ELECTROSTATICS

ELECTRIC CHARGE
Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of charge.

Types of charge:

(i) Positive charge: It is the deficiency of electrons as compared to proton.

(ii) Negative charge: It is the excess of electrons as compared to proton.

SI unit of charge: ampere second i.e. Coulomb Dimension: [A T]

Practical units of charge are ampere hour (=3600 C) and faraday (= 96500 C)

- Millikan calculated quanta of charge by 'Highest common factor' (H.C.F.) method and it is equal to charge of electron.
- \(1 \text{ C} = 3 \times 10^9 \text{ stat coulomb}, 1 \text{ absolute - coulomb} = 10 \text{ C}, 1 \text{ Faraday} = 96500 \text{ C.}\)

SPECIFIC PROPERTIES OF CHARGE

- **Charge is a scalar quantity**: It represents excess or deficiency of electrons.

- **Charge is transferable**: If a charged body is put in contact with another body, then charge can be transferred to another body.

- **Charge is always associated with mass**: Charge cannot exist without mass though mass can exist without charge.
  - So the presence of charge itself is a convincing proof of existence of mass.
  - In charging, the mass of a body changes.
  - When body is given positive charge, its mass decreases.
  - When body is given negative charge, its mass increases.

- **Charge is quantised**: The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e. Thus charge \(q\) of a body is always given by
  \[ q = ne \]
  \(n = \) positive integer or negative integer

The quantum of charge is the charge that an electron or proton carries.

**Note**: Charge on a proton = (-) charge on an electron = \(1.6 \times 10^{-19} \text{ C}\)

- **Charge is conserved**: In an isolated system, total charge does not change with time, though individual charge may change i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.

- **Charge is invariant**: Charge is independent of frame of reference, i.e. charge on a body does not change whatever be its speed.

**Accelerated charge radiates energy**

<table>
<thead>
<tr>
<th>(v = 0) (i.e. at rest)</th>
<th>(v = \text{constant})</th>
<th>(v = \text{constant (i.e. time varying)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\nabla\text{Q}] produces only (\vec{E}) (electric field)</td>
<td>[\nabla\text{Q}] produces both (\vec{E}) and (\vec{B}) (magnetic field) but no radiation</td>
<td>[\nabla\text{Q}] produces (\vec{E}), (\vec{B}) and radiates energy</td>
</tr>
</tbody>
</table>

- **Attraction – Repulsion**: Similar charges repel each other while dissimilar attract
METHODS OF CHARGING

- **Friction**: If we rub one body with other body, electrons are transferred from one body to the other.

Transfer of electrons takes places from lower work function body to higher work function body.

<table>
<thead>
<tr>
<th>Positive charge</th>
<th>Negative charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass rod</td>
<td>Silk cloth</td>
</tr>
<tr>
<td>Woolen cloth</td>
<td>Rubbers, Amber, Plastic objects</td>
</tr>
<tr>
<td>Dry hair</td>
<td>Comb</td>
</tr>
<tr>
<td>Flannel or cat skin</td>
<td>Ebonite rod</td>
</tr>
</tbody>
</table>

*Note: Clouds become charged by friction*

- **Electrostatic Induction**

If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'.

- Charging a body by induction (in four successive steps)

Some important facts associated with induction:

(i) Inducing body neither gains nor loses charge

(ii) The nature of induced charge is always opposite to that of inducing charge

(iii) Induction takes place only in bodies (either conducting or non-conducting) and not in particles.

- **Conduction**

The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

- The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)

- The charge gained by the uncharged body is always lesser than initial charge present on the charged body.

- Flow of charge depends upon the potential difference of both bodies. [No potential difference ⇒ No conduction].

- Positive charge flows from higher potential to lower potential, while negative charge flows from lower to higher potential.
GOLDEN KEY POINTS

- Charge differs from mass in the following sense.
  
  (i) In SI units, charge is a derived physical quantity while mass is fundamental quantity.
  
  (ii) Charge is always conserved but mass is not (Note : Mass can be converted into energy E=mc²)
  
  (iii) The quanta of charge is electronic charge while that of mass it is yet not clear.
  
  (iv) For a moving charged body mass increases while charge remains constant.

- True test of electrification is repulsion and not attraction as attraction may also take place between a charged and an uncharged body and also between two similarly charged bodies.

- For a non relativistic (i.e. v << c) charged particle, specific charge \( \frac{q}{m} \) =constant

- For a relativistic charged particle \( \frac{q}{m} \) decreases as v increases, where v is speed of charged body.

Example

When a piece of polythene is rubbed with wool, a charge of \(-2\times 10^{-7}\) C is developed on polythene. What is the amount of mass, which is transferred to polythene.

Solution

From \( Q = ne \), So, the number of electrons transferred \( n = \frac{Q}{e} = \frac{2\times 10^{-7}}{1.6\times 10^{-19}} = 1.25 \times 10^{12} \)

Now mass of transferred electrons = \( n \) mass of one electron = \( 1.25 \times 10^{12} \times 9.1 \times 10^{-31} = 11.38 \times 10^{-19} \) kg

Example

\( 10^{12} \alpha \) - particles (Nuclei of helium) per second falls on a neutral sphere, calculate time in which sphere gets charged by \( 2 \mu C \).

Solution

Number of \( \alpha \) - particles falling in t second = \( 10^{12} t \)

Charge on \( \alpha \) - particle = \(+2e\), So charge incident in time \( t = (10^{12} t)(2e) \)

\[ \therefore \text{Given charge is } 2 \mu C \quad \therefore 2 \times 10^{-6} = (10^{12} t)(2e) \quad \Rightarrow t = \frac{10^{-18}}{1.6 \times 10^{-19}} = 6.25 \text{s} \]

COULOMB'S LAW

The electrostatic force of interaction between two static point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

If two points charges \( q_1 \) and \( q_2 \) separated by a distance \( r \). Let \( F \) be the electrostatic force between these two charges. According to Coulomb's law,

\[ F \propto q_1 q_2 \text{ and } F \propto \frac{1}{r^2} \]

\[ F = \frac{kq_1 q_2}{r^2} \quad \text{where} \quad k = \frac{1}{4\pi \varepsilon_0} = 9 \times 10^9 \text{Nm}^2\text{C}^{-2} \]

\( k = \text{coulomb's constant or electrostatic force constant} \)
Coulomb's law in vector form

\[ \vec{F}_{12} = \text{force on } q_1 \text{ due to } q_2 = \frac{kq_1q_2}{r^2} \hat{r}_{12} \]

\[ \vec{F}_{21} = \frac{kq_1q_2}{r^2} \hat{r}_{12} \quad (\text{here } \hat{r}_{12} \text{ is unit vector from } q_1 \text{ to } q_2) \]

Coulomb’s law in terms of position vector

\[ \vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) \]

Principle of superposition

The force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to number of point charges is the resultant of forces due to individual point charges, i.e., \( \vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots \)

Note: Nuclear force is many body interaction, so principle of superposition is not valid in case of nuclear force.

