Ray Optics - 07

Magnification

Linear/Transverse Magnification: \( m \)

Lateral

\[
m = \frac{\text{Height of Image}}{\text{Height of Object}} = \frac{h_i}{h_o}
\]

\[
m = \frac{h_i}{h_o} \rightarrow \text{with sign}
\]

* Height above principal axis is +ve
  " below "  "  is -ve

\[
m = \frac{h_i}{h_o} = -\frac{v}{u}
\]

Proof: for \( m = -\frac{v}{u} \)

\[
\frac{A'B'}{AB} = \frac{PB'}{PB}
\]

\[
-\frac{h_i}{h_o} = -\frac{v}{u}
\]

\[
\frac{h_i}{h_o} = -\frac{v}{u}
\]

\[
\Rightarrow m = -\frac{v}{u}
\]
Note:

1. \( m = +ve \) \( \Rightarrow \) Upright image \( \Rightarrow \) Virtual image
   \( m = -ve \) \( \Rightarrow \) Inverted image \( \Rightarrow \) Real image

2. \(|m| > 1\) \( |\theta_{i} > |\theta_{o}\) Magnified image
   \(|m| < 1\) \( |\theta_{i} < |\theta_{o}\) Diminished image
(1) Find the position, size & characteristics of image formed by a concave mirror of focal length 10cm, when an object 2cm long is kept in front of mirror at a distance of

a) 15cm  

b) 5cm  

c) 30cm

\[ \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \]

\[ V = \frac{uf}{u-f} = \frac{-15 \times -10}{-15+10} = \frac{150}{-5} = -30cm \]

image at 30cm in front of mirror

\[ m = -\frac{v}{u} = -\left(\frac{-30}{15}\right) = -2 \]

\[ m = -2 \rightarrow |hi| = 2|h_o| \]

\[ m = \frac{hi}{ho} \quad \text{hi & ho have opp. sign} \]

\[ -2 = \frac{hi}{2} \]

\[ hi = -4cm \quad \text{size of image is} \ 4cm. \]
\[ \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \]

\Rightarrow \quad \frac{v}{u} = \frac{mf}{u-f} = \frac{-5x-10}{-5+10} \quad \frac{v}{u} = \frac{50}{5} = 10\text{cm}

Image formed at 10 cm inside mirror.

\[ m = -\frac{v}{u} = -\frac{10}{-5} = +2 \]

\[ m = +2 \rightarrow |\text{hi}| = 2|\text{ho}| \quad \text{Magnified.} \]

\( \text{hi} \& \text{ho} \) are of same sign, upright & virtual image.

\[ m = \frac{\text{hi}}{\text{ho}} \]

\[ +2 = \frac{\text{hi}}{2} \]

\[ \text{hi} = +4\text{cm} \]
A concave mirror has \( R = 24 \text{cm} \). How far is an object from the mirror if an image is formed that is

a) virtual & 3 times the size of the object
b) real & 3 times the size of the object
c) real & \( \frac{1}{3} \) the size of the object.

a) \( f = \frac{R}{2} = -12 \text{cm} \) as \( R = -24 \) concave mirror

virtual & upright image \( m = +3 \)

\[-\frac{1}{u} = +3 \]

\[v = -3u\]

\[\frac{1}{f} = \frac{1}{v} + \frac{1}{u}\]

\[-\frac{1}{12} = -\frac{1}{3u} + \frac{1}{u} \Rightarrow -\frac{1}{12} = \frac{1}{u} \left(1 - \frac{1}{3}\right)\]

\[-\frac{1}{12} = \frac{1}{u} \times \frac{2}{3}\]

\[-\frac{1}{12} = u \times \frac{2}{3}\]

\[u = -8 \text{cm}\]

Object is placed 8 cm in front of mirror.
b) Real & Inverted \[ m = -3 \]
\[ \frac{-v}{u} = -3 \]
\[ v = 3u \]

\[ \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \]
\[ -\frac{1}{12} = \frac{1}{3u} + \frac{1}{u} \Rightarrow -\frac{1}{12} = \frac{1}{u} \left( \frac{1}{3} + 1 \right) \]

\[ u = -12 \times \frac{4}{3} = -16 \text{ cm} \]

16 cm in front of mirror

c) Real & Inverted \[ m = -\frac{1}{3} \]

\[ \frac{-v}{u} = -\frac{1}{3} \]

\[ \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \]
\[ v = \frac{u}{3} \]

\[ -\frac{1}{12} = \frac{3}{u} + \frac{1}{u} \Rightarrow -\frac{1}{12} = \frac{4}{u} \]

\[ u = -48 \text{ cm} \]

48 cm in front of mirror
Find the distance of object from a concave mirror of focal length 10 cm so that image size is four times the size of the object.

Solution \[ \frac{1}{f} \cdot 4 = \frac{1}{h_0} \Rightarrow |m| = 4 \] Magnified image

Concave can form

i) Real, Inverted Magnified image

ii) Virtual, Upright Magnified image

i) Real Inverted \[ m = -\frac{4}{y} \]
\[ \frac{-y}{u} = -4 \]
\[ v = 4u \]
\[ f = \frac{1}{v} + \frac{1}{u} \]
\[ \frac{-10}{4} = \frac{1}{u} + \frac{1}{u} \Rightarrow \frac{-10}{4} = \frac{1}{2u} (\frac{1}{4} + 1) \]
\[ u = -10 \times \frac{5}{4} = -\frac{50}{4} = -12.5 \text{ cm} \quad \text{(Object between C & F)} \]

ii) Virtual upright \[ m = +\frac{4}{y} \]
\[ \frac{-v}{u} = 4 \Rightarrow v = -4u \]
\[ f = \frac{1}{v} + \frac{1}{u} \]
\[ \frac{-10}{4} = -\frac{4}{u} + \frac{1}{u} \Rightarrow u = -7.5 \text{ cm} \quad \text{(Object between F & P)} \]
2) A spherical mirror of focal length 10 cm forms an upright virtual image of half the size of object. Find nature of mirror & object distance: \( f = +10 \text{ cm} \)

Solution

Note: Virtual, upright & diminished image is formed by convex mirror.

(i) Convex mirror always form virtual, upright & diminished image.

\[ m = + \frac{1}{2} \]

\[ \frac{-v}{u} = + \frac{1}{2} \quad \Rightarrow \quad v = -\frac{u}{2} \]

\[ \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \]

\[ \frac{1}{10} = -\frac{2}{u} + \frac{1}{u} \quad \Rightarrow \quad \frac{1}{10} = -\frac{1}{u} \]

\[ u = -10 \text{ cm} \quad ; \quad v = -\frac{u}{2} = -5 \text{ cm} \]

Object is at 10 cm from mirror.