Bohr Atomic Model - Part I

1911, Rutherford gave Nuclear Atomic Model

\[ \text{e}^- \text{ is a charged particle} \]
\[ \text{e}^- \text{ is accelerated} (\text{direction of velocity changes}) \]
So an e\(^-\) must emit radiation & loose energy. Such an e\(^-\) will be pulled towards Nucleus & collide with Nucleus in \(10^{-8}\) s & atom won't exist.

Drawbacks:

1. He cannot explain stability of atom

According to classical Electromagnetism, an accelerated charged particle emits radiation (energy)

2. Discrete Atomic Spectra cannot be explained.

If a gas (atoms) is excited by passing an electric current through it, it emits radiation having certain specific wavelengths. (Discrete emission spectra)

If white light passes through a gas, the transmitted light have some missing wavelengths in its spectrum. (Discrete absorption spectrum)
Bohr accepted Rutherford's model & studied in his laboratory for 1 year.

In 1913, Bohr gave his own Atomic model with some modifications in Rutherford's Model.

Bohr's Postulates:

1. An e⁻ in an atom can revolve only in certain stable orbits with fixed energy. These orbits are called Energy Level or Stationary (constant energy) States. These were named as $K, L, M, N, \ldots$ Energy levels

2. While revolving in a particular Energy level, an e⁻ do not radiate any radiations.
   (Classical electromagnetism & Classical Mechanics cannot be applied on atomic level in all aspects)

   While revolving in a stationary energy state, an e⁻ has the same energy as that of the state. An e⁻ neither loses nor gains energy in a particular energy level. If it does so, it moves to another energy level.

3. The Energy states are defined as →

   e⁻ can revolve only in those orbits for which the orbital Angular Momentum is integral multiple of $\frac{\hbar}{2\pi}$ (Bohr Quantisation Condition)

   $\hbar = \text{Planck's Constant} = \frac{\hbar}{2\pi}, \frac{2\hbar}{2\pi}, \frac{3\hbar}{2\pi} \ldots = \frac{\pi n \hbar}{2\pi}$

   $\approx 6.626 \times 10^{-34} \text{ Js}$
Angular Momentum of $e^-$ in

\( L = \frac{nh}{2\pi} \)

i) 1st energy level \( (1) \Rightarrow L = \frac{h}{2\pi} \)

ii) 2nd " " \( (L) \Rightarrow L = \frac{2h}{2\pi} \)

iii) 3rd " " \( (M) \Rightarrow L = \frac{3h}{2\pi} \) & so on.

*** Angular Momentum \( \vec{L} = \vec{r} \times \vec{p} = |\vec{r}| m v \sin \theta \)

\[ \Rightarrow \angle \text{between } \vec{r} \text{ & } \vec{v} \]

\[ L = \vec{r} \times mv \sin 90^\circ = mv \]

\[ \Rightarrow \left\{ \begin{array}{l}
\frac{mv \tau}{2\pi} = \frac{nh}{2\pi} \\
(\tau)
\end{array} \right. \]

(4) From Rutherford's Model:

An $e^-$ moves in a circle, so it needs centripetal force. This centripetal force is provided by the electromagnetic force between $e^-$ & nucleus.
\( Fe = \frac{Kq_1 q_2}{\gamma^2} = \frac{K(e)(Ze)}{\gamma^2} \)

Nuclear Charge \( (z) = \) Atomic Number

\( K = \frac{1}{4\pi\varepsilon_0} \)constant

\( F_e = \frac{mv^2}{\gamma} \)

\( F_e = F_c \)

\( \frac{mv^2}{\gamma} = \frac{KZe^2}{\gamma^2} \) (ii)

(i)^2 \div (ii)

\( \frac{m^2 v^2 \gamma^2}{mv^2} = \frac{\eta^2 h^2}{4\pi^2} \)

\( \frac{KZe^2}{\gamma^2} \)

\( m \gamma^3 = \frac{\eta^2 h^2}{4\pi^2} \frac{1}{4\pi\varepsilon_0 \gamma^2} ZX e^2 \)

\( m \gamma^3 = \frac{\eta^2 h^2 \eta\varepsilon_0 \gamma^2}{4\pi^2 Ze^2} \)

\( \gamma = \frac{\eta^2 h^2 \varepsilon_0}{11 mZe^2} \) radius of \( n^{th} \) energy level
\[ \gamma = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2} \]

for \( h = 6.626 \times 10^{-34} \text{ Js} \)
\( m = 9.1 \times 10^{-31} \text{ kg} \)
\( \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \)
\( \pi = 3.142 \)
\( e = 1.6 \times 10^{-19} \text{ C} \)

\[ \gamma = 0.529 \frac{x n^2}{Z} \]
\( n \rightarrow \text{shell number} \)
\( \text{energy level} \)
\( Z \rightarrow \text{atomic number} \)

for H-atom \( Z = 1 \)

\[ \gamma_1 = 0.529 \text{A}^0 = a_0 \text{ (Bohr's Radius)} \]
\[ \gamma_2 = 4 \times 0.529 \text{A}^0 \]
\[ \gamma_3 = 9 \times 0.529 \text{A}^0 \]

Using value of \( \gamma = \frac{n^2 h^2 \varepsilon_0}{\pi m Z e^2} \); we can find

velocity of e\(^-\) in a particular energy level

putting value of \( \gamma \) in (i)
\[ \frac{\hbar}{m_n} \frac{n^2 \hbar^2 e^2}{2 \hbar} = \frac{n^2 \hbar^2}{2\pi} \]

\[ V = \frac{Ze^2}{2\hbar \varepsilon_0 n} \]  velocity of e\(^{-}\) in \(n\)th orbit.