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually

\[ F = \frac{kq_0q_1}{r_1^2} + \frac{kq_0q_2}{r_2^2} + \ldots + \frac{kq_0q_i}{r_i^2} + \ldots + \frac{kq_0q_n}{r_n^2} \quad \text{in vector form } \vec{F} = kq_0 \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i \]

**SOME IMPORTANT POINTS REGARDING COULOMB’S LAW AND ELECTRIC FORCE**

- The law is based on physical observations and is not logically derivable from any other concept. Experiments till today reveal its universal nature.

- The law is analogous to Newton’s law of gravitation: \( F = G \frac{m_1m_2}{r^2} \) with the difference that:
  
  (a) Electric force between charged particles is much stronger than gravitational force, i.e., \( F_e >> F_G \). This is why when both \( F_e \) and \( F_G \) are present, we neglect \( F_G \).
  
  (b) Electric force can be attractive or repulsive while gravitational force is always attractive.
  
  (c) Electric force depends on the nature of medium between the charges while gravitational force does not.
  
  (d) The force is an action-reaction pair, i.e., the force which one charge exerts on the other is equal and opposite to the force which the other charge exerts on the first.

- The force is conservative, i.e., work done in moving a point charge once round a closed path under the action of Coulomb’s force is zero.

- The net Coulomb’s force on two charged particles in free space and in a medium filled up to infinity are

  \[ F = \frac{1}{4\pi\varepsilon_0} \frac{q_1q_2}{r^2} \quad \text{and} \quad F' = \frac{1}{4\pi\varepsilon} \frac{q_1q_2}{r^2} \quad \text{So} \quad \frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0} = K, \]

- Dielectric constant (\( K \)) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium filled up to infinity.

- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its ‘centre’ as this is true only for spherical charged body, that too for external point.

*Although net electric force on both particles change in the presence of dielectric but force due to one charge particle on another charge particle does not depend on the medium between them.*

- Electric force between two charges does not depend on neighbouring charges.
Example

If the distance between two equal point charges is doubled and their individual charges are also doubled, what would happen to the force between them?

Solution

\[ F = \frac{1}{4\pi \varepsilon_0} \frac{q \times q}{r^2} \quad \text{...(i)} \]

Again, \( F' = \frac{1}{4\pi \varepsilon_0} \frac{(2q)(2q)}{(2r)^2} \) or \( F' = \frac{1}{4\pi \varepsilon_0} \frac{4q^2}{4r^2} = \frac{1}{4\pi \varepsilon_0} \frac{q^2}{r^2} = F \)

So, the force will remain the same.

Example

A particle of mass \( m \) carrying charge \( +q_1 \) is revolving around a fixed charge \( -q_2 \) in a circular path of radius \( r \). Calculate the period of revolution.

Solution

\[ \frac{1}{4\pi \varepsilon_0} \frac{q_1 q_2}{r^2} = m r \omega^2 = \frac{4\pi^2 m r}{r^2} \]

\[ T^2 = \frac{(4\pi \varepsilon_0 \rho r^2)(4\pi^2 m r)}{q_1 q_2} \]

or \( T = 4\pi \sqrt{\frac{4\pi \varepsilon_0 m r}{q_1 q_2}} \)

where \( \vec{r} \) is the vector drawn from source charge is test charge.

Example

The force of repulsion between two point charges is \( F \), when these are at a distance of 1 m. Now the point charges are replaced by spheres of radii 25 cm having the charge same as that of point charges. The distance between their centres is 1 m, then compare the force of repulsion in two cases.

Solution

In 2nd case due to mutual repulsion, the effective distance between their centre of charges will be increased (\( d' > d \)) so force of repulsion decreases as \( F \propto \frac{1}{d^2} \)

\[ \begin{array}{c}
\text{EQUILIBRIUM OF CHARGED PARTICLES} \\
\text{In equilibrium net electric force on every charged particle is zero. The equilibrium of a charged particle, under the action of Colombian forces alone can never be stable.}
\end{array} \]

\[ \text{Equilibrium of three point charges} \]

(i) Two charges must be of like nature as \( F = \frac{K Q_1 q}{x^2} + \frac{K Q_2 q}{(r-x)^2} = 0 \)

(ii) Third charge should be of unlike nature as \( F = \frac{K Q_1 Q_2}{r^2} + \frac{K Q_1 q}{x^2} = 0 \)

Therefore \( x = \frac{\sqrt{Q_1}}{\sqrt{Q_1} + \sqrt{Q_2}} r \) and \( q = \frac{-Q_1 Q_2}{(\sqrt{Q_1} + \sqrt{Q_2})^2} \)
Equilibrium of symmetric geometrical point charged system

Value of Q at centre for which system to be in state of equilibrium

(i) For equilateral triangle $Q = \frac{-q}{\sqrt{3}}$

(ii) For square $Q = \frac{-q(2\sqrt{2} + 1)}{4}$

Equilibrium of suspended point charge system

For equilibrium position

$T \cos \theta = mg$ and $T \sin \theta = F_e = \frac{kQ^2}{x^2}$ \Rightarrow $\tan \theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$

- If $\theta$ is small then tan

$\theta \approx \sin \theta = \frac{x}{2\ell} \Rightarrow \frac{x}{2\ell} = \frac{kQ^2}{x^2 mg} \Rightarrow x^3 = \frac{2kQ^2\ell}{mg} \Rightarrow x = \left[ \frac{Q^2\ell}{2\pi \epsilon_0 mg} \right]^{\frac{1}{3}}$

- If whole set up is taken into an artificial satellite ($g_{eff} = 0$) then $T = F_e = \frac{kq^2}{4\ell^2}$

Example

For the system shown in figure find Q for which resultant force on q is zero.

