Torque on a current loop in magnetic field.

\[ F = iBA \]

\[ \vec{F} = i \vec{B} \vec{a} \]

\[ \text{Net} = 0 \]

\[ \text{Net} = 0 \text{ (Torque)} \]

(since line of action of opposite forces are same)

(Note that torque = 0 when angle between magnetic field \( \vec{B} \) & magnetic moment \( \vec{M} \) is 0° here)

(Here \( \vec{M} \) is inward — using right hand curl rule)

If we rotate the loop by angle \( \theta \).

\[ \text{Net} = 0 \text{ (still)} \]

But

\[ \text{Net} \neq 0 \]

Horizontal forces have different line of action

\[ \vec{F} \]

(Note: Now angle between \( \vec{B} \) & \( \vec{M} \) is also 0°)

See next page
When loop's plane rotate by \( \theta \) angle, Magnetic Moment \( \mathbf{M} \) (towards loop's plane always) rotates \( \theta \) from initial direction

\[ \mathbf{M} \] is at \( \theta \) angle w.r.t \( \mathbf{B} \)

Consider Top View of Loop.

\[
\mathbf{F} = iB\mathbf{a}
\]

\[
F = iBA\sin \theta
\]

Torque = \[ F \times l \mathbf{d} + F \times l \mathbf{d} \]

\[
= \frac{iBA}{2} b\sin \theta + \frac{iBA}{2} b\sin \theta
\]

\[
= (iBA b\sin \theta) x z
\]

\[
= iB (ab) \sin \theta \quad (ab) = \text{Area of Loop} = A
\]

\[
= iAB \sin \theta
\]

\[
\mathbf{\mathbf{M}} = MB \sin \theta
\]

\[
\mathbf{\mathbf{M}} = iA
\]
\[ \vec{\tau} = \vec{M} \times \vec{B} \quad \Rightarrow \quad \vec{\tau} = MB \sin \theta \]

See that \( \vec{M} \times \vec{B} \) is in direction of Torque whereas \( \vec{B} \times \vec{M} \) isn't.

**Note:** Current loop behaves as Magnetic Dipole for electric dipole in Electric Field

\[ \vec{\tau} = \vec{P} \times \vec{E} \]

Electric dipole Moment \( \vec{P} \)
Electric Field \( \vec{E} \)

Similarly for Magnetic Dipole (currents) in Magnetic Field

\[ \vec{\tau} = \vec{M} \times \vec{B} \quad \Rightarrow \quad \vec{\tau} = MB \sin \theta \]

\( \tau \) is maximum when \( \sin \theta \to \) max \( \theta = 90^\circ \)

\[ \frac{\vec{M}}{\vec{B}} \]

\( \tau \) is zero when \( \sin \theta \to \) min \( \theta = 0^\circ \) or \( 180^\circ \)

\( \vec{M} \) is along \( \vec{B} \) or \( \vec{M} \) is opposite to \( \vec{B} \)

\( \theta = 0^\circ \) \hspace{1cm} \text{Equilibrium} \hspace{1cm} \theta = 180^\circ \)

\[ \text{Net} = 0 \quad \text{Net} = 0 \]

\( \downarrow \) \hspace{1cm} \text{Stable} \hspace{1cm} \text{Unstable}

\text{Equilibrium} \hspace{1cm} \text{Equilibrium}
there are $N$ turns in loop

$\mathbf{v} = N \mathbf{A} \mathbf{B} \sin \theta$

$\mathbf{v} = \mathbf{M} \mathbf{B} \sin \theta$

$\mathbf{v} = \mathbf{M} \times \mathbf{B}$

$M' = N \mathbf{A}$