Photo Electric Effect

Photon theory

According to Planck's quantum theory, light consists of photons as energy packets having following properties:

(i) Each photon is of energy \( E = h\nu = \frac{hc}{\lambda} \).
   Where \( h \) is Planck's constant. Where \( h = 6.63 \times 10^{-34} \text{ J-sec} = 4.14 \times 10^{-15} \text{ eV -sec} \)
(ii) All photons travel in straight line with the speed of light in vacuum.
(iii) Photons are electrically netural.
(iv) Photons have zero rest mass.
(v) Photons are not deflected by electric and magnetic fields.
(vi) The equivalent mass of a photon while moving is given by

\[
m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}
\]

(vii) Momentum of the photon

\[
p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}.
\]

(viii) Number of photons of wavelength \( \lambda \) emitted in t second from a lamp of power P is.

\[
n = \frac{Pt\lambda}{hc}
\]

Illustration:

Violet light (\( \lambda = 4000 \text{ A} \)) of intensity 4 watt/m² falls normally on a surface of area 10 cm \( \times \) 20 cm.
Find
(a) the energy received by the surface per second.
(b) the number of photons hitting the surface per second.
(c) If surface is tilted such that plane of the surface makes an angle 30° with light beam, find the number of photons hitting the surface per second.

Sol. (a) Energy received per second per unit area
\[
E = I A \cos \theta = 4 \times 0.02 \text{ J} \times \cos 0° = 0.08 \text{ J}.
\]

(b) \( n h (c/\lambda) = E \)

\[
\Rightarrow n = \frac{0.08 \times 4000 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^8}
\]

\[
= \frac{32 \times 10^{17}}{19.89} = 1.609 \times 10^{17}
\]

(c) \( n = \frac{I A \cos 60° \times \lambda}{hc} \)

\[
= \frac{1}{2} \times \frac{32 \times 10^{17}}{19.89} = 0.805 \times 10^{17}.
\]
Practice Exercise

Q.1 Visible light has wavelengths in the range of 400 nm to 780 nm. Calculate the range of energy of the photons of visible light.

Q.2 Calculate the number of photons emitted per second by a 10 W sodium vapour lamp. Assume that 60% of the consumed energy is converted into light. Wavelength of sodium light = 590 nm.

Answers

Q.1 $2.56 \times 10^{-19}$ J to $5.00 \times 10^{-19}$ J

Q.2 $1.77 \times 10^{19}$

Radiation Pressure

The electromagnetic wave transports not only energy but also momentum, and hence can exert a radiation pressure on a surface due to the absorption and reflection of the momentum.

Illustration:

A photon of wavelength 6630 Å is incident on a totally reflecting surface. Find the momentum delivered by the photon.

Sol. The momentum of the incident radiation is given as $P = \frac{h}{\lambda}$

When the light is totally reflected normal to the surface the direction of the ray is reversed. That means it reverses the direction of its momentum without changing its magnitude.

$\Rightarrow$ Change in momentum has a magnitude

$\Delta P = 2P = \frac{2h}{\lambda}$

$\Rightarrow \Delta P = \frac{2(2.63 \times 10^{-14} \text{ J - sec})}{(6630 \times 10^{-14} \text{ m})} = 2 \times 10^{-7} \text{ kgm/s}$

Illustration:

A parallel beam of monochromatic light of wavelength $\lambda$ is incident normally on a surface. The intensity of the beam is $I$. Find the force exerted by the light beam on the surface if surface is (i) perfectly reflecting (ii) perfectly absorbing

Sol. Energy incident per unit time = $IA$

Momentum incident per unit time = $\frac{IA}{c}$
(i) momentum transported to the wall per unit time = \( \frac{2IA}{c} \)

\[ \therefore F = \frac{2IA}{c} \]

\[ \therefore \text{pressure} = \frac{2I}{c} \]

(ii) momentum transported to the wall per unit time = \( \frac{IA}{c} \)

\[ \therefore F = \frac{IA}{c} \]

\[ \therefore \text{pressure} = \frac{I}{c} \]

---

**Practice Exercise**

Q.1 A 100 W light bulb is placed at the centre of a spherical chamber of radius 20 cm. Assume that 60% of the energy supplied to the bulb is converted into light and that the surface of the chamber is perfectly absorbing. Find the pressure exerted by the light on the surface of the chamber.

Q.2 A totally reflecting, small plane mirror placed horizontally faces a parallel beam of light as shown in the figure. The mass of the mirror is 20 g. Assume that there is no absorption in the lens and that 30% of the light emitted by the source goes through the lens. Find the power of the source needed to support the weight of the mirror. Take \( g = 10 \text{ m/s}^2 \).