Solution

For force on q to be zero, charges q and Q must be of opposite of nature.

Net attraction force on q due to charges Q = Repulsion force on q due to q

$\sqrt{2} F_A = F_r \Rightarrow \sqrt{2} \frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \Rightarrow q = 2\sqrt{2} Q$ Hence $q = -2\sqrt{2} Q$

Example

Two identically charged spheres are suspended by strings of equal length. The strings make an angle of $30^0$ with each other. When suspended in a liquid of density 0.8 g/cc the angle remains same. What is the dielectric constant of liquid. Density of sphere = 1.6 g/cc.
Solution

When set up shown in figure is in air, we have \( \tan 15^\circ = \frac{F}{mg} \) When set up is immersed in the medium as shown in figure, the electric force experienced by the ball will reduce and will be equal to \( \frac{F}{\varepsilon_r} \) and the effective gravitational force will become \( mg \left( 1 - \frac{\rho_i}{\rho_s} \right) \). Thus we have \( \tan 15^\circ = \frac{F}{mg \varepsilon_r \left( 1 - \frac{\rho_i}{\rho_s} \right)} \), \( = \frac{F}{mg} \Rightarrow \varepsilon_r = \frac{1}{1 - \frac{\rho_i}{\rho_s}} \).

Example

Given a cube with point charges \( q \) on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.

Solution

The net force on particle \( A \) can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb’s law \( \vec{F} = \frac{kq_1q_2}{|\vec{r}_1 - \vec{r}_2|^3} (\vec{r}_1 - \vec{r}_2) \).

From the figure the different forces acting on \( A \) are given as \( \vec{F}_{A_1} = \frac{kq^2 (-a\hat{i} - a\hat{j})}{a^3} \),

\( \vec{F}_{A_2} = \frac{kq^2 (-a\hat{i} - a\hat{j} - a\hat{k})}{(\sqrt{2a})^3} \), \( \vec{F}_{A_3} = \frac{kq^2 (-a\hat{i} - a\hat{k})}{(\sqrt{3a})^3} \),

\( \vec{F}_{A_4} = \frac{kq^2 (-a\hat{i})}{a^3} \), \( \vec{F}_{A_5} = \frac{kq^2 (-a\hat{i} - a\hat{j})}{(\sqrt{2a})^3} \), \( \vec{F}_{A_6} = \frac{kq^2 (-a\hat{j})}{a^3} \),

\( \vec{F}_{A_7} = \frac{kq^2 (-a\hat{i} - a\hat{j} - a\hat{k})}{(\sqrt{3a})^3} \).

The net force experienced by \( A \) can be given as

\( \vec{F}_{net} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_3} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_6} + \vec{F}_{A_7} = \frac{-kq^2}{a^2} \left[ \left( \frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1 \right) (\hat{i} + \hat{j} + \hat{k}) \right] \).
Example

Five point charges, each of value \( +q \) are placed on five vertices of a regular hexagon of side \( L \) m. What is the magnitude of the force on a point charge of value \( -q \) coulomb placed at the centre of the hexagon?

Solution

If there had been a sixth charge \( +q \) at the remaining vertex of hexagon force due to all the six charges on \( -q \) at \( O \) will be zero (as the forces due to individual charges will balance each other).

Now if \( \vec{f} \) is the force due to sixth charge and \( \vec{F} \) due to remaining five charges.

\[
\vec{F} + \vec{f} = 0 \quad \Rightarrow \quad \vec{F} = -\vec{f} \quad \Rightarrow \quad F = f = \frac{1}{4\pi\varepsilon_0} \frac{q \times q}{L^2} = \frac{q^2}{4\pi\varepsilon_0 L^2}
\]

ELECTRIC FIELD

In order to explain ‘action at a distance’, i.e., ‘force without contact’ between charges it is assumed that a charge or charge distribution produces a field in space surrounding it. So the region surrounding a charge (or charge distribution) in which its electrical effects are perceptible is called the electric field of the given charge.

Electric field at a point is characterized either by a vector function of position \( \vec{E} \) called ‘electric intensity’ or by a scalar function of position \( V \) called ‘electric potential’. The electric field in a certain space is also visualized graphically in terms of ‘lines of force.’ So electric intensity, potential and lines of force are different ways of describing the same field.

Intensity of electric field due to point charge

Electric field intensity is defined as force on unit test charge.

\[
\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{ka}{r^2} \hat{r} = \frac{ka}{r^3} \hat{r}
\]

Note: Test charge \( (q_0) \) is a fictitious charge that exerts no force on nearby charges but experiences forces due to them.

Properties of electric field intensity:

(i) It is a vector quantity. Its direction is the same as the force experienced by positive charge.

(ii) Electric field due to positive charge is always away from it while due to negative charge always towards it.

(iii) Its unit is Newton/coulomb

(iv) Its dimensional formula is \([\text{ML}^{-1}\text{T}^{-3}\text{A}^{-1}]\)

(v) Force on a point charge is in the same direction of electric field on positive charge and in opposite direction on a negative charge.

\[
\vec{E} = q\vec{E}
\]

(vi) It obeys the superposition principle that is the field intensity point due to charge distribution is vector sum of the field intensities due to individual charge.
GOLDEN KEY POINTS

- Charged particle in an electric field always experiences a force either it is at rest or in motion.
- In presence of a dielectric, electric field decreases and becomes \( \frac{1}{\varepsilon_r} \) times of its value in free space.
- Test charge is always a unit (+ve) charge. \( \vec{E} = \frac{\vec{F}_{\text{test}}}{\text{test charge}} \)
- If identical charges are placed on each vertices of a regular polygon, then \( \vec{E} \) at centre = zero.

