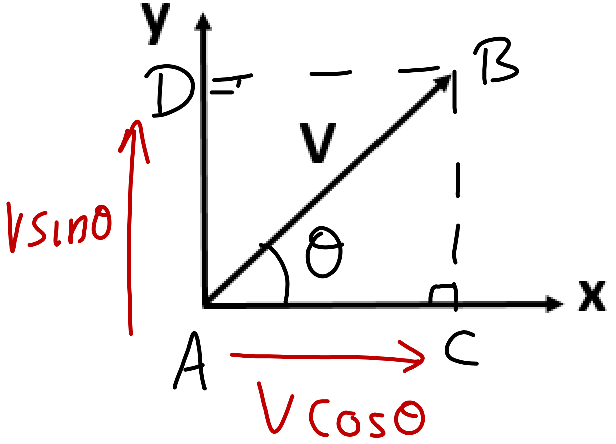
$$\cos\theta = \frac{AC}{V}$$

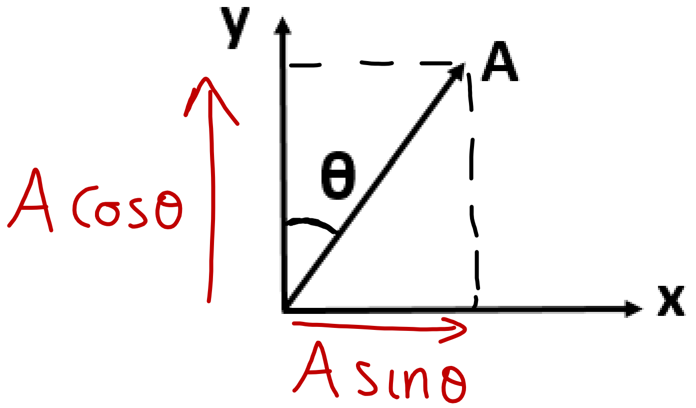
$$AC = V\cos\theta$$

$$\sin\theta = \frac{BC}{V}$$

$$BC = V\sin\theta$$

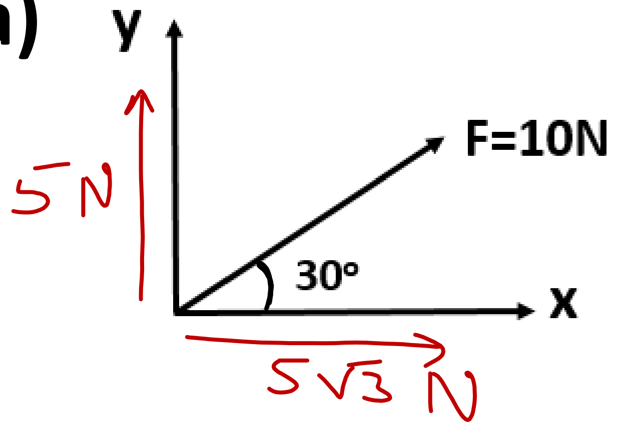
$$AD = BC = V\sin\theta$$



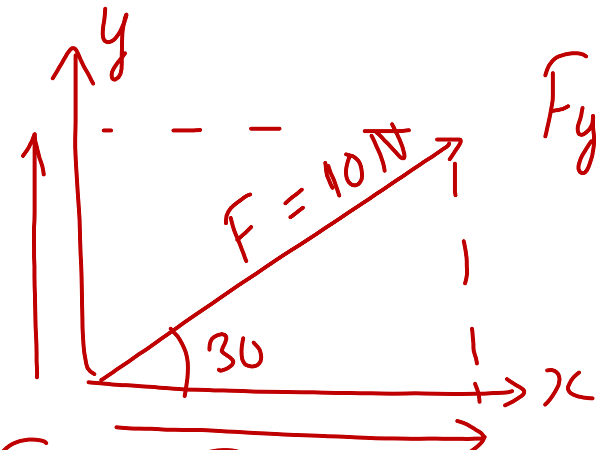

y
A
θ
x
A cosθ
A sinθ

Q1) Find the x & y components of vector shown.

a)



