Nuclear Physics

It exists at the centre of an atom, containing entire positive charge and almost whole of mass. The electron revolve around the nucleus to form an atom. The nucleus consists of protons (+ve charge) and neutrons. A proton has positive charge equal in magnitude to that of an electron (+1.6 × 10⁻¹⁹ C) and a mass equal to 1840 C) and a mass equal to 1840 times that of an electron. A neutron has no charge and mass is approximately equal to that of proton.

Properties of a nucleus

(1) **Nuclear Mass**

As we know that every nucleus contains protons and neutrons and so every nucleus has a definite mass. However, since the mass of electron is negligible so atomic mass is roughly equal to nuclear mass.

Atomic masses are measured in atomic mass unit (a.m.u.) defined as

\[ 1 \text{ amu} = 1.6604 \times 10^{-27} \text{ kg} \]

\[ \Rightarrow 1 \text{ u} = 931.478 \text{ MeV}/c^2 \]

and its energy equivalent is 931.48 MeV.

The number of protons in a nucleus of an atom is called as the atomic number (Z) of that atom. The number of protons plus neutrons (called as Nucleus) in a nucleus of an atom is called as mass number (A) of that atom.

A particular set of nucleons forming an atom is called as nuclide. It is represented as \( ZX^A \). The nuclides having same number of protons (Z), but different number of nucleons (A) are called as isotopes. The nuclide having same number of nucleons (A), but different number of protons (Z) are called as isobars. The nuclide having same number of neutrons (A–Z) are called as isotones.

(2) **Nuclear charge**

Since nucleus contain +vely charged protons (charge = 1.6 × 10⁻¹⁹ C) and neutrons (neutral) so every nucleus has a net +ve charge.

(3) **Nuclear radius**

A rough estimate of nuclear size suggests us that the radius of the nucleus of an atom having mass number 'A' is given by

\[ R = R_0 A^{1/3} \]

Where \( R_0 \) is a constant found to be equal to

\[ R_0 = 1.4 \times 10^{-15} \text{ m} = 1.4 \text{ fm} \]

(4) **Nuclear Density**

In spite of the fact that nuclear radius depends on mass number of the atom but nuclear density is independent of mass number because if neutrons are supposed to be of almost the same mass as that of protons then the total mass of a nucleus is proportional to A. If each nucleon are
supposed to have a mass \( m \) then nuclear density is given by

\[
\rho = \frac{mA}{4\pi R_0^3} = \frac{3m}{4\pi R_0^3}
\]

\textit{(Which is independent of } A)\]

(5) **Nuclear spin and magnetic moment**

Like orbital electrons in an atom, nucleons inside nucleus have well defined quantum states.

Correspondingly they have angular momentum and hence a magnetic moment. Like electrons nucleons also have intrinsic angular momentum and ‘magnetic’ moment corresponding to their spin.

**Nuclear Forces**

If only the electrostatic and gravitational forces existed in the nucleus, then it would be impossible to have stable nuclei composed of protons and neutrons. The gravitational forces are much too small to hold the nucleons together compared to the electrostatic forces repelling the protons. Since stable atoms of neutrons and protons do exist, there must be another attractive force acting within the nucleus. This force is called the nuclear force.

**Properties of nuclear force**

(1) They are charge independent. The nuclear force between two proton is same as that between two neutrons or between a neutron and proton. This is known as charge independent character of nuclear forces.

(2) They may be repulsive may be attractive (Repulsive at exceedingly small separation between two nucleons appreciably smaller than \( 10^{-13} \) cm i.e. \( 10^{-11} \) m

(3) It is a short range force. Its radius of action is of the order of \( 10^{-11} \) cm.

(4) The nuclear force is of saturation character. Each nucleon in nucleus interacts with a limited number of nucleons.

(5) Nuclear force are much stronger than electromagnetic force or gravitational attractive forces. It is the strongest of all the forces. This is why it is called strong interaction.

(6) Nuclear force is spin dependent. If two interacting nucleons are having parallel spins then nuclear force operative between them is comparatively stronger and if their spins are antiparallel, nuclear interaction is comparatively weaker.

(7) Nuclear force is a non-central force. They can not be represented as directed along the st. line connecting the centres of the interacting nucleons. Its non central nature is due to the fact that it depends also on the orientation of the nucleon spins.

**Mass defect**

It is observed that the mass of a nucleus is slightly less than the sum of the masses of constituent nucleons. Suppose a nucleus consists of ‘Z’ protons and ‘N’ neutrons. Mass of a proton, a neutron and the resulting nucleus are respectively \( m_p \), \( m_n \) and \( M \) then mass defect of the nucleus is given by

\[
\Delta m = Zm_p + Nm_n - M
\]
If \( A \) is the mass number of the nucleus
\[
\Delta m = [Zm_p + (A - Z)m_n - M]
\]
In terms of atomic masses we may also write mass defect as
\[
\Delta m = [Zm(\text{^1}_H) + Nm(\text{^0}_n) - m(\text{^A}_X)]
\]
Where \( m(\text{^1}_H) = \) mass of one hydrogen atom.
\( m(\text{^A}_X) = \) mass of atom having atomic no. \( Z \) and mass no. \( A \)
\( \text{e.g.,} \)
mass of \( \text{^1}_H = 1.00784 \text{ u} \)
mass of neutron = 1.00874 u
Expected mass of deuterium = 2.01654 u
but measured mass = 2.0141 u.
mass defect,
\( \Delta m = 0.00244 \text{ u} \).

**Nuclear Stability**

Figure shows pot of \( N \) vs. \( Z \) for known nuclides. The stable nuclides are indicated by the black dots. Non-stable nuclides decay by emission of particles, or electromagnetic radiation, in a process called radioactivity.