ELECTRIC FIELD INTENSITIES DUE TO VARIOUS CHARGE DISTRIBUTIONS

Due to discrete distribution of charge

Field produced by a charge distribution for discrete distribution:

By principle of superposition intensity of electric field due to \( i^{\text{th}} \) charge \( \vec{E}_i = \frac{kq_i}{r_i^3} \)

\[ \therefore \text{ Net electric field due to whole distribution of charge } \vec{E}_p = \sum_{i=1}^{n} \vec{E}_i \]

Continuous distribution of charge

Treating a small element as particle \( \vec{E} = \frac{1}{4\pi \varepsilon_0} \int \frac{dq}{r^3} \)

Due to linear charge distribution \( E = k \int \frac{\lambda \, dl}{r^2} \) \[ \lambda = \text{charge per unit length} \]

Due to surface charge distribution \( E = k \int \frac{\sigma \, ds}{r^2} \) \[ \sigma = \text{charge per unit area} \]

Due to volume charge distribution \( E = k \int \frac{\rho \, dv}{r^2} \) \[ \rho = \text{charge per unit volume} \]

Electric field strength at a general point due to a uniformly charged rod

As shown in figure, if \( P \) is any general point in the surrounding of rod, to find electric field strength at \( P \), we consider an element on rod of length \( dx \) at a distance \( x \) from point \( O \) as shown in figure. Now if \( dE \) be the electric field at \( P \) due to the element, then

\[ dE = \frac{kq}{(x^2 + r^2)^{3/2}} \]

Here \( dq = \frac{Q}{L} \, dx \)

Electric field strength in \( x \)-direction due to \( dq \) at \( P \) is

\[ dE_x = dE \sin \theta = \left[ \frac{kq}{(x^2 + r^2)^{3/2}} \right] \sin \theta = \frac{kQ \sin \theta}{L (x^2 + r^2)} \, dx \]

Here we have \( x = r \tan \theta \) and \( dx = r \sec^2 \theta \, d\theta \)

Thus \( dE_x = \frac{kQ}{L} \frac{r \sec^2 \theta \, d\theta}{r^2 \sec^2 \theta} \sin \theta \)

\[ \text{Strength} = \frac{kQ}{L} \sin \theta \, d\theta \]
Net electric field strength due to dq at point P in x-direction is

\[ E_x = \int dE_x = \frac{kQ}{Lr} \int_{-\theta_2}^{\theta_1} \sin \theta \, d\theta = \frac{kQ}{Lr} \left[ -\cos \theta \right]_{-\theta_2}^{\theta_1} = \frac{kQ}{Lr} \left[ \cos \theta_2 - \cos \theta_1 \right] \]

Similarly, electric field strength at point P due to dq in y-direction is

\[ dE_y = dE \cos \theta = \frac{kQ}{Lr} \frac{dQ}{r^2 + x^2} \cos \theta \]

Again we have \( x = r \tan \theta \) and \( dx = r \sec^2 \theta \, d\theta \). Thus we have

\[ dE_y = \frac{kQ}{L} \cos \theta \frac{r \sec^2 \theta}{r^2 \sec^2 \theta} \, d\theta = \frac{kQ}{L} \cos \theta \, d\theta \]

Net electric field strength at P due to dq in y-direction is

\[ E_y = \int dE_y = \frac{kQ}{Lr} \int_{-\theta_2}^{\theta_1} \cos \theta \, d\theta = \frac{kQ}{Lr} \left[ +\sin \theta \right]_{-\theta_2}^{\theta_1} = \frac{kQ}{Lr} \left[ \sin \theta_1 + \sin \theta_2 \right] \]

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles \( \theta_1 \) and \( \theta_2 \) at the two corners of rod can be given as

in x-direction: \( E_x = \frac{kQ}{Lr} \left( \cos \theta_2 - \cos \theta_1 \right) \) and in y-direction \( E_y = \frac{kQ}{Lr} \left( \sin \theta_1 - \sin \theta_2 \right) \)

Electric field due to a uniformly charged ring

**Case - I : At its centre**

![Diagram of a uniformly charged ring with AB and CD segments]

Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. The electric field strength at centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is \( E_0 = 0 \)

**Case II : At a point on the axis of Ring**

![Diagram showing the electric field at a point on the axis of a uniformly charged ring]
Here we’ll find the electric field strength at point $P$ due to the ring which is situated at a distance $x$ from the ring centre. For this we consider a small section of length $d\ell$ on ring as shown. The charge on this elemental section is $dq = \frac{Q}{2\pi R} \frac{d\ell}{l}$ [Q= total charge of ring]

Due to the element $d\ell$, electric field strength $dE$ at point $P$ can be given as $dE = \frac{Kdq}{(R^2 + x^2)^{3/2}}$

The component of this field strength $dE \sin \alpha$ which is normal to the axis of ring will be cancelled out due to the ring section opposite to $d\ell$. The component of electric field strength along the axis of ring $dE \cos \alpha$ due to all the sections will be added up. Hence total electric field strength at point $P$ due to the ring is

$$E_P = \int dE \cos \alpha = \int_0^{2\pi R} \frac{kQx}{2\pi R (R^2 + x^2)^{3/2}} \frac{x}{\sqrt{R^2 + x^2}} d\ell = \frac{kQx}{2\pi R (R^2 + x^2)^{3/2}}$$

Electric field strength due to a charged circular arc at its centre:

Figure shows a circular arc of radius $R$ which subtend an angle $\phi$ at its centre. To find electric field strength at $C$, we consider a polar segment on arc of angular width $d\theta$ at an angle $\theta$ from the angular bisector $XY$ as shown.

![Diagram](image)

The length of elemental segment is $Rd\theta$, the charge on this element $d\ell$ is $dq = \frac{Q}{\phi} d\theta$

Due to this $dq$, electric field at centre of arc $C$ is given as $dE = \frac{kdq}{R^2}$

Now electric field component due to this segment $dE \sin \theta$ which is perpendicular to the angular bisector gets cancelled out in integration and net electric field at centre will be along angular bisector which can be calculated by integrating $dE \cos \theta$ within limits from $-\frac{\phi}{2}$ to $\frac{\phi}{2}$. Hence net electric field strength at centre $C$ is $E_C = \int dE \cos \theta$

$$= \int_{-\phi/2}^{\phi/2} \frac{kQ}{\phi R^2} \cos \theta d\theta = \frac{kQ}{\phi R^2} \left[ \sin \theta \right]_{-\phi/2}^{\phi/2} = \frac{kQ}{\phi R^2} \left[ \sin \frac{\phi}{2} + \sin \frac{-\phi}{2} \right] = \frac{2kQ \sin \left(\frac{\phi}{2}\right)}{\phi R^2}$$
Electric field strength due to a uniformly surface charged disc:

If there is a disc of radius $R$, charged on its surface with surface charge density $\sigma$, we wish to find electric field strength due to this disc at a distance $x$ from the centre of disc on its axis at point $P$ shown in figure.