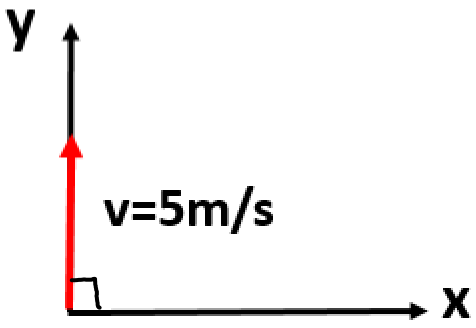
$F_y = F\sin 30$



$F_y = F\sin 30 = 10 \times \frac{1}{2} = 5N$

$F_x = F\cos 30 = 10\cos 30 = \frac{10\sqrt{3}}{2} = 5\sqrt{3}N$

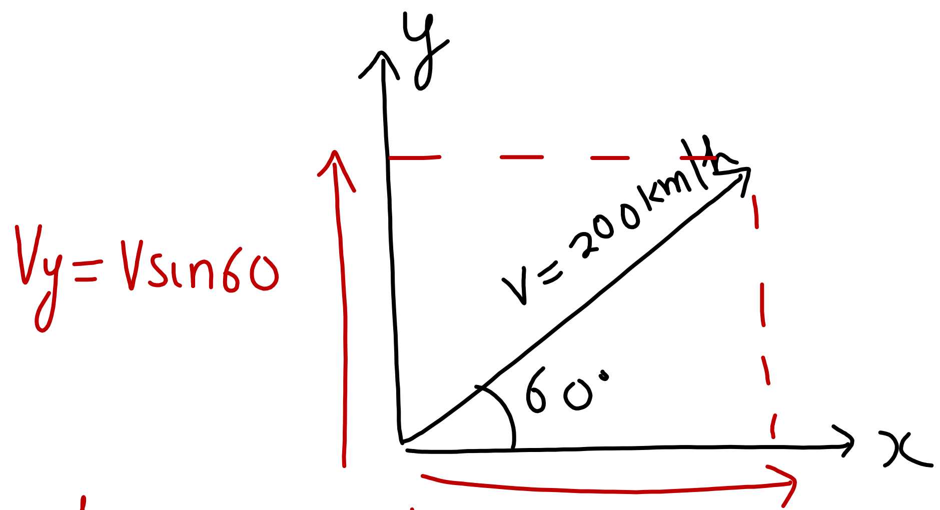
b)



$V_y = V\sin 90 = 5 \times 1 = 5 m/s$

$V_x = V\cos 90 = 0$

Q2) An aeroplane takes off at an angle of 60º to the horizontal . If the velocity of the plane is 200 km/hr, calculate its horizontal and vertical component of its velocity.