---

**Answers**

Q.1 \( 4.0 \times 10^{-7} \text{ Pa} \)  
Q.2 100 MW
Photoelectric Effect

The photoelectric effect is a process where electrons are ejected from a surface by the action of light (electromagnetic radiation). The process was discovered by Heinrich Hertz in 1887. Attempts to explain the effect by classical electromagnetic theories failed. In 1905, Albert Einstein presented an explanation based on the quantum concept of Max Planck.

Observation of the experiments on Photo-Electric Effect:
(i) The emission of photoelectrons is instantaneous.
(ii) The number of photoelectrons emitted per second is proportional to the intensity of the incident light.
(iii) The maximum velocity with which electrons emerge is dependent only on the frequency and not on the intensity of the incident light.
(iv) There is always a lower limit of frequency called threshold frequency below which no emission takes place, however high the intensity of the incident radiation may be.

Explanation

Einstein suggested that when a light beam is incident on a metal surface, the free electrons of the metal absorb the entire energy of an incident photon during its collision with it. If this electron gets sufficient energy in this manner to do work against the surface adhesion of the given metallic surface and escape then it leaves the metal and a photoelectron is found.

For an electron to escape from a metallic surface by doing work against its attractive force and get out of the force field of the metallic surface, a minimum amount of energy is required to be supplied to electrons. This minimum energy required for an electron to escape from a metallic surface is called the work function of the given metal which is characteristic of the material and is hence different for different metals. Work function of a given metal is generally represented by the symbol $\phi$.

The minimum frequency of light corresponding to which the energy of a photon is equal to the work function of a given metal is called the Threshold frequency of that metal and the corresponding wavelength is called Threshold wavelength.

$$h\nu_0 = \phi \text{ or } \nu_0 = \frac{\phi}{h}$$
Where $\nu_0$ is called threshold frequency.

So, \[ \frac{hc}{\lambda_0} = \phi \quad \text{or} \quad \lambda_0 = \frac{hc}{\phi} \]

Where $\lambda_0$ is called threshold wavelength.

Clearly, when a light beam of frequency less than $\nu_0$ or wavelength greater than $\lambda_0$ is incident then no photoelectrons can be emitted, no matter how high is the intensity of the incident beam.

Suppose, a photon transfers energy more than the work function of the given metal then the photoelectron may be ejected with a kinetic energy.

\[ K_{\text{max}} = (h\nu - \phi) \]

or less than that because part or all of the extra energy may be lost during several collisions that the electron makes before emission.

If the frequency of the photon is $\nu$ and threshold frequency for the metal is $\nu_0$, then

\[ K_{\text{max}} = h (\nu - \nu_0) \]

If the wavelength of the photon is $\lambda$ and threshold wavelength for the metal is $\lambda_0$, then

\[ K_{\text{max}} = hc \left( \frac{1}{\lambda_0} - \frac{1}{\lambda} \right) \]

**Stopping Potential**

If the polarity of the battery is reversed and the applied potential is gradually increased, the photocurrent starts decreasing. This is because the electrons are retarded, and most of the electrons are unable to reach the opposite electrode. It is observed that when the applied retarding potential is increased, the photocurrent eventually becomes zero. This potential is known as the stopping potential and depends only on the material of the photocathode and the frequency of light.

If $V_s$ be the stopping potential, then

\[ eV_s = h\nu - \phi \]

The stopping potential $V_s$ depends only on the metal and does not depend on the intensity of incident light. $a$, $b$, $c$ - different intensities.
Illustration:

When a metallic surface is illuminated with monochromatic light of wavelength $\lambda$ the stopping potential for photoelectric current is $3V_0$. When the same metallic surface is illuminated with a light of wavelength $2\lambda$, the stopping potential is $V_0$. Find the threshold wavelength for the surface.

**Sol.** Einstein's photoelectric equation:

$$\frac{hc}{\lambda} = e(3V_0) + W \quad \ldots(i)$$

Here $W =$ work function

and $$\frac{hc}{2\lambda} = e(V_0) + W \quad \ldots(ii)$$

Solving these equations

$$\Rightarrow \frac{hc}{\lambda} = \frac{3hc}{2\lambda} + W - 3W \Rightarrow 2W = \frac{hc}{2\lambda}$$

$$\Rightarrow \frac{hc}{\lambda_0} = \frac{hc}{4\lambda}$$

$$\Rightarrow \lambda_0 = 4\lambda \quad \text{When } \lambda_0 = \text{threshold wavelength}$$

---

**Practice Exercise**

**Q.1** The work function of a metal is $2.5 \times 10^{-10}$ J. (a) Find the threshold frequency for photoelectric emission. (b) If the metal is exposed to a light beam of frequency $6.0 \times 10^{14}$Hz, what will be the stopping potential?