To find electric field at point $P$ due to this disc, we consider an elemental ring of radius $y$ and width $dy$ in the disc as shown in figure. The charge on this elemental ring is $dq = \sigma 2\pi ydy$ [Area of elemental ring $ds = 2\pi y dy$].

Now we know that electric field strength due to a ring of radius $R$, charge $Q$, at a distance $x$ from its centre on its axis can be given as

$$E = \frac{kQx}{\left(x^2 + R^2\right)^{3/2}}$$

Here due to the elemental ring electric field strength $dE$ at point $P$ can be given as

$$dE = \frac{k\sigma 2\pi y dy x}{\left(x^2 + y^2\right)^{3/2}} = \frac{k\sigma 2\pi y dy x}{\left(x^2 + y^2\right)^{3/2}}$$

Net electric field at point $P$ due to this disc is given by integrating above expression from $0$ to $R$ as

$$E = \int_0^R dE = \int_0^R \frac{k\sigma 2\pi y dy x}{\left(x^2 + y^2\right)^{3/2}} = k\sigma \pi x \int_0^R \frac{2y dy}{\left(x^2 + y^2\right)^{3/2}} = 2k\sigma \pi x \left[ \frac{1}{\sqrt{x^2 + y^2}} \right]_0^R = \frac{\sigma}{2e_0} \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

**Example**

Calculate the electric field at origin due to infinite number of charges as shown in figures below.

![Figures](image)

**Solution**

(a) $E_0 = kq \left[ \frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \cdots \right] = \frac{kq\cdot1}{(1-1/4)} = \frac{4kq}{3}$ \[ \therefore S_0 = \frac{a}{1-r} \quad a = 1 \text{ and } r = \frac{1}{4} \]

(b) $E_0 = kq \left[ \frac{1}{1} - \frac{1}{4} + \frac{1}{16} - \cdots \right] = \frac{kq\cdot1}{(1-(-1/4))} = \frac{4kq}{5}$

**Example**

A charged particle is kept in equilibrium in the electric field between the plates of millikan oil drop experiment. If the direction of the electric field between the plates is reversed, then calculate acceleration of the charged particle.

**Solution**

Let mass of the particle = $m$, Charge on particle = $q$

Intensity of electric field in between plates = $E$. Initially $mg = qE$

After reversing the field $ma = mg + qE \Rightarrow ma = 2mg$

\[ \therefore \text{Acceleration of particle } \Rightarrow a = 2g \]
Example

Calculate the electric field intensity \( E \) which would be just sufficient to balance the weight of an electron.
If this electric field is produced by a second electron located below the first one what would be the
distance between them? \([\text{Given: } e = 1.6 \times 10^{-19} \text{ C}, m = 9.1 \times 10^{-31} \text{ kg and } g = 9.8 \text{ m/s}^2]\)

Solution

As force on a charge \( e \) in an electric field \( E \)

\[ F_e = eE \]

So according to given problem

\[ F_e = W \quad \Rightarrow eE = mg \]

\[ \Rightarrow E = \frac{mg}{e} = \frac{9.1 \times 10^{-31} \times 9.8}{1.6 \times 10^{-19}} = 5.57 \times 10^{-11} \text{ V/m} \]

As this intensity \( E \) is produced by another electron \( B \), located at a distance \( r \) below \( A \).

\[ E = \frac{1}{4\pi\varepsilon_0} \frac{e}{r^2} \Rightarrow r = \sqrt[2]{\frac{e}{4\pi\varepsilon_0 E}} \quad \text{So,} \quad r = \left[ \frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{5.57 \times 10^{-11}} \right] = 5 \text{ m} \]

Example

A block having mass \( m = 4 \text{ kg} \) and charge \( q = 50 \mu\text{C} \) is connected to a spring having a force constant
\( k = 100 \text{ N/m} \). The block lies on a frictionless horizontal track and a uniform electric field
\( E = 5 \times 10^5 \text{ V/m} \) acts on the system as shown in figure. The block is released from rest when the spring
is unstretched (at \( x = 0 \))

(a) By what maximum amount does the spring expand?

(b) What is the equilibrium position of the block?

(c) Show that the block’s motion is simple harmonic and determine the amplitude and time period of the motion.

Solution

(a) As \( x \) increases, electric force \( qE \) will accelerate the block while elastic force in the spring \( kx \) will
oppose the motion. The block will move away from its initial position \( x = 0 \) till it comes to rest, i.e., work
done by the electric force is equal to the energy stored in the spring.

So if \( x_{\text{max}} \) is maximum stretch of the spring.

\[ \frac{1}{2}kx_{\text{max}}^2 = (qE)x_{\text{max}} = \frac{2qE}{k} \quad \Rightarrow x_{\text{max}} = \frac{2 \times (50 \times 10^{-6}) \times (5 \times 10^5)}{100} = 0.5 \text{ m} \]

(b) In equilibrium position \( F_r = 0 \), so if \( x_o \) is the stretch of the spring in equilibrium position

\[ kx_o = qE \quad \Rightarrow x_o = (qE/k) = \frac{1}{2}x_{\text{max}} = 0.25 \text{ m} \]

(c) If the displacement of the block from equilibrium position \( (x_o) \) is \( x \), restoring force will be

\[ F = k(x \pm x_o) = \pm qE = kx \quad [\text{as } kx_o = qE] \]

and as the restoring force is linear the motion will be simple harmonic with time period

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{4}{100}} = 0.4\pi \text{ s} \]

and amplitude \( = x_{\text{max}} - x_o = 0.5 - 0.25 = 0.25 \text{ m} \)
ELECTRIC LINES OF FORCE

Electric lines of electrostatic field have following properties

(i) Imaginary
(ii) Can never cross each other
(iii) Can never be closed loops
(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system \(1/e_o\) electric lines are associated with unit charge, so if a body encloses a charge \(q\), total lines of force associated with it (called flux) will be \(q/e_o\).
(v) Lines of force ends or starts normally at the surface of a conductor.
(vi) If there is no electric field there will be no lines of force.
(vii) Lines of force per unit area normal to the area at a point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field.
(viii) Tangent to the line of force at a point in an electric field gives the direction of intensity. So a positive charge free to move follow the line of force.