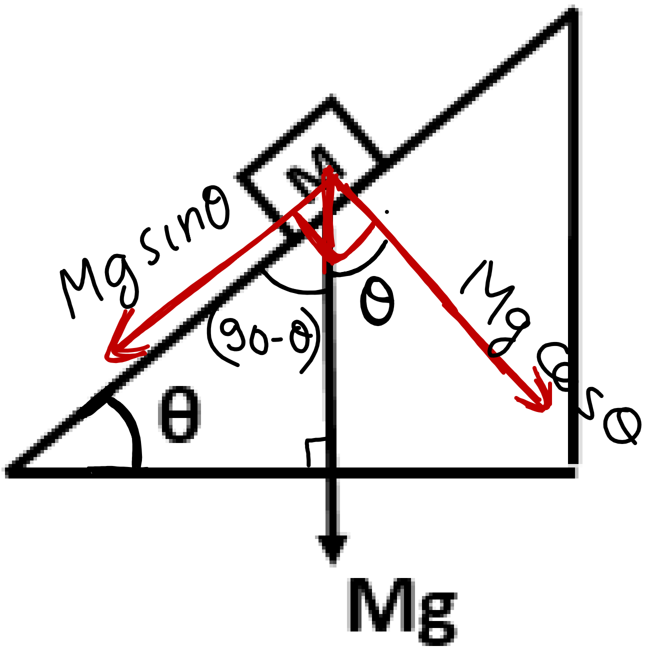


Vertical Component

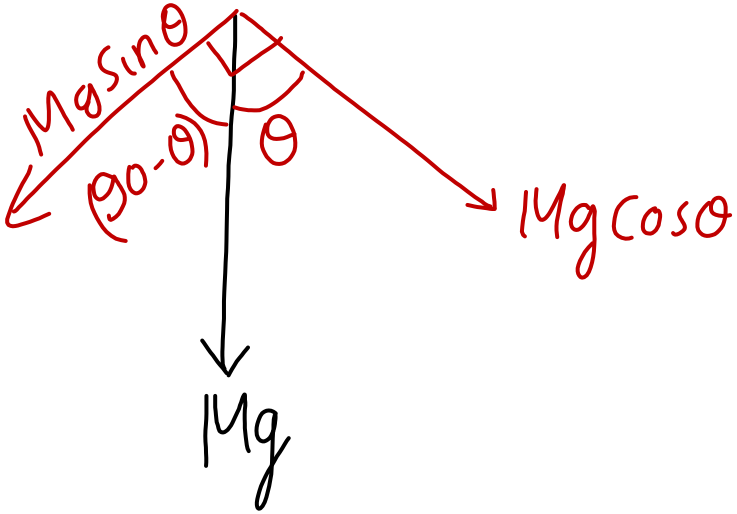
$V_y = V\sin 60 = \cancel{200}^{100} \times \frac{\sqrt{3}}{\cancel{2}} = 100\sqrt{3}\,km/h$

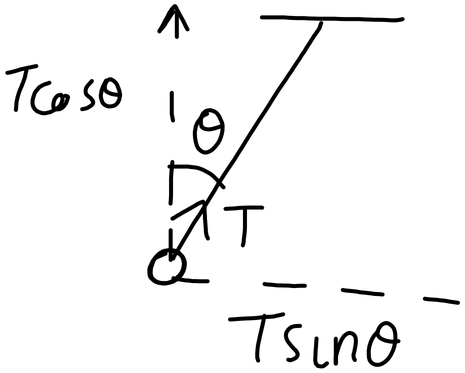
horizontal Component $V_x = V\cos 60° = 200 \times \frac{1}{2} = 100\,km/h$

We can have two mutually perpendicular components of a vector other than x-direction or y-direction.



Along the Incline
Perpendicular to Incline





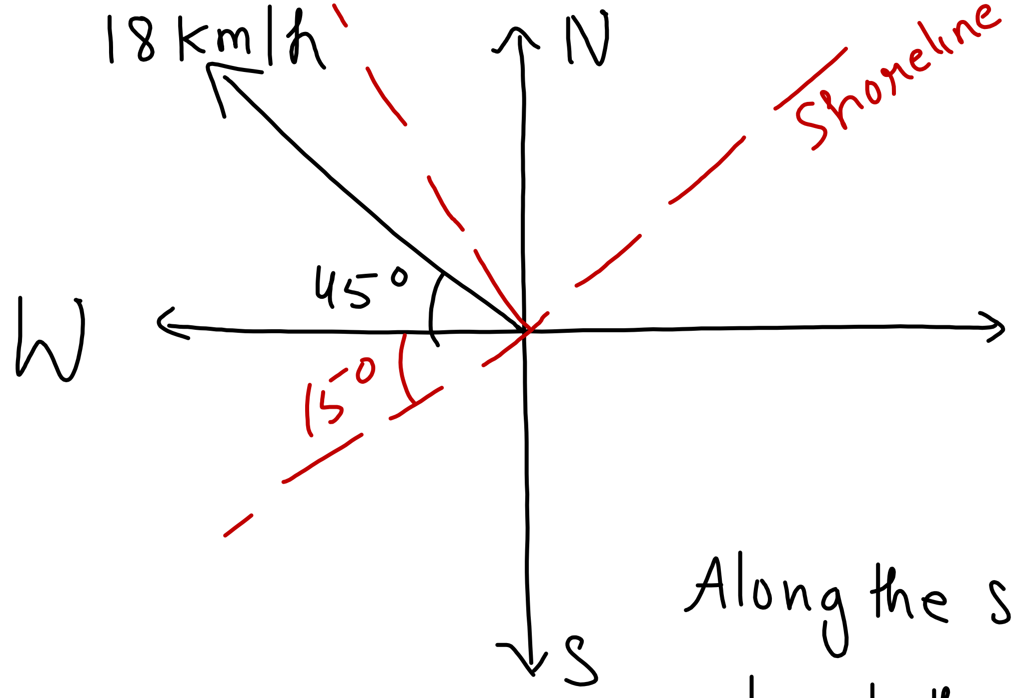
Q3) A man rows a boat with a speed of 18 km/hr in the north- west direction. The shoreline makes an angle of 15° south of west. Obtain the components of the velocity of the boat along the shoreline and perpendicular to the shoreline