For:
\[ \hbar = 6.626 \times 10^{-34} \text{Js} \]
\[ e = 1.6 \times 10^{-19} \text{J} \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2/\text{Nm}^2 \]

\[ V = \frac{2.18 \times 10^6 Z}{n} \text{m/s} \]

For H-atom \(Z = 1\)
\[ V_1 = \frac{2.18 \times 10^6}{n} \text{m/s} \]
\[ V_2 = \frac{2.18 \times 10^6}{2} \text{m/s} \]

We can also calculate Potential, Kinetic & Total Energy of an e\(^{-}\) in an orbit (or energy level)

K.E.:
\[ K.E. = \frac{1}{2} m v^2 = \frac{1}{2} m \left( \frac{Ze^2}{2\hbar \varepsilon_0 n} \right)^2 \]

\[ K.E. = \frac{m z^2 e^4}{8 \hbar^2 \varepsilon_0^2 n^2} \]
\[ U = k \frac{Q_1 Q_2}{r} \quad k = \frac{1}{4 \pi \varepsilon_0} \]

\[ U = \frac{1}{4 \pi \varepsilon_0} \frac{m^2 e^2}{\pi m^2 e^2} \]

\[ U = -\frac{m^2 e^4}{4 n^2 h^2 \varepsilon_0^2} \]

\[ \begin{aligned} U &= -\frac{m^2 e^4}{8 n^2 h^2 \varepsilon_0^2} \quad U = -2kE \\
\end{aligned} \]

\[ T.E. = K.E + U \]

\[ \begin{aligned} \varepsilon_0 \text{ in an orbit} &= \frac{m^2 e^4}{8 n^2 h^2 \varepsilon_0^2} - 2 \frac{m^2 e^4}{8 n^2 h^2 \varepsilon_0^2} \\
\end{aligned} \]

\[ E = T.E. = -\frac{m^2 e^4}{8 n^2 h^2 \varepsilon_0^2} \quad \text{This is also the energy of that orbit in which } e^- \text{ is present} \]

\[ E = -\frac{m^2 e^4}{8 n^2 h^2 \varepsilon_0^2} \]
Note that \[ U = -2k \cdot E = 2E \]

For \( m = 0.1 \times 10^{-31} \text{ kg} \)
\( E = 1.6 \times 10^{-19} \text{ J} \)
\( h = 6.626 \times 10^{-34} \text{ Js} \)
\( E_0 = 8.85 \times 10^{-12} \text{ e}^2/\text{Nm}^2 \)

\[
E = -13.6 \times \frac{Z^2 \text{ eV}}{n^2}
\]
Energy of nth level
\( \text{eV} = \text{electron volt} \)

For H-atom \( Z = 1 \)

\( E_1 = -13.6 \text{ eV} \)
\( E_2 = -13.6 \times \frac{1}{4} = -3.4 \text{ eV} \)
\( E_3 = -13.6 \times \frac{1}{9} = -1.51 \text{ eV} \)
\( E_4 = -13.6 \times \frac{1}{16} = -0.85 \text{ eV} \)

\( E_5 = -0.54 \text{ eV} \)
\( E_6 = -0.37 \text{ eV} \)
\( E_7 = -0.27 \text{ eV} \)

Note that total energy of an \( e^- \) in an orbit is \( -\text{ve} \) which indicates that \( e^- \) is bounded to the nucleus (like planets are drawn to sun)

\[
U = 2E = -2 \times 13.6 \times \frac{Z^2 \text{ eV}}{n^2}
\]

\[
k = -E = 13.6 \times \frac{Z^2 \text{ eV}}{n^2}
\]
5) An $e^-$ while orbiting in a lower energy level may absorb a specific photon (energy) & jumps [excites] to a higher energy level. The energy of such a photon is equal to the energy difference of two orbits.

$$\Delta E = E_n^2 - E_n^1$$

$$\frac{hc}{\lambda} = E_n^2 - E_n^1$$
ii) An e\textsuperscript{−} in a higher energy level may release a specific photon (energy) \& Sums [\textit{deexcite}] [makes transition] to a lower energy level. The Energy of emitted photon is equal to the Energy difference of two orbits:

\[ \Delta E = E_{n_2} - E_{n_1} \]

\[ \frac{hc}{\lambda} = E_{n_2} - E_{n_1} \]

**Note:** e\textsuperscript{−} absorbs \& release only those photons whose energy is exactly equal to energy difference of two orbits.

\[ \Rightarrow \text{This explains the existence of emission \& absorption Discrete Spectrum} \] (study in detail in next lecture)

**Drawbacks in Rutherford’s Model**
Drawbacks in Bohr Model

(i) Model was applicable only for H-atom and H-like atoms. Single e- species like He\textsuperscript{+}, Be\textsuperscript{++}, B\textsuperscript{+++}
(ii) It could not explain spectra of multi-electron atoms
(iii) Wave nature of electron was not taken into consideration (inconsistent with De-Broglie hypothesis)
(iv) It violates Heisenberg's Uncertainty Principle & simultaneously defines position (value of \(x\)) and momentum (value of \(p\)) of electron correctly.
(v) It cannot explain splitting of spectral lines in
   a) Electric Field \(\rightarrow\) Stark effect
   b) Magnetic Field \(\rightarrow\) Zeeman effect