GOLDEN KEY POINTS

- Lines of force starts from (\(+ve\)) charge and ends on (\(-ve\)) charge.
- Lines of force start and end normally on the surface of a conductor.
- The lines of force never intersect each other due to superposition principle.
- The property that electric lines of force contract longitudinally leads to explain attraction between opposite charges.
- The property that electric lines of force exert lateral pressure on each other leads to explain repulsion between like charges.

Electric flux (\(\phi\))

The word "flux" comes from a Latin word meaning "to flow" and you can consider the flux of a vector field to be a measure of the flow through an imaginary fixed element of surface in the field.

Electric flux is defined as \(\phi = \int \vec{E} \cdot \, d\vec{A}\)

This surface integral indicates that the surface in question is to be divided into infinitesimal elements of area \(d\vec{A}\) and the scalar quantity \(\vec{E} \cdot \, d\vec{A}\) is to be evaluated for each element and summed over the entire surface.

Important points about electric flux:

(i) It is a scalar quantity
(ii) Units (V-m) and N - m²/C
(iii) The value of \(\phi\) does not depend upon the distribution of charges and the distance between them inside the closed surface.
Electric Flux through a circular Disc:

Figure shows a point charge $q$ placed at a distance $l$ from a disc of radius $R$. Here we wish to find the electric flux through the disc surface due to the point charge $q$. We know a point charge $q$ originates electric flux in radially outward direction. The flux is originated in cone shown in figure passes through the disc surface.

To calculate this flux, we consider on elemental ring an disc surface of radius $x$ and width $dx$ as shown. Area of this ring (strip) is $dS = 2\pi x \, dx$. The electric field due to $q$ at this elemental ring is given as $E = \frac{kq}{(x^2 + l^2)}$

If $d\phi$ is the flux passing through this elemental ring, then

$$d\phi = EdS \cos \theta = \frac{kq}{(x^2 + l^2)} \cdot \frac{l}{\sqrt{x^2 + l^2}} \cdot 2\pi x \, dx = \frac{2\pi kq \ell x \, dx}{(\ell^2 + x^2)^{3/2}}$$

$$\phi = \int d\phi = \frac{q \ell}{2 \varepsilon_0} \int_0^R \frac{x \, dx}{(\ell^2 + x^2)^{3/2}} = \frac{q \ell}{2 \varepsilon_0} \left[ -\frac{1}{\sqrt{\ell^2 + x^2}} \right]_0^R = \frac{q \ell}{2 \varepsilon_0} \left[ 1 - \frac{1}{\sqrt{\ell^2 + x^2}} \right]_0^R$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's law.

Electric flux through the lateral surface of a cylinder due to a point charge:

Figure shows a cylindrical surface of length $L$ and radius $R$. On its axis at its centre a point charge $q$ is placed. Here we wish to find the flux coming out from the lateral surface of this cylinder due to the point charge $q$. For this we consider an elemental strip of width $dx$ on the surface of cylinder as shown. The area of this strip is $dS = 2\pi R \, dx$.
The electric field due to the point charge on the strip can be given as \( E = \frac{kq}{(x^2 + R^2)^{1/2}} \). If \( d\phi \) is the electric flux through the strip, then \( d\phi = EdS \cos \theta = \frac{Kq}{(x^2 + R^2)^{1/2}} \cdot \frac{R}{\sqrt{x^2 + R^2}} \cdot 2\pi \cdot Rdx = 2\pi \cdot KqR^2 \cdot \frac{dx}{(x^2 + R^2)^{3/2}} \).

Total flux through the lateral surface of cylinder \( \phi = \int d\phi = \frac{qR^2}{2\varepsilon_0} \int_{-L/2}^{+L/2} \frac{dx}{(x^2 + R^2)^{3/2}} = \frac{q\varepsilon_0 \ell}{\sqrt{\ell^2 + 4R^2}} \).

This situation can also be easily handled by using the concepts of Gauss's law.

**GAUSS'S LAW**

It relates with the total flux of an electric field through a closed surface to the net charge enclosed by that surface and according to it, the total flux linked with a closed surface is \( (1/\varepsilon_0) \) times the charge enclosed by the closed surface i.e., \( \int_S \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0} \).

![Diagram of electric field](image)

**REGARDING GAUSS'S LAW IT IS WORTH NOTING THAT:**

**Note:**

(i) Flux through gaussian surface is independent of its shape.

(ii) Flux through gaussian surface depends only on charges present inside gaussian surface.

(iii) Flux through gaussian surface is independent of position of charges inside gaussian surface.

(iv) Electric field intensity at the gaussian surface is due to all the charges present (inside as well as outside).

(v) In a close surface incoming flux is taken negative while outgoing flux is taken positive.

(vi) In a gaussian surface \( \phi = 0 \) does not employ \( E = 0 \) but \( E = 0 \) employs \( \phi = 0 \).

(vii) Gauss's law and Coulomb's law are equivalent, i.e., if we assume Coulomb's law we can prove Gauss's law and vice-versa. To prove Gauss's law from Coulomb's law consider a hypothetical spherical surface [called Gaussian-surface] of radius \( r \) with point charge \( q \) at its centre as shown in figure. By Coulomb's law intensity at a point \( P \) on the surface will be, \( \vec{E} = \frac{1}{4\pi\varepsilon_0 r^2} \hat{r} \). And hence electric flux linked with area \( d\vec{s} \Rightarrow \vec{E} \cdot d\vec{s} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \cdot d\vec{s} \).

Here direction of \( \hat{r} \) and \( d\vec{s} \) are same, i.e., \( \oint_S \vec{E} \cdot d\vec{s} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \oint_S d\vec{s} = \frac{1}{4\pi\varepsilon_0} \frac{1}{r^2} \left(4\pi r^2\right) \oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0} \).

Which is the required result. Though here in proving it we have assumed the surface to be spherical, it is true for any arbitrary surface provided the surface is closed.
(viii) (a) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non-uniform) total flux linked with it will be zero.

(b) If a closed body encloses a charge \( q \), then total flux linked with the body will be \( \oint_S \vec{E} \cdot d\vec{s} = \frac{q}{\varepsilon_0} \).