**Q.2** The electric field associated with a light wave is given by

$$E = E_0 \sin \left((1.57 \times 10^9 \text{ m}^{-1}) (x - ct) \right)$$

Find the stopping potential when this light is used in an experiment on photoelectric effect with the emitter having work function $1.9$ eV.

**Q.3** A monochromatic light source of intensity $5$ mW emits $8 \times 10^{15}$ photons per second. This light ejected photoelectrons from a metal surface. The stopping potential for this setup is $2.0$ V. Calculate the work function of the metal.

**Q.4** A photographic film is coated with a silver bromide layer. When light falls on this films, silver bromide molecules dissociate and the field records the light there. A minimum of $0.6$ eV is needed to dissociate a silver bromide molecule. Find the maximum wavelength of light that can be recorded by the film.

**Q.5** A small metal plate (work function $\phi$) is kept at a distance $d$ from a singly ionized, fixed ion. A monochromatic light beam is incident on the metal plate and photoelectrons are emitted. Find the maximum wavelength of the light beam so that some of the photoelectrons may go round the ion along a circle.
Wave Particle Duality

Electromagnetic radiation is an emission with a dual nature, i.e., it has both wave and particle aspects. In particular, the energy conveyed by an electromagnetic wave is always carried in packets whose magnitude is proportional to frequency of the wave. These packets of energy are called photons.

Energy of photon is \( E = hf \), where \( h \) is Planck's constant, and \( f \) is frequency of wave.

**de-Broglie idea**

As wave behaves like material particles, similarly matter also behaves like waves. According to him, a wavelength of the matter wave associated with a particle is given by \( \lambda = \frac{h}{p} = \frac{h}{mv} \), where \( m \) is the mass and \( v \) is velocity of the particle.

If an electron is accelerated through a potential difference of \( V \) volt,

\[
\frac{1}{2} m_e v^2 = eV \quad \text{or} \quad V = \sqrt{\frac{2eV}{m_e}}
\]

\[
\therefore \quad \lambda = \frac{h}{m_e v} = \frac{h}{\sqrt{2eV} m_e}
\]

(It is assumed that the voltage \( V \) is not more than several tens of Kilovolt)

**Illustration:**

Find the ratio of de-Broglie wavelength of molecules of hydrogen and helium in two gas jars kept separately at temperature 27°C and 127°C respectively.

**Sol.**

de-Broglie wavelength \( \lambda = \frac{h}{mv} \)

where the speed (r.m.s) of a gas particle at the given temperature \( (T) \) is given as

\[
\frac{1}{2} m v^2 = \frac{3}{2} KT
\]

\[
\Rightarrow v = \sqrt{\frac{3KT}{m}}, \text{ where } K = \text{Boltzmann's constant, } m = \text{mass of the gas particle and } T = \text{temperature of gas in K}
\]

\[
\Rightarrow mv = \sqrt{3mKT}
\]

\[
\Rightarrow \lambda = \frac{h}{mv} = \frac{h}{\sqrt{3mKT}} \quad \therefore \quad \lambda_{\text{H}} = \frac{m_{\text{H}} T_{\text{H}}}{m_{\text{He}} T_{\text{He}}}
\]

\[
= \frac{(4\text{amu})(273 + 127)}{(2\text{amu})(273 + 27)} \cdot \left( \frac{8}{3} \right) = \frac{8}{3}
\]
Illustration :

If the stationery proton and $\alpha$-particle are accelerated through same potential difference. Find the ratio of de-Broglie wavelength.

Sol. The gain in K.E. of a change particle after moving through a potential difference of $V$ is given as $qV$, that is also equal to $\frac{1}{2}mv^2$, where $v$ is the velocity of the charged particle.

$$\frac{1}{2}mv^2 = qV \quad \Rightarrow \quad v = \sqrt{\frac{2qV}{m}}$$

$\therefore \quad mv = \sqrt{2mqV}$

$\Rightarrow$ de-Broglie wavelength $= \lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mqV}}$

$\therefore \quad \frac{\lambda_p}{\lambda_a} = \sqrt{\frac{m_a q_a V_a}{m_p q_p V_p}}$

Putting $V_a = V_p \cdot \frac{\lambda_p}{\lambda_a} = \sqrt{\frac{(4)(2)}{(1)(1)}} = 2\sqrt{2}$