**Binding energy**

To break a nucleus into its constituent nuclei some energy is required to be supplied. This energy is called Binding Energy of the given nucleus or the energy equivalent of the missing mass of a nucleus is called the binding energy of the nucleus.
BE = (Δm)c^2 = [Zm_p + (A-Z)m_n - M]c^2,

BE = Δm(in amu)×931 MeV  

Where Δm = mass defect

Binding energy per nucleon is a measure of the stability of the nucleus. If there be n nucleons which is equal to A,

\[
\frac{\text{Binding Energy}}{\text{Nucleon}} = \frac{\text{B.E.}}{A}
\]

From the plot of B.E./ nucleons Vs mass number (A), we observe that:

1. Binding energy per nucleon has low value for both heavy and light nuclei i.e. Heavy as well as light nuclei, both are unstable. B.E./nucleons increases on an average and reaches a maximum of about 8.7 MeV for A = 50 – 80. For more heavy nuclei, B.E./nucleons decreases slowly as A increases. For the heaviest natural element U^{238} it drops to about 7.5 MeV. From above observation, it follows that nuclei in the region of atomic masses 50 – 80 are most stable.

2. The intermediate nuclei have large value of binding energy per nucleon so they are more stable.

3. Binding energy per nucleon increases rapidly up to mass number 20 but there are peaks corresponding to \(^{4}\text{He}, \, ^{12}\text{C}, \, ^{16}\text{O}\) which indicates that these nuclei are more stable than neighbours. The reason is that they may be considered to possess magic numbers i.e. their mass number is divisible by 4 and these nuclei may have \(^{4}\text{He}\) as their constituents.

4. The minimum value of the BE/Nucleon is in the case of deuteron that is 1.11 Mev.

5. The maximum value of the BE/Nucleon is 8.79 Mev for the nuclide \(^{56}\text{Fe}\) which is therefore the most stable nucleus.
Illustration:

Using the following plot of BE/nucleon vs mass number, mention the condition for which the energy is absorbed or released for the reaction

\[ z_1 P^{A_1} + z_2 Q^{A_2} \rightarrow z_3 P^{A_1} \]

**Sol.** Binding energy for reactant is \((xE_a + yE_p)\) and that for product is \(zE_f\)

**Case-I:**

\[
if \ (A_1E_f + A_2E_f) > A_3E_f
\]

Energy is absorbed.

**Case-II:**

\[
if \ (A_1E_f + A_2E_f) < A_3E_f
\]

Energy is released.

**Note**

(i) If we split a heavy nucleus into two medium sized nuclei and total binding energy of new nuclei is greater than parent nuclei, then energy is released (Nuclear fission)

(ii) If two nuclei of small mass number combine to form a single medium size nucleus for which binding energy is greater than the constituent nuclei, then energy is released (Nuclear fusion)

**Nuclear Fission**

The breaking of a heavy nucleus into two or more fragments of comparable mass, with the release of tremendous energy is called as nuclear fission.

The most typical fission reaction occurs when slow moving neutrons strike \(^{235}\)U. The following nuclear reaction takes place.

\[ ^{235}\text{U} + _0\text{n}^1 \rightarrow ^{92}\text{Kr} + ^{141}\text{Ba} + 3_0\text{n}^1 + 200 \text{ MeV} \]

If more than one of the neutrons produced in the above fission reaction are capable of inducing a fission reaction (provided \(^{235}\)U is available), then the number of fission taking place at successive stages goes increasing at a very brisk rate and this generates a series of fission. This is known as chain reaction. The chain reaction takes place only if the size of the fissionable material (\(^{235}\)U) is greater than a certain size called the critical size.
If the number of fission in a given interval of time goes on increasing continuously, then a condition of explosion is created. In such cases, the chain reaction is known as uncontrolled chain reaction. This forms the basis of atomic bomb.

In a chain reaction, the fast moving neutrons are absorbed by certain substances known as moderators (like heavy water), then the number of fissions can be controlled and the chain reaction is such cases is known as controlled chain reaction. This forms the basis of a nuclear reactor.

**Nuclear Fusion**

The process in which two or more light nuclei are combined into a single nucleus with the release of tremendous amount of energy is called as nuclear fusion. Like a fission reaction, the sum of masses before the fusion (i.e. of bigger nucleus) and this difference appears as the fusion energy. The most typical fusion reaction is the fusion of two deuterium nuclei into helium.

$$^1\text{H}^2 + ^1\text{H}^2 \rightarrow ^2\text{He}^4 + 21.6 \text{ MeV}$$

For the fusion reaction to occur, the light nuclei are brought closer to each other (with a distance of $10^{-14} \text{ m}$). This is possible only at very high temperature to counter the repulsive force between nuclei. Due to this reason, the fusion reaction is very difficult to perform. The inner core of sun is at very high temperature, and is suitable for fusion. In fact the source of sun’s and other star’s energy is the nuclear fusion reaction.

**Conservation laws in nuclear reaction**

Nuclear reaction processes have led to the formulation of useful conservation principles. The four principles of most interest in this module are discussed below.

(i) Conservation of electric charge implies that charges are neither created nor destroyed. Single positive and negative charges may, however, neutralize each other. It is also possible for a neutral particle to produce one charge of each sign.

(ii) Conservation of mass number does not allow a net change in the number of nucleons i.e. total number of protons and neutrons should also remain same on both sides of a nuclear reaction. However, the conversion of a proton to a neutron and vice versa is allowed.

(iii) Conservation of mass and energy implies that the total of the kinetic energy and the energy equivalent of the mass in a system must be conserved in all decays and reactions. Mass can be converted to energy and energy can be converted to mass, but the sum of mass and energy must be constant. In nuclear reactions, sum of masses before reaction is greater than the sum of masses after the reaction. The difference in masses appears in form of energy following the Law of inter-conversion of mass & energy. The energy released in a nuclear reaction is called as Q Value of a reaction and is given as follows.