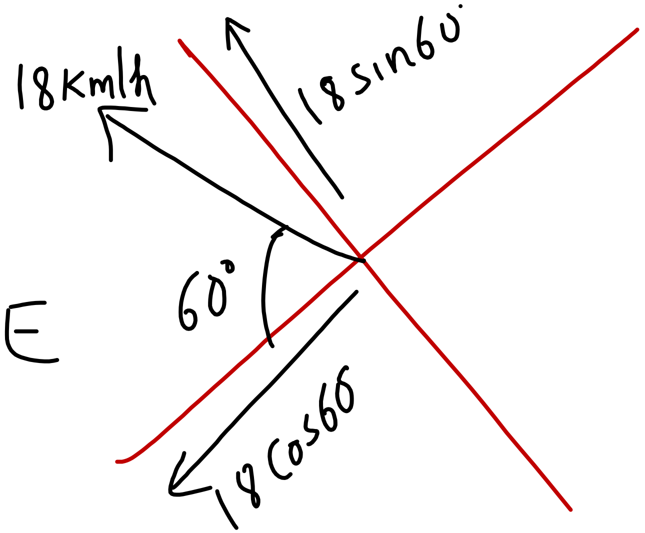
a) $3\sqrt{3}, 3$

b) $9\sqrt{3}, 9$

c) $3, 3\sqrt{3}$

d) $9, 9\sqrt{3}$ ✓





Along the SL $\rightarrow$ $18\cos 60 = 18 \times \frac{1}{2} = 9$ km/h

$\perp$ to the SL $\rightarrow$ $18\sin 60 = 18\,\frac{\sqrt{3}}{2} = 9\sqrt{3}$ km/h

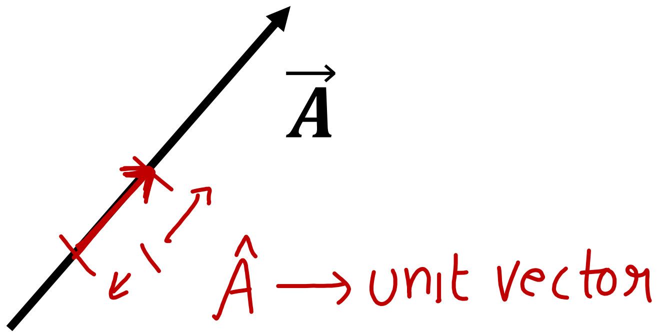
Unit Vector

$$\left(\hat{A} = \frac{\vec{A}}{|A|}\right)$$

A unit vector is a dimensionless vector having a magnitude of exactly 1 & it gives direction