From this expression it is clear that the flux linked with a closed body is independent of the shape and size of the body and position of charge inside it. [figure]

**Note**: So in case of closed symmetrical body with charge at its centre, flux linked with each half will be

\[
\frac{1}{2}(\Phi_E) = \left( \frac{q}{2\varepsilon_0} \right)
\]

and the symmetrical closed body has \( n \) identical faces with point charge at its centre, flux linked with each face will be

\[
\left( \frac{\Phi_E}{n} \right) = \left( \frac{q}{n\varepsilon_0} \right)
\]

(ix) Gauss's law is a powerful tool for calculating electric intensity in case of symmetrical charge distribution by choosing a Gaussian surface in such a way that \( \vec{E} \) is either parallel or perpendicular to its various faces. As an example, consider the case of a plane sheet of charge having charge density \( \sigma \). To calculate \( E \) at a point \( P \) close to it consider a Gaussian surface in the form of a 'pill box' of cross-section \( S \) as shown in figure.

The charge enclosed by the Gaussian-surface = \( \sigma S \) and the flux linked with the pill box = \( ES + 0 + ES = 2ES \) (as \( E \) is parallel to curved surface and perpendicular to plane faces)

So from Gauss's law, \( \Phi_E = \frac{1}{\varepsilon_0} (q) \), \( 2ES = \frac{1}{\varepsilon_0} (\sigma S) \) \( \Rightarrow E = \frac{\sigma}{2\varepsilon_0} \)
If $\vec{E} = 0$, $\phi = \oint \vec{E} \cdot d\vec{s} = 0$, so $q = 0$ but if $q = 0$, $\oint \vec{E} \cdot d\vec{s} = 0$. So $\vec{E}$ may or may not be zero.

If a dipole is enclosed by a closed surface then, $q = 0$, so $\oint \vec{E} \cdot d\vec{s} = 0$, but $\vec{E} \neq 0$

Note: If instead of plane sheet of charge, we have a charged conductor, then as shown in figure (B) $E_0 = 0$.

So $\phi_E = ES$ and hence in this case $E = \frac{\sigma}{\varepsilon_0}$. This result can be verified from the fact that intensity at the surface of a charged spherical conductor of radius $R$ is, $E = \frac{1}{4\pi \varepsilon_0} \frac{q}{R^2}$ with $q = 4\pi R^2 \sigma$

So for a point close to the surface of conductor, $E = \frac{1}{4\pi \varepsilon_0 R^2} \left(4\pi R^2 \sigma\right) = \frac{\sigma}{\varepsilon_0}$

Example

If a point charge $q$ is placed at the centre of a cube.

What is the flux linked (a) with the cube? (b) with each face of the cube?

Solution

(a) According to Gauss's law, flux linked with a closed body is $\left(\frac{1}{\varepsilon_0}\right)$ times the charge enclosed and hence the closed body cube is enclosing a charge $q$ so, $\phi_T = \frac{1}{\varepsilon_0}(q)$

(b) Now as cube is a symmetrical body with 6-faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_F = \frac{1}{6}(\phi_T) = \frac{q}{6\varepsilon_0}$

Note: (i) Here flux linked with cube or one of its faces is independent of the side of cube.

(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

Example

If a point charge $q$ is placed at one corner of a cube, what is the flux linked with the cube?

Solution

In this case by placing three cubes at three sides of given cube and four cubes above, the charge will be in the centre.

So, the flux linked with each cube will be one-eighth of the flux $\frac{q}{\varepsilon_0}$. :: Flux associated with given cube = $\frac{q}{8\varepsilon_0}$
### Flux Calculation Using Gauss Law

<table>
<thead>
<tr>
<th>Shape</th>
<th>Electric Field</th>
<th>Flux Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder</td>
<td>$E = \frac{kq}{R^2}$</td>
<td>( \phi = 2\pi R^2 \times \frac{q}{4\pi \varepsilon_0 R^2} = \frac{q}{2\varepsilon_0} )</td>
</tr>
<tr>
<td>Sphere</td>
<td>$E = \frac{kq}{R^2}$</td>
<td>( \phi_{\text{hemisphere}} = \frac{q}{2\varepsilon_0} )</td>
</tr>
<tr>
<td>Cube</td>
<td></td>
<td>( \phi_{\text{cube}} = \frac{q}{2\varepsilon_0} )</td>
</tr>
</tbody>
</table>

**Note:** Here, the electric field is radial.

- For a cylindrical shape, the flux is given by $\phi = 2\pi R^2 \times \frac{q}{4\pi \varepsilon_0 R^2} = \frac{q}{2\varepsilon_0}$.
- For a spherical shape, the flux through a hemisphere is $\phi_{\text{hemisphere}} = \frac{q}{2\varepsilon_0}$.
- For a cubic shape, the flux through a cube is $\phi_{\text{cube}} = \frac{q}{2\varepsilon_0}$.
Example

As shown in figure a closed surface intersects a spherical conductor. If a negative charge is placed at point P. What is the nature of the electric flux coming out of the closed surface?

Solution

Point charge Q induces charge on conductor as shown in figure.

Net charge enclosed by closed surface is negative so flux is negative.

Example

Consider $\vec{E} = 3 \times 10^3 \widehat{i}$ (N/C) then what is the flux through the square of 10 cm side, if the normal of its plane makes 60° angle with the X axis.

Solution

$\phi = E \cos 0 = 3 \times 10^3 \times [10 \times 10 \times 10^3]^2 \times \cos 60 = 3 \times 10^3 \times 10^{-2} \times \frac{1}{2} = 15 \text{ Nm}^2/\text{C}$

Example

Find the electric field due to an infinitely long cylindrical charge distribution of radius R and having linear charge density $\lambda$ at a distance half of the radius from its axis.

Solution

$r = \frac{R}{2}$ point will be inside so

$E = \frac{2 \kappa \lambda}{R^2} = \frac{2 \kappa \lambda}{2 R^2} = \frac{\lambda}{4 \pi \epsilon_0 R}$

ELECTRIC FIELD DUE TO SOLID CONDUCTING OR HOLLOW SPHERE

- For outside point ($r > R$)

Using Gauss's theorem $\oint \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\epsilon_0}$

:. At every point on the Gaussian surface $\vec{E} || d\vec{s}$ : $\vec{E} \cdot d\vec{s} = E \cdot ds \cos 0 = E \cdot ds$

$. \oint E \cdot ds = \frac{\Sigma q}{\epsilon_0}$ [E is constant over the gaussian surface] $\Rightarrow E \times 4 \pi r^2 = \frac{q}{\epsilon_0} \Rightarrow E = \frac{q}{4 \pi \epsilon_0 R^2}$

- For surface point $r = R$:

$E_S = \frac{q}{4 \pi \epsilon_0 R^2}$

- For Inside point ($r < R$): Because charge inside the conducting sphere or hollow is zero.