If difference in mass before and after the reaction is $\Delta m \text{ amu}$ ($\Delta m =$ mass of reactants minus mass of products) then

$$Q \text{ value} = \Delta m (931) \text{ MeV}$$

(iv) Conservation of momentum is responsible for the distribution of the available kinetic energy among product nuclei, particles, and/or radiation. The total amount is the same before and after the reaction even though it may be distributed differently among entirely different nuclides and/or particles.
**Illustration:**

In the sun about 4 billion kg of matter is converted to energy each second. Find the power output of the sun in watts.

\[
\frac{m}{t} = 4 \times 10^8 \text{ kgs}^{-1}
\]

\[
E = mc^2
\]

\[
\Rightarrow \frac{E}{t} = \left(\frac{m}{t}\right)c^2
\]

\[
\Rightarrow \frac{E}{t} = 4 \times 10^8 \times 9 \times 10^{16}
\]

\[
\Rightarrow \frac{E}{t} = 3.6 \times 10^{23} \text{ J s}^{-1}
\]

\[
\Rightarrow \frac{E}{t} = 3.6 \times 10^{23} \text{ W}
\]

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**Illustration:**

A neutron breaks into a proton and electron. Calculate the energy produced in this reaction in MeV. Mass of an electron = \(9 \times 10^{-31}\) kg, Mass of Proton = \(1.6725 \times 10^{-27}\) kg, Mass of neutron = \(1.6747 \times 10^{-27}\) kg. Speed of light = \(3 \times 10^8\) m/sec.

\[
\beta^+ \rightarrow p + e^0
\]

\[
\Delta m = [\text{Mass of neutron} - (\text{mass of proton} + \text{mass of electron})]
\]
\[
= [1.6747 \times 10^{-27} - (1.6725 \times 10^{-27} + 9 \times 10^{-31})]
\]
\[
= 0.0013 \times 10^{-27} \text{ kg}
\]

\[
\therefore \text{Energy released}
\]

\[
E = \Delta mc^2 = (0.0013 \times 10^{-27}) \times (3 \times 10^8)^2 = 1.17 \times 10^{-13} \text{ joule}
\]

\[
= \frac{1.17 \times 10^{-13}}{1.6 \times 10^{-19}} \text{ eV} = 0.73 \times 10^6 \text{ eV} = 0.73 \text{ MeV}
\]

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**Illustration:**

The nuclei involved in the nuclear reaction \(A_1 + A_2 \rightarrow A_3 + A_4\) have the binding energies \(E_1, E_2, E_3\) and \(E_4\). Find the energy released (Q value) of this reaction.

**Sol.** Suppose \(M_1, M_2, M_3, M_4\) are the rest masses of the nuclei \(A_1, A_2, A_3\) and \(A_4\) participating in the reaction

\[
A_1 + A_2 \rightarrow A_3 + A_4 + Q
\]

Here \(Q\) is the energy released. Then by conservation of energy:

\[
Q = (M_1 + M_2 - M_3 - M_4)c^2
\]

Now \(M_ic^2 = c^2 (Z_i m_i + (A_i - Z_i) m_\text{p}) - E_1\) etc. and

\[
Z_1 + Z_2 = Z_3 + Z_4 \text{ (conservation of charge)}
\]

\[
A_1 + A_2 = A_3 + A_4 \text{ (conservation of mass number)}
\]

Here \(Q = (E_3 + E_4) - (E_1 + E_2)\)
Practice Exercise

Q.1 Calculate the electric potential energy due to the electric repulsion between two nuclei of $^{12}$C when they 'touch' each other at the surface.

Q.2 Find the binding energy of $^{26}$Fe$^{56}$. Atomic mass of $^{26}$Fe$^{56}$ is 55.9349 u and that of $^1$H is 1.00783 u. Mass of neutron is 1.00867u.

Q.3 Calculate the Q-value in the following decay
$$^{25}\text{Al} \rightarrow ^{25}\text{Mg} + e^- + \nu$$

Q.4 Find the maximum energy that a beta particle can have in the following decay
$$^{176}\text{Lu} \rightarrow ^{176}\text{Hf} + e + \overline{\nu}$$

Atomic mass of $^{176}\text{Lu}$ is 175.942694 u and that of $^{176}\text{Hf}$ is 175.941420 u

Answers

Q.1 10.2 MeV.  Q.2 492 MeV.  Q.3 3.254 MeV  Q.4 0.2806 MeV

Radioactivity

Among about 2500 known nuclides, fewer than 300 are stable. The others are unstable structures that decay to form other nuclides by spontaneously emitting particles and electromagnetic radiation, a process called radioactivity. The time scale of these decay processes ranges from a small fraction of a microsecond to billions of years. The substances which emit these radiations are called as radioactive substances. It was discovered by Henry Becquerel for atoms of Uranium. Later it was discovered that many naturally occurring compounds of heavy elements like radium, thorium etc also emit radiations.

At present, it is known that all the naturally occurring elements having atomic number greater than 82 are radioactive. For example some of them are : radium, polonium, thorium, actinium, uranium, radon etc. Later on Rutherford found that emission of radiation always accompanied by transformation of one element (transmutation) into another. In actual radioactivity is the result of disintegration of an unstable nucleus. Rutherford studied the nature of these radiations and found that these mainly consist of $\alpha$, $\beta$, $\gamma$ particles (rays).

$\alpha$-Particles ($^2\text{He}^4$)

These carry a charge of $+2e$ and mass equal to $4m_p$. These are nuclei of helium atoms. The energies of $\alpha$-particles vary from 5 MeV to 9 MeV; their velocities vary from $0.01 \sim 0.1$ times of c (velocity of light). They can be deflected by electric and magnetic field and have lower penetrating power but high ionising power.
\(\beta\)-Particles : \((-1e^0)\)

These are fast moving electrons having charge equal to \(e\) and mass \(m_e = 9.1 \times 10^{-31}\) kg. Their velocities vary from 1% to 99% of the velocity of light (c). They can also be deflected by electric and magnetic fields. They have low ionising power but high penetrating power.

\(\gamma\)-Radiations : \((0\gamma^0)\)

These are electro-magnetic waves of nuclear origin and of very short wavelength. They have no mass. They have maximum penetrating power and minimum ionising power. The energy released in a nuclear reaction is mainly emitted in from these \(\gamma\)-radiations.

**Radioactive decays**

**\(\alpha\)-decay**

Nuclides decay by emitting \(\alpha\)-particles. \(\alpha\)-particles are generally emitted by very heavy nuclei containing to many nucleons to remain stable. The emission of such a nucleon cluster as a whole rather than the emission of single nucleon is energetically more advantageous because of the particularly high binding energy of alpha-particles. The parent nucleus \((Z,A)\) is transformed as

\[
_zX^A \longrightarrow \rightarrow_2He^4 + z_2Y^{A-4}
\]

\[
\Delta m = \left[ M_2He^4 + M_{z-2}Y^{A-4} \right] - \left[ M_zX^A \right]
\]

**Note**

(i) Nuclear mass is different from atomic mass because nucleus is without electrons.