$$\vec{A} = \text{Magnitude of } A \times \text{Direction of } A$$

$$\vec{A} = |A| \times \hat{A}$$



$$\vec{A} = |A|\,\hat{A}$$

$$\boxed{\hat{A} = \frac{\vec{A}}{|A|}}$$

$$\boxed{\hat{A} = \frac{\vec{A}}{|A|}}$$

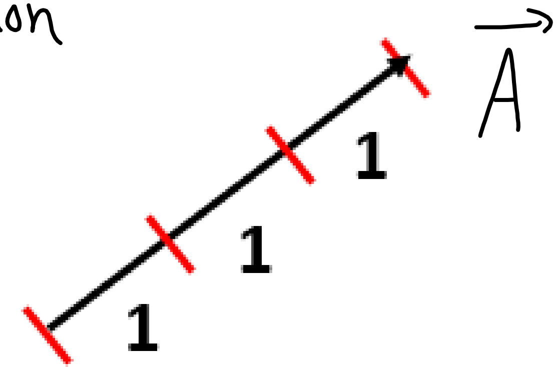
$$\boxed{\vec{A} = |A|\hat{A}}$$

Magnitude direction

$\vec{A} = ?$

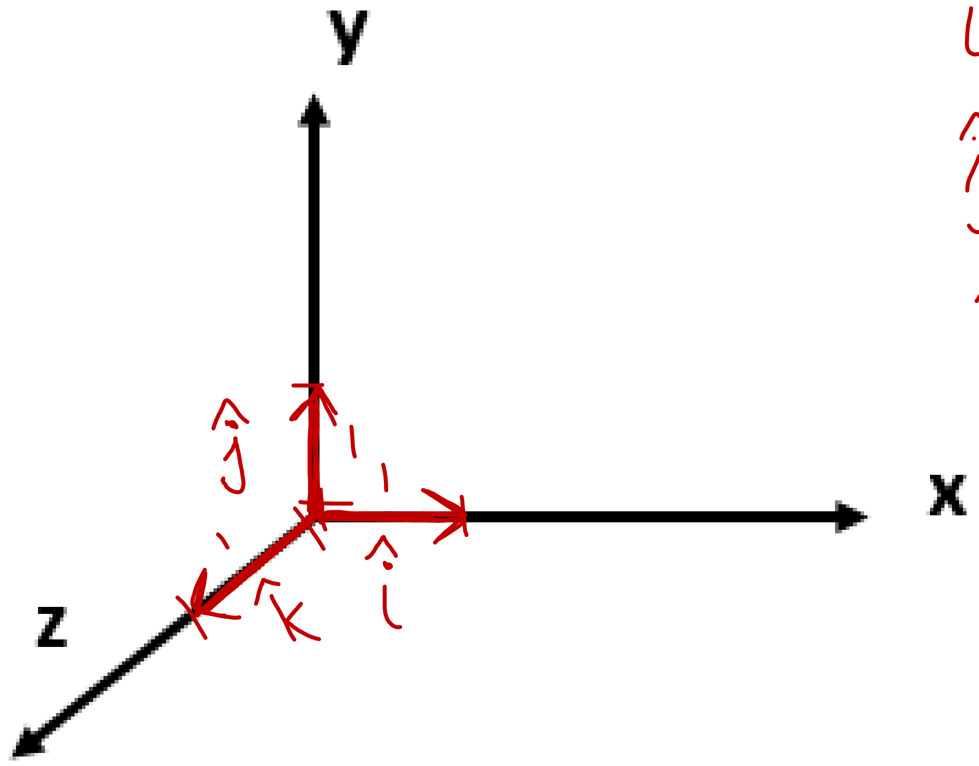
$\vec{A} = |A|\,\hat{A}$

$\vec{A} = 3\,\hat{A}$



Orthogonal Unit Vectors: $\hat{\imath}$, $\hat{\jmath}$, $\hat{k}$

mutually
$\perp^{r}$



$\hat{i} \longrightarrow$ Unit vector in +x-direction

$\hat{j} \longrightarrow$ " " in +y direction

$\hat{k} \longrightarrow$ " " in +z direction

Q4)Represent the following vectors

1. Vector $\vec{A}$ has magnitude 5 units and is in the direction of +x axis.

$$\vec{A} = |A|\,\hat{A} = 5\hat{i}$$

2. Vector $\vec{B}$ has magnitude 4 units and is in the direction of +z axis.

$$\vec{B} = |B|\,\hat{B} = 4\hat{k}$$