(i.e. $\Sigma q = 0$) So $\oint \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\epsilon_0} = 0 \Rightarrow E_S = 0$
ELECTRIC FIELD DUE TO SOLID NON CONDUCTING SPHERE

- **Outside** \((r > R)\)
  
  From Gauss’s theorem
  
  \[
  \oint \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\varepsilon_0} \Rightarrow E \times 4\pi r^2 = \frac{q}{\varepsilon_0} \Rightarrow E_r = \frac{q}{4\pi \varepsilon_0 r^2}
  \]

- **At surface** \((r = R)\)
  
  \[E_s = \frac{q}{4\pi \varepsilon_0 R^2}\]  
  Put \(r = R\)

- **Inside** \((r < R)\)

  From Gauss’s theorem
  
  \[
  \oint \vec{E} \cdot d\vec{s} = \frac{\Sigma q}{\varepsilon_0}
  \]

  Where \(\Sigma q\) charge contained within Gaussian surface of radius \(r\)

  \[E(4\pi r^2) = \frac{\Sigma q}{\varepsilon_0} \Rightarrow E = \frac{\Sigma q}{4\pi \varepsilon_0 r^2} \ldots (i)\]

  As the sphere is uniformly charged, the volume charge density (charge/volume) \(\rho\) is constant throughout the sphere

  \[\rho = \frac{q}{\frac{4}{3}\pi R^3} \Rightarrow \text{charge enclosed in gaussian surface } \Sigma q = \rho \left(\frac{4}{3}\pi r^3\right) = \left(\frac{q}{(4/3)\pi R^3}\right) \left(\frac{4}{3}\pi r^3\right) \Rightarrow \Sigma q = \frac{qr^3}{R^3}\]

  Put this value in equation \((i)\)

  \[E = \frac{1}{4\pi \varepsilon_0} \frac{qr}{R^3}\]

ELECTRIC FIELD DUE TO AN INFINITE LINE DISTRIBUTION OF CHARGE

Let a wire of infinite length is uniformly charged having a constant linear charge density \(\lambda\).

\(P\) is the point where electric field is to be calculated.

Let us draw a coaxial Gaussian cylindrical surfaces of length \(\ell\).

From Gauss’s theorem

\[
\int \vec{E} \cdot d\vec{s}_1 + \int \vec{E} \cdot d\vec{s}_2 + \int \vec{E} \cdot d\vec{s}_3 = \frac{q}{\varepsilon_0}
\]

\(\vec{E} \perp d\vec{s}_1\) so \(\vec{E} \cdot d\vec{s}_1 = 0\) and \(\vec{E} \perp d\vec{s}_2\) so \(\vec{E} \cdot d\vec{s}_2 = 0\)

\[E \times 2\pi r\ell = \frac{q}{\varepsilon_0} \quad \Rightarrow \quad \vec{E} \parallel d\vec{s}_3\]

Charge enclosed in the Gaussian surface

\[q = \lambda \ell\]

So

\[E \times 2\pi r\ell = \frac{\lambda \ell}{\varepsilon_0} \Rightarrow E = \frac{\lambda}{2\pi \varepsilon_0 r} \quad \text{or} \quad E = \frac{2k\lambda}{r} \quad \text{where} \quad k = \frac{1}{4\pi \varepsilon_0}\]

\((f)\) Charged cylindrical nonconductor of infinite length

Electric field at outside point

\[\vec{E}_A = \frac{2k\lambda}{r}\hat{r} \quad \text{for} \quad r > R\]

Electric field at inside point

\[\vec{E}_B = \frac{\lambda \hat{r}}{2\pi \varepsilon_0 R^2} \quad \text{for} \quad r < R\]
DIELECTRIC IN ELECTRIC FIELD

Let \( \vec{E}_0 \) be the applied field. Due to polarisation, electric field is \( \vec{E}_p \).

The resultant field is \( \vec{E} \). For homogeneous and isotropic dielectric, the direction of \( \vec{E}_p \) is opposite to the direction of \( \vec{E}_0 \).

So, Resultant field is \( \vec{E} = \vec{E}_0 - \vec{E}_p \).

**GOLDEN KEY POINTS**

- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero (\( E \neq 0 \) if non-zero charge is inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. \( x \gg R \).

**Example**

For infinite line distribution of charge draw the curve between \( \log E \) and \( \log r \).

**Solution**

\[
E = \frac{\lambda}{2\pi \varepsilon_0 r} = \frac{A}{r} \quad \text{where} \quad A = \frac{\lambda}{2\pi \varepsilon_0} = \text{constant}
\]

take log on both side \( \log E = \log A - \log r \)

**Example**

A point charge of 0.009 \( \mu C \) is placed at origin.

Calculate intensity of electric field due to this point charge at point \( \left( \sqrt{2}, \sqrt{7}, 0 \right) \).

**Solution**

\[
\vec{E} = \frac{q\vec{r}}{4\pi \varepsilon_0 r^3} ; \quad \text{where} \quad \vec{r} = x\hat{i} + y\hat{j} = \sqrt{2}\hat{i} + \sqrt{7}\hat{j}, \quad \vec{E} = \frac{9 \times 10^5 \times 9 \times 10^{-9}(\sqrt{2}\hat{i} + \sqrt{7}\hat{j})}{(3)^3} = \left(3\sqrt{2}\hat{i} + 3\sqrt{7}\hat{j}\right) \text{NC}^{-1}
\]

**ELECTROSTATIC POTENTIAL ENERGY**

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles. Electrostatic potential energy is defined in two ways.

(i) Interaction energy of charged particles of a system
(ii) Self energy of a charged object

- **Electrostatic Interaction Energy**

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to the given configuration. When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system, hence final potential energy of system will be positive. If the force between the particle is attractive, work will be done by the system and final potential energy of system will be negative.