(ii) Released energy converts into kinetic energy.

(iii) In nucleus, Atomic energy is 13.6 eV small atomic binding energy has been neglected.

(iv) Released energy is shared as kinetic energy by products and outgoing particles.

**Calculation of Kinetic Energy**

Momentum of \(\alpha\) particle + momentum of daughter nuclei = 0

\[
(m_a, \vec{v}) + (\vec{p}_D)
\]

assuming parent nuclei to be at rest initially

\[
\vec{p}_a + \vec{p}_D = 0
\]

\[
|\vec{p}_a| = |\vec{p}_D|
\]

If \(Q\) is released energy or \(Q\) value of reaction.

\[
K_a + K_d = Q
\]
\[ \Rightarrow \quad K_a + \frac{P_n^2}{2m_D} = Q \]
\[ \Rightarrow \quad K_a + \frac{P_a^2}{2m_D} = Q \]
but
\[ \Rightarrow \quad K_a + \frac{2K_a \cdot m_a}{2m_D} = Q \]
\[ \Rightarrow \quad K_a \left[ 1 + \frac{m_a}{m_D} \right] = Q \]
\[ K_a = \frac{m_D \times Q}{m_D + m_a} \]
\[ K_a = \frac{(A-4)m}{4m+(A-4)m} Q \]
\[ \Rightarrow \quad K_a = \left[ \frac{A-4}{A} \right] Q \]

**\( \beta \) decay**

Another way in which nuclides decay radioactively is by the emission of \( \beta \) particles. When neutron-proton ratio inside a nucleus is not suitable for it to be stable (either less or more) then \( \beta \)-decay takes place. Due to a special type of interaction called weak interaction a neutron gets converted into a proton and a proton gets converted into a neutron and a positron. Electrons or positrons are emitted from the nucleus just after their creation. This emission of electron or positron from nucleus is called \( \beta \)-decay. Emission of positron (of the order of MeV) is called \( \beta^- \)-decay and emission of electron (of the order of MeV) is called \( \beta^- \)-decay.

(i) **Negative \( \beta \) decay (\( \beta^- \) decay)**

Neutron inside nucleus is transformed into proton.

\[ n \longrightarrow p + e^- + \bar{\nu} \quad (\text{Antineutrino}) \]
\[ X^A \longrightarrow_{-e} Y^A + e^- + \bar{\nu} \quad + \text{energy released.} \]

Equation corresponding to nuclear mass
\[ \Delta m = M[0, X^A] - \{ M[z, Y^A] + Me \} \]
Equation corresponding to atomic mass
\[ \Delta m = \left[ M[z, X^A] - M[z+1, Y^A] \right] \]
energy released
\[ E = \Delta mc^2 \]
(ii) Positive $\beta$ decay($\beta^+$ decay)

Proton inside nucleus is transformed into neutron.

\[ p \longrightarrow n + e^- \text{ (Positron)} + \nu \text{ (neutrino)} \]

Positron is anti-particle of electron. It is highly reactive.

\[ _zX^A \longrightarrow _{z-1}Y^A \rightarrow e^+ + \nu + \text{energy released.} \]

Equation corresponding to nuclear mass

\[ \Delta m = M^A_z x - \left[ M^A_{z-1} Y^A \right] + M^\nu \]

Equation corresponding to atomic mass

\[ \Delta m = M^+ z^A x - \left[ M^+ z_{-1} Y^A - 2 M^\nu \right] \]

energy released

\[ E = \Delta mc^2 \]

Experiments show that $\beta$-particles are emitted with continuous range of kinetic energy.

![Intensity of $\beta$ particles vs. K.E.](image)

(iii) Electron capture

Nuclei having an excess of protons may capture an electron from one of the inner orbits which immediately combines with a proton in the nucleus to form a neutron. This process is called electron capture (EC). The electron is normally captured from the innermost orbit (the K-shell), and, consequently, this process is sometimes called K-capture.

The process is observed from the emission of the characteristic X-rays produced, when an orbiting electron from an outer shell makes a downward transition into a K shell vacancy. The X-rays are characteristic of daughter nuclei not of the parent because x-ray emission taken place 'K-capture'

\[ e^- + Be^7 \longrightarrow Li^7 + \nu \quad (A \text{ simple } K\text{-capture equation}) \]
**Neutrino and anti-neutrino**

1. It has zero electric charge, hence shows no electromagnetic interaction.

2. Rest mass is possibly zero. Recent experiments show that mass of neutrino is less than \( \frac{7}{c^2} \text{ eV} \).

3. It travels with the speed of light.

4. It has spin quantum number \( 1/2 \). A spin of \( 1/2 \) satisfies the law of conservation of angular momentum when applied to \( \beta \)-decay.

5. It shows very weak interactions with matter.

**\( \gamma \)-Decay**

As we know that like the discrete energy orbits of electrons in an atom. Nucleons in an atom inside the nucleus also have well defined energy state or discrete quantum state. After every \( \alpha \) or \( \beta \) emission a nucleus is in the excited state correspondingly subsequent to every \( \alpha \)-or \( \beta \)-emission a nucleus emits electromagnetic radiation (of the order of MeV) to come to ground state. The frequency or the wavelength of the emitted radiations lie in \( \gamma \)-region and is called \( \gamma \)-emission.

\[
\begin{array}{c}
B^{12} \\
\downarrow \\
C^{12} \\
\downarrow \\
e \\
\downarrow \\
4.4 \text{ MeV} \\
\end{array}
\]

\[
13.4 \text{ MeV}
\]

**Group-Displacement Law**

(i) When a nuclide emits one \( \alpha \)-particle (\( _2\text{He}^4 \)), its mass number (\( A \)) decreases by 4 units and atomic number (\( Z \)) decreases by two units.

\[ \gamma X^A \rightarrow z_{z-2} Y^{A-4} + _2\text{He}^4 + \text{Energy} \]

(ii) When a nuclide emits a \( \beta \) particle, its mass number remains unchanged but atomic number increases by one unit.

\[ \gamma X^A \rightarrow z_{z+1} Y^A + _{-1}e^0 + \bar{\nu} + \text{Energy} \quad (\bar{\nu} \text{ is antineutrino}) \]

(iii) When a nuclide emits a \( \beta^+ \) particle, its mass number remains unchanged but atomic number decreases by one unit.

\[ \gamma X^A \rightarrow z_{z-1} Y^A + _{-1}e^0 + \nu + \text{Energy} \quad (\nu \text{ is neutrino}) \]

(iv) When a \( \gamma \) particle is produced, both atomic and mass number remain constant.