$$\vec{C} = |C|\,\hat{C} = 2(-\hat{i}) = -2\hat{i}$$

3. Vector $\vec{C}$ has magnitude 2 units and is in the direction of –x axis.

4. Vector $\vec{D}$ has magnitude 3 units in +x direction & 4 units in +y direction.

$\vec{D} = 3\hat{i} + 4\hat{j}$ $\longrightarrow$ Cartesian form of vector

5. Vector $\vec{E}$ has magnitude 7 units in +y direction & 5 units in –z direction.

$\vec{E} = 7\hat{j} - 5\hat{k}$

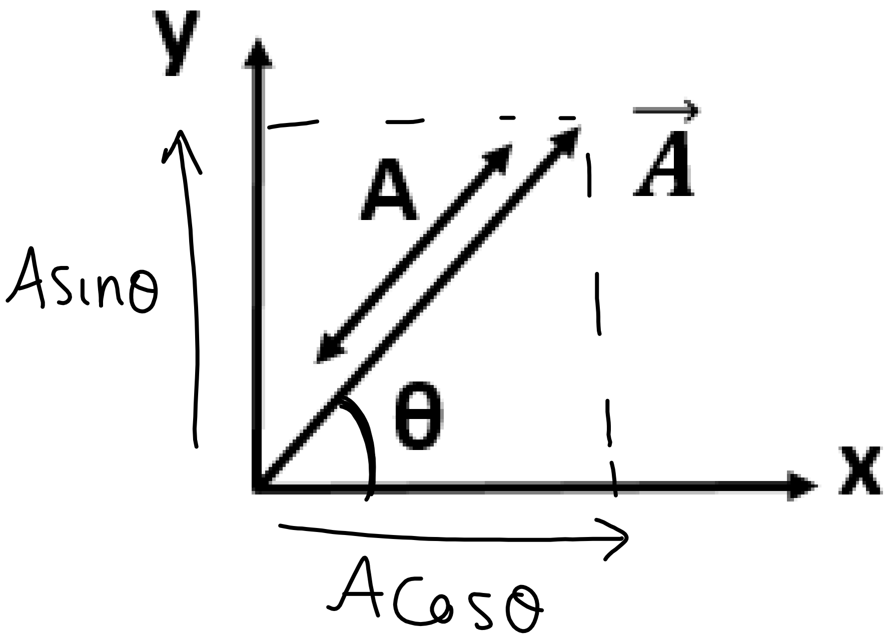
Refer to the figure shown & Find:

i. **Ax** $= A\cos\theta$

ii. **Ay** $= A\sin\theta$

iii. $\overrightarrow{\mathbf{Ax}} = |A_x|\,\hat{A}_x = A\cos\theta\,\hat{i}$

iv. $\overrightarrow{\mathbf{Ay}} = |A_y|\,\hat{A}_y = A\sin\theta\,\hat{j}$



$\boxed{|A| = \sqrt{A_x^2 + A_y^2}}$

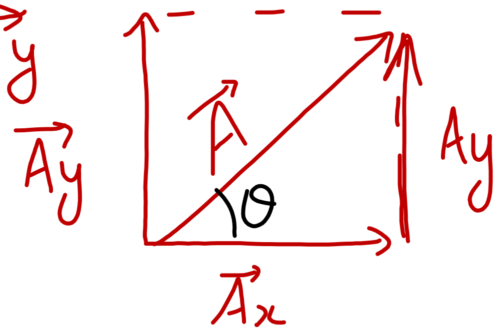
cartesian form

v. $\vec{A}$

$\vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$

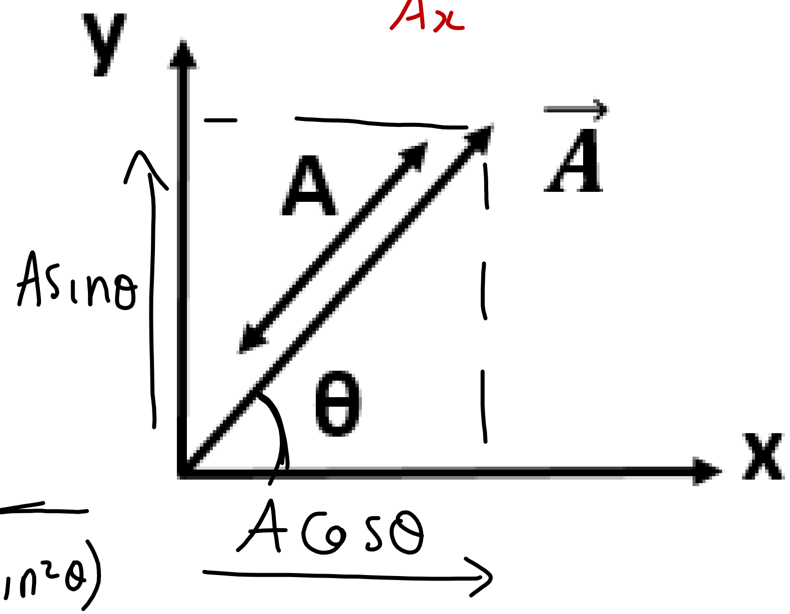
$\boxed{\vec{A} = \vec{A}_x + \vec{A}_y}$