**Rutherford-Soddy Law (Statistical Law)**

The disintegration of a radioactive substance is random and spontaneous.

Radioactive decay is purely a nuclear phenomenon and is independent of any physical and chemical conditions.
The radioactive decay follows first order kinetics, i.e., the rate of decay is proportional to the number of undecayed atoms in a radioactive substance at any time t. If \( \frac{dN}{dt} \) be the number of atoms (nuclei) disintegrating in time \( dt \), the rate of decay is given as \( \frac{dN}{dt} \). From first order kinetic rate law:

\[
\frac{dN}{dt} = -\lambda N
\]

where \( \lambda \) is called as decay or disintegration constant.

Let \( N_0 \) be the number of nuclei at time \( t = 0 \) and \( N_t \) be the number of nuclei after time \( t \), then according to integrated first order rate law, we have:

\[
N_t = N_0 e^{-\lambda t}
\]

\[
\Rightarrow \lambda t = \ln \frac{N_0}{N_t} = 2.303 \log \frac{N_0}{N_t}
\]

The half life \( (t_{1/2}) \) period of a radioactive substance is defined as the time in which one-half of the radioactive substance is disintegrated. If \( N_0 \) be the number of nuclei at \( t = 0 \), then in half life \( T \), the number of nuclei decayed will be \( N_0/2 \)

\[
N_t = N_0 e^{-\lambda t}
\]

\[
\Rightarrow \frac{N_0}{2} = N_0 e^{-\lambda T}
\]

from (i) & (ii)

\[
\frac{N_t}{N_0} = \left( \frac{1}{2} \right) T = \left( \frac{1}{2} \right)^n
\]

\[n: \text{number of half lives}\]

The half life \( (T) \) and decay constant \( (\lambda) \) are related as:

\[
T = \frac{0.693}{\lambda}
\]

The mean life \( (T_m) \) of a radioactive substance is equal to the sum of life times of all atoms divided by the number of all atoms and is given follows

\[
T_m = \frac{\int t dN}{\int dN} = \frac{\int t \lambda e^{-\lambda t} dt}{\int \lambda e^{-\lambda t} dt}
\]

\[
T_m = \frac{1}{\lambda}
\]
**Activity of a Radioactive Isotope**

The activity of a radioactive substance (or radioisotope) means the rate of decay per second or the number of nuclei disintegrating per second. It is generally denoted by $A$.

\[
A = \frac{dN}{dt}
\]

If at time $t = 0$, the activity of a radioactive substance be $A_0$ and after time $t = t$ sec., activity be $A_t$ then:

\[
A_0 = \left[ \frac{dN}{dt} \right]_{t=0} = \lambda N_0
\]

\[
A_t = \left[ \frac{dN}{dt} \right]_{t=t} = \lambda N_t
\]

\[
A_t = A_0 e^{-\lambda t}
\]

**Unit of activity**

The activity is measured in terms of curie (Ci). 1 curie is the activity of 1 gram of a freshly prepared sample of radium Radium ($t_{1/2} = 1602$ yrs.)

1 curie $1 \text{Ci} = 3.7 \times 10^{10}$ dps (disintegration per second)

1 dps is also known as 1 bq (Becquerel) $\Rightarrow 1 \text{Ci} = 3.7 \times 10^{10}$ bq

**Note**

All the equations discussed above is valid only when the number of nuclei are very large

**Survival probability and decay probability for a finite time interval**

The probability of survival (i.e. not decaying) in time $t$ is

\[
P_{\text{survival}} = \frac{N(t) e^{-\lambda t}}{N_0} = e^{-\lambda t}.
\]

Hence the probability of decay is

\[
P_{\text{decay}} = 1 - e^{-\lambda t}.
\]

**Successive disintegration and secular equilibrium**

Suppose $A \rightarrow B \rightarrow C \ldots \ldots$ (i.e., radioactive nucleus A decays to B and B decays to C)

Let number of radioactive nucleus A (Parent nucleus) at time $t = 0$ be $N_0$ and that of B = 0.

For A

\[
\frac{dN_A}{dt} = -\lambda_A N_A
\]

\[
N_A = N_0 e^{-\lambda_A t}
\]
For B
\[ \frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \]
\[ \Rightarrow \quad \frac{dN_B}{dt} = \lambda_A N_B e^{-\lambda_A t} - \lambda_B N_B \]

Multiplying both sides of this equation by \( e^{\lambda_B t} \) dt, we get
\[ e^{\lambda_B t} \cdot dN_B + \lambda_B N_B e^{\lambda_B t} dt = \lambda_A N_0 e^{(\lambda_A - \lambda_B)t} dt \]
\[ \Rightarrow \quad \frac{d}{dt} \left( N_B e^{\lambda_B t} \right) = e^{\lambda_B t} \frac{dN_B}{dt} + N_B \lambda_B e^{\lambda_B t} dt \]
\[ \Rightarrow \quad d \left( N_B e^{\lambda_B t} \right) = e^{\lambda_B t} dN_B + N_B \lambda_B e^{\lambda_B t} dt \]
\[ \Rightarrow \quad \int d \left( N_B e^{\lambda_B t} \right) = \lambda_A N_0 \int e^{(\lambda_A - \lambda_B)t} dt \]
\[ \Rightarrow \quad N_B e^{\lambda_B t} = \frac{\lambda_A N_0}{(\lambda_B - \lambda_A)} e^{(\lambda_A - \lambda_B)t} + C \]

Where C is constant of integration.

at \( t=0, N_B=0 \)
\[ \Rightarrow \quad C = \frac{-\lambda_A N_0}{(\lambda_B - \lambda_A)} \]

Hence,
\[ N_B e^{\lambda_B t} = \frac{\lambda_A N_0}{\lambda_B - \lambda_A} \left[ e^{(\lambda_A - \lambda_B)t} - 1 \right] \]
\[ \Rightarrow \quad N_B = \frac{\lambda_A N_0}{\lambda_B - \lambda_A} \left[ e^{-\lambda_A t} - e^{-\lambda_B t} \right] \]

Suppose the parent nucleus A is long lived i.e. the half life of the parent nucleus A is much larger in comparison to the half life of the daughter nucleus B
\[ \Rightarrow \quad t_{1/2 A} >> t_{1/2 B} \quad \Rightarrow \quad \lambda_A << \lambda_B \quad \Rightarrow \quad \lambda_B - \lambda_A \approx \lambda_B \]
\[ \Rightarrow \quad e^{-\lambda_A t} \text{ is negligible in comparison to } e^{-\lambda_B t} \]
\[ \Rightarrow \quad N_B = \frac{\lambda_A}{\lambda_B} N_0 e^{-\lambda_B t} \]
\[ \Rightarrow \quad N_B = \frac{\lambda_A}{\lambda_B} N_A \]
\[ \Rightarrow \quad N_A \lambda_A = N_B \lambda_B \]
i.e., after a time much longer in comparison to the half-life of the daughter nucleus B but much shorter in comparison to the half-life of parent nucleus A, we have \( N_A \lambda_A = N_B \lambda_B \). This state is called secular equilibrium.

**Illustration:**

The mean lives of an radioactive substance are 1620 and 405 years for \( \alpha \)-emission and \( \beta \)-emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both by \( \alpha \)-emission and \( \beta \)-emission simultaneously.

**Sol.** When a substance decays by \( \alpha \) and \( \beta \) emission simultaneously, the equivalent rate of disintegration \( \lambda_{eq} \) is given by:

\[
\lambda_{eq} = \lambda_\alpha + \lambda_\beta
\]

where \( \lambda_\alpha = \text{disintegration constant for } \alpha \)-emission only

\( \lambda_\beta = \text{disintegration constant for } \beta \)-emission only

Mean life is given by:

\[
T_{eq} = \frac{1}{\lambda_{eq}}
\]

\[
\Rightarrow \lambda_{eq} = \lambda_\alpha + \lambda_\beta = \frac{1}{T_{eq}} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1620} + \frac{1}{405} = 308 \times 10^{-3}
\]

\[
\lambda_{eq} t = 2.303 \log \frac{N_0}{N_t}
\]

\[
\Rightarrow (3.08 \times 10^{-1}) t = 2.303 \log \frac{100}{25}
\]

\[
\Rightarrow t = 2.303 \times \frac{1}{3.08 \times 10^{-1}} \log 4 = 449.24 \text{ years}
\]

**Illustration:**

Two radioactive materials \( A_1 \) and \( A_2 \) have decay constants of 10 \( \lambda_0 \) and \( \lambda_0 \). If initially they have same number of nuclei, find the time after which the ratio of number their undecayed nuclei will be \((1/e)\)

**Sol.**

\[
\frac{N_A}{N_B} = \frac{e^{10\lambda_0 t}}{e^{-10\lambda_0 t}} = e^{20\lambda_0 t} = \frac{1}{e} = e^{-1}
\]

\[
\Rightarrow 9 \lambda_0 t = 1
\]

or \( t = \frac{1}{9\lambda_0} \)
**Illustration:**

The weight based ratio for U$^{238}$ and Pb$^{206}$ in a sample of rock is 4 : 3. If the half life of U$^{238}$ is $4.5 \times 10^8$ year, then find the age of rock.

**Sol.** Let initial no. of U-atoms = $N_0$

After time $t$, (age of rock), let no. of atoms remaining undecayed = $N$

\[
\frac{238N}{26(N_0 - N)} = \frac{4}{3}
\]

\[
N_0 \cdot N = 1.79
\]

\[
t = \frac{T \log N_0/N}{\log 2}
\]

\[
= \frac{4.5 \times 10^8 \times \log 1.79}{0.301}
\]

**Illustration:**

A count rate-meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minutes. Five minutes later it shows 2700 counts per minutes. Find:

(a) decay constant  
(b) the half life of the sample

**Sol.** Initial activity = $A_0 = dN/dt$ at $t = 0$

Final activity = $A_1 = dN/dt$ at $t = t$

\[
\frac{dN}{dt} \bigg|_{t=0} = \lambda N_0 \text{ and } \frac{dN}{dt} \bigg|_{t=5} = \lambda N_1
\]

\[
\Rightarrow \frac{4750}{2700} = \frac{N_0}{N_1}
\]

Using $\lambda t = 2.303 \log \frac{N_0}{N_1}$

\[
\Rightarrow \lambda(5) = 2.303 \log \frac{4750}{2700}
\]

\[
\Rightarrow \lambda = \frac{2.303}{5} \log \frac{4750}{2700} = 0.1129 \text{ min}^{-1}
\]

\[
\Rightarrow t_{1/2} = \frac{0.693}{0.1129} = 6.14 \text{ min}
\]
Illustration:

A small amount of solution containing $\text{Na}^{24}$ radionuclide with activity $A = 2.0 \times 10^3$ disintegrations per second was injected in the bloodstream of a man. The activity of 1 cm$^3$ of blood sample taken 1 = 5.0 hours later turned out to be $A' = 16$ disintegrations per minute per cm$^3$. The half-life of the radionuclide is $T = 15$ hours. Find the volume of the man's blood.

Sol. Let $V$ = volume of blood in the body of the human being. Then the total activity of the blood is $AV$. Assuming all this activity is due to the injected $\text{Na}^{24}$ and taking account of the decay of this radionuclide, we get

$$VA' = Ae^{-\lambda t}$$

Now

$$\lambda = \frac{\ln 2}{15} \text{ per hour, } t = 5 \text{ hour}$$

Thus

$$V = \frac{A}{A'} e^{-\ln \frac{2}{3}} = \frac{2.0 \times 10^3}{(16/60)} e^{-\ln \frac{2}{3}} \text{ cc} = 5.95 \text{ litre}$$

Practice Exercise

Q.1 The half-life of $^{198}$Au is 2.7 days. Calculate
(a) the decay constant,
(b) the average-life and
(c) the activity of 1.00 mg of $^{198}$Au.
Take atomic weight of $^{198}$Au to be 198 g/mol.