$\vec{A} = \vec{A}_x + \vec{A}_y$



vi. |A|

$|A| = A$



vii. $\sqrt{Ax^2 + Ay2}$

$= \sqrt{(A\cos\theta)^2 + (A\sin\theta)^2}$

$= \sqrt{A^2\cos^2\theta + A^2\sin^2\theta} = \sqrt{A^2(\cos^2\theta + \sin^2\theta)}$

$= A$

viii. tanθ

$\boxed{\tan\theta = \frac{A_y}{A_x}}$

$\vec{B} = 5\hat{i} - 12\hat{j}$

$$\boxed{\vec{A} = Ax\,\hat{i} + Ay\,\hat{j}}$$

$\vec{A} = 3\hat{i} + 4\hat{j}$

$|B| = \sqrt{(5)^2 + (-12)^2}$

$|B| = 13$

$$\boxed{|A| = \sqrt{(Ax)^2 + (Ay)^2}}$$

$|A| = \sqrt{(3)^2 + (4)^2}$

$= 5$

Similarly we can say, in 3-D

$\vec{R} = 2\hat{i} + 3\hat{j} - 4\hat{k}$

$$\boxed{\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}$$

$|R| = \sqrt{(2)^2 + (3)^2 + (-4)^2}$

$$\boxed{|A| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2}}$$

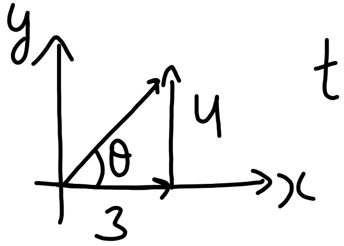
$= \sqrt{4 + 9 + 16}$

$|R| = +\sqrt{29}$

Q5) Find the magnitude and angle with +x-axis of the following vectors

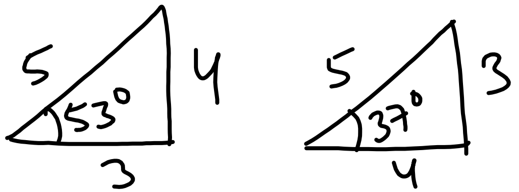
i. $\vec{a} = 3\hat{i} + 4\hat{j}$

$|a| = \sqrt{(3)^2 + (4)^2} = 5$



$\tan\theta = \frac{4}{3}$

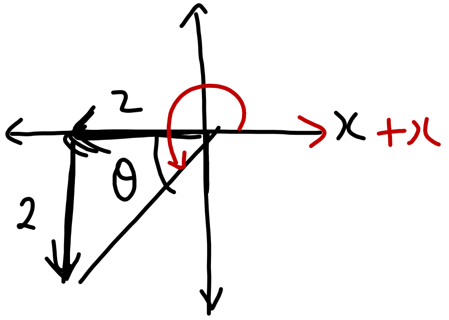
$\theta = 53^\circ$



ii. $\vec{b} = -2\hat{i} - 2\hat{j}$

$|b| = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

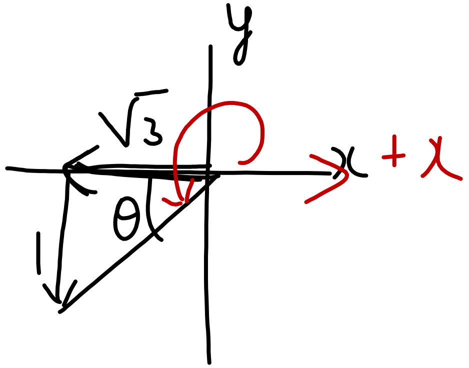
$\tan\theta = \frac{2}{2} = 1 \quad \theta = 45^\circ$