Q.2 A radioactive sample has $6 \times 10^{18}$ active nuclei at a certain instant. How many of these nuclei will still be in the same active state after two half-lives.

Q.3 The activity of a radioactive sample falls from 600 s$^{-1}$ to 500 s$^{-1}$ in 40 minutes. Calculate its half-life.

Q.4 The number of $^{238}$U atoms in an ancient rock equals the number of $^{206}$Pb atoms. The half-life of decay of $^{238}$U is $4.5 \times 10^9$ y. Estimate the age of the rock assuming that all the $^{206}$Pb atoms are formed from the decay of $^{238}$U.

Q.5 A radioactive nucleus can decay by two different processes. The half-life for the first process is $t_1$ and that for the second process is $t_2$. Find the effective half-life $t$ of the nucleus.

Q.6 A radioactive sample decays with an average-life of 20 ms. A capacitor of capacitance 100 μF is charged to some potential and then the plates are connected through a resistance R. What should be the value of R so that the ratio of the charge on the capacitor to the activity of the radioactive sample remains constant in time?
Q.7  Suppose, the daughter nucleus in a nuclear decay is itself radioactive. Let $\lambda_p$ and $\lambda_d$ be the decay constants of the parent and the daughter nuclei. Also, let $N_p$ and $N_d$ be the number of parent and daughter nuclei at time $t$. Find the condition for which the number of daughter nuclei becomes constant.

## Answers

| Q.1 | (a) $2.9 \times 10^{-6} \text{ s}^{-1}$, (b) 3.9 days, (c) 240 Ci. |
| Q.2 | $1.5 \times 10^{18}$ |
| Q.3 | 152 min. |
| Q.4 | $4.5 \times 10^9 \text{ y}$ |
| Q.5 | $\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2}$ |
| Q.6 | 200 $\Omega$. |
| Q.7 | $\lambda_p N_p = \lambda_d N_d$ |
Q.1 A point source of light is placed at the centre of curvature of a hemispherical surface. The radius of curvature is $r$ and the inner surface is completely reflecting. Find the force on the hemisphere due to the light falling on it if the source emits a power $W$.

Sol. The energy emitted by the source per unit time, i.e. $W$ fall on a area $4\pi r^2$ at a distance $r$ in unit time. Thus, the energy falling per unit area per unit time is $\frac{W}{4\pi r^2}$. Consider a small area $dA$ at the point $P$ of the hemisphere (figure).

The energy falling per unit time on it is

$$P = \frac{WdA}{4\pi r^2}$$

The corresponding momentum incident on this area per unit time is

$$= \frac{WdA}{4\pi r^2 c}$$

Suppose the radius OP through the area $dA$ makes an angle $\theta$ with the symmetry axis OX. The force on $dA$ is along this radius.

$$dF = \frac{2WdA}{4\pi r^2 c}$$

By symmetry, the resultant force on the hemisphere is along OX. The component of $dF$ along OX is

$$dF \cos \theta = \frac{2WdA}{4\pi r^2 c} \cos \theta$$

$$= \frac{2W}{4\pi r^2 c} (\text{projection the area } dA \text{ on the plane containing the rim})$$

The net force along OX is

$$F = \frac{2W}{4\pi r^2 c} \Sigma (\text{projection the area } dA \text{ on the plane containing the rim})$$

$$= \frac{2W}{4\pi r^2 c} (\pi r^2) = \frac{W}{2c}$$

Q.2 Find the maximum kinetic energy of photo-electron liberated from the surface of lithium ($\phi = 2.39$ eV) by electromagnetic radiation whose electric component varies with time as $E = a \left(1 + \cos \omega t\right) \cos \omega_0 t$, where $'a'$ is a constant. $\omega = 6 \times 10^{14}$ rad/sec and $\omega_0 = 3.60 \times 10^{15}$ rad/s.

Sol. $E = a \left(1 + \cos \omega t\right) \cos \omega_0 t = a \cos \omega_0 t + \cos \omega_0 t + \frac{1}{2} a \cos \left(\omega + \omega_0\right) t$ $\Rightarrow E = a \cos \omega_0 t + \frac{1}{2} a \cos \left(\omega + \omega_0\right) t$

This is a complex vibration consisting of harmonic components of frequencies $\omega_0$, $(\omega + \omega_0)$ and $(\omega - \omega_0)$. 
The highest angular frequency is \( (\omega + \omega_b) \).

Now, \( h \nu = \phi + k_{max} \)

So, \( k_{max} = \frac{h}{2\pi} (\omega + \omega_b) - \phi \)

\[
\begin{align*}
&= \frac{6.6 \times 10^{-34}}{2\pi} (6 \times 10^{14} + 3.6 \times 10^{13}) - 2.39 \times 1.6 \times 10^{-19} \\
&= 4.41 \times 10^{-19} - 3.82 \times 10^{-19} = 0.59 \times 10^{-19} J = 0.37 \text{ eV}
\end{align*}
\]

Q.3 Find the ratio of de-Broglie wavelength of an \( \alpha \)-particle to that of a proton being subjected to the same magnetic field so that the radii of their paths are equal to each other, assuming that the field induction vector \( \vec{B} \) is perpendicular to the velocity vectors of the \( \alpha \)-particle and the proton.