Angle $= 180 + 45 = 225^\circ$

iii. $\vec{c} = -\sqrt{3}\hat{i} - \hat{j}$

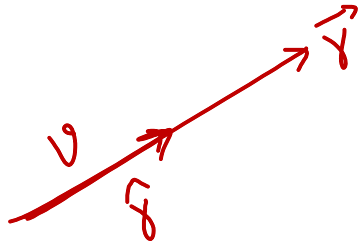
$|c| = \sqrt{(-\sqrt{3})^2 + (-1)^2} = \sqrt{3 + 1} = 2$



Angle $= 180 + 30 = 210^\circ$

$\tan\theta = \frac{1}{\sqrt{3}} \quad \theta = 30^\circ$

Q) Find the velocity vector of a particle moving with 10 m/s in the direction of $\vec{r}$ $\quad \vec{r} = 2\hat{i} - 3\hat{j} + 4\hat{k}$



Ans)

$$\vec{v} = \underset{\downarrow\atop 10}{|v|}\ \underset{\downarrow\atop \hat{r}}{\hat{v}} = 10\,\hat{r}$$

$$= 10\left(\frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}\right)$$

$$= 10\left(\frac{2}{\sqrt{29}}\hat{i} - \frac{3}{\sqrt{29}}\hat{j} + \frac{4}{\sqrt{29}}\hat{k}\right)$$

$$\hat{r} = \frac{\vec{r}}{|r|} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{2^2 + 3^2 + 4^2}}$$

$$\hat{r} = \frac{2\hat{i} - 3\hat{j} + 4\hat{k}}{\sqrt{29}}$$

Q) Find a unit vector in the direction of

$$\hat{A} = \frac{\vec{A}}{|A|}$$

i) $\vec{A} = 3\hat{i} - 4\hat{j}$

$$|A| = \sqrt{3^2 + (-4)^2} = 5$$

$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{3\hat{i} - 4\hat{j}}{5}$$

$$\hat{A} = \frac{3}{5}\hat{i} - \frac{4}{5}\hat{j}$$

ii) $\vec{B} = 5\hat{i} + 12\hat{j}$

$$|B| = \sqrt{(5)^2 + (12)^2} = 13$$

$$\hat{B} = \frac{\vec{B}}{|B|} = \frac{5\hat{i} + 12\hat{j}}{13}$$

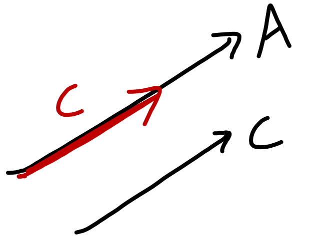
$$\hat{B} = \frac{5}{13}\hat{i} + \frac{12}{13}\hat{j}$$

Q6) If $\vec{A} = 3\hat{\imath} + 4\hat{\jmath}$ & $\vec{B} = 7\hat{\imath} + 24\,\hat{\jmath}$. Find a vector having the same magnitude as B and parallel to $\vec{A}$ (The reqd vector is in the direction of $\vec{A}$)

$$\vec{C} = |C|\,\hat{C} = 25\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = 5(3\hat{i}+4\hat{j})$$

$$= 15\hat{i}+20\hat{j}$$

$|C| \to |B|$, $\hat{C} \to \hat{A}$



$$|B| = \sqrt{7^2+24^2} = \sqrt{49+576} = \sqrt{625} = 25$$

$$\hat{A} = \frac{\vec{A}}{|A|} = \frac{3\hat{i}+4\hat{j}}{\sqrt{3^2+4^2}} = \frac{3\hat{i}+4\hat{j}}{5}$$

Thank You

Download DPP and lecture notes of this lecture right after this session from

www.physicswallahalakhpandey.com