Sol. When a charged particle of charge \( q \), mass \( m \) enters perpendicularly to the magnetic induction \( \vec{B} \) of a magnetic field, it will experience a magnetic force

\[ F = q (\vec{v} \times \vec{B}) = qvB \sin 90^\circ = qvB \] that will provide a centripetal acceleration \( \frac{v^2}{r} \)

\[ \Rightarrow qvB = \frac{mv^2}{r} \]
\[ \Rightarrow mv = qBr \]

\[ \Rightarrow \text{The de-Broglie wavelength} \quad \lambda = \frac{h}{mv} = \frac{h}{qBr} \]

\[ \Rightarrow \frac{\lambda_{\alpha\text{-particle}}}{\lambda_{\text{proton}}} = \frac{q_{\alpha}r_{\alpha}}{q_{p}r_{p}} \]

Since \( \frac{r_{\alpha}}{r_{p}} = 1 \) and \( \frac{q_{\alpha}}{q_{p}} = 2 \)

\[ \Rightarrow \frac{\lambda_{\alpha}}{\lambda_{p}} = 1/2 \]

Q.4 A particle of mass \( m \) moves along a circular orbit in a centrosymmetric potential field \( U = \frac{kr^2}{2} \). Using Bohr’s quantization condition. Find (a) radius of \( n \)th orbit (b) Energy of \( n \)th orbit

Sol. \[ F = -\frac{dU}{dr} = -kr \]

so \[ kr = \frac{mv^2}{r} \] \[ ...(i) \]

and \[ mvr = \frac{nh}{2\pi} \] \[ ...(ii) \]
Solving we get \( r = \left( \frac{\hbar^2 n^2}{4\pi \alpha m} \right)^{1/2} \)

Total energy \( E_n = KE_n + PE_n \)

\[
= \frac{1}{2} mv^2 + \frac{kr^2}{2} = kr^2
\]

\[
= K \sqrt{\frac{n^2 \hbar^2}{4\pi^2 mK}} = \sqrt{\frac{n^2 \hbar^2 K^2}{4\pi^2 mK}}
\]

\[
= \frac{nhK}{2\pi \sqrt{mK}}
\]

Q.5 Compare the radii and energy of ground state of H-atom and p-atom considering the motion of nucleus.

Sol. If we consider the motion of nucleus mass of \( e^- \) in all the expressions will be replaced by \( \mu \). Where

\[
\mu_H = \frac{mM}{m + M} ; \quad M = \text{Mass of Proton or neutron} \]

\[
m = \text{mass of electron.}
\]

and \( \mu_D = \frac{m(2M)}{m + 2M} \)

hence \( \mu_D > \mu_H \)

radius of \( n^{th} \) orbit \( r_n \propto \frac{1}{\mu} \) so \( r_H > r_D \)

Energy of \( n^{th} \) orbit \( E_n \propto \mu \) so \( E_H < E_D \).

Q.6 What lines of atomic hydrogen absorbing spectrum fall with in the wave length range from 94.5 nm to 130 nm.

Sol. Absorption lines are always corresponding to Lyman series.

\[
\text{Wave length of Lyman series} \quad \frac{1}{\lambda} = R \left[ \frac{1}{12} - \frac{1}{n^2} \right]
\]

for \( n = 2, \lambda_1 = 121 \) nm
for \( n = 3, \lambda_2 = 102.2 \) nm
for \( n = 4, \lambda_3 = 96.9 \) nm
for \( n = 5, \lambda_4 = 94.64 \) nm
for \( n = 6, \lambda_5 = 93.45 \) nm

hence 121.1 nm 102.2 nm 96.9 nm and 94.64 nm.
Q.7 A stationary H-atom emits a photon corresponding to the first line of Lyman series. What velocity does the atom acquire? \(M_H = 1.67\times10^{-19} \text{ kg}\)

**Sol.**

![Image showing H-atom and photon]

Applying momentum and energy conservation,

\[
mv = \frac{hv}{c} \quad \text{and} \quad \Delta E = \frac{1}{2}mv^2 + hv
\]

when \(\Delta E = 10.2 \times 1.6 \times 10^{-19} \text{ Joule}\)

we get \(\Delta E = \frac{1}{2}mv^2 + mvC\)

\[
\Delta E = mvC \left[ \frac{V}{2C} + 1 \right] \approx mvC
\]

\[
v = \frac{DV}{mC} = \frac{10.2 \times 1.6 \times 10^{-19}}{1.6 \times 10^{-27} \times 3 \times 10^8}
\]

\[
= 3.25 \text{ m/sec}
\]

Q.8 The BE of an electron in ground state of the atom is equal to \(E_g = 24.6 \text{ ev}\). Find the energy required to remove both electron from the atom.

**Sol.**

Ionisation energy of He\(^+\) atom = 54.4 ev

hence to remove both electrons from He-atom

we require = 24.6 + 54.4 = 79 ev

Q.9 An X-ray tube with a copper target is found the emit lines other than those due to copper. The \(K_\lambda\) line of copper is known to have a wavelength 1.5405 Å and the other two \(K_\lambda\) lines observed have wavelengths 0.7092 Å and 1.6578 Å. (Identify the impurities (find the value of Z, atomic number.).

**Sol.**

According to Moseley's equation for \(K_\lambda\) radiation

\[
\frac{1}{\lambda} = R(Z - 1)^2 \left[ \frac{1}{1^2} - \frac{1}{2^2} \right] \text{ where } \lambda = \text{wavelength of copper}
\]

Let \(\lambda_1\) and \(\lambda_2\) be the two other unknown wavelengths, then

\[
\frac{\lambda_1}{\lambda} = \frac{(Z_1 - 1)^2}{(Z_1 - 1)^2} = \frac{0.7092}{1.5405}
\]

Solving we get \(Z_1 = 42\)

Similarly

\[
\frac{\lambda_2}{\lambda} = \frac{(28)^2}{(28 - 1)^2} = \frac{1.6578}{1.5405}
\]

Solving we get \(Z_2 = 28\)
Q.10 When 0.50 Å X-rays strike a material, the electrons from the k shell are observed to move in a circle of radius 23 mm in a magnetic field of $2 \times 10^2$ T. What is the binding energy of K-shell electrons?

Sol. The velocity of the photoelectrons is found from $F = ma$:

$$e\nu B = m\frac{\nu^2}{R} \quad \text{or} \quad \nu = \frac{e}{m} BR$$

The kinetic energy of the photoelectrons is then

$$K = \frac{1}{2} mu^2 = \frac{1}{2} \frac{e^2 B^2 R^2}{m}$$

$$= \frac{1}{2} \frac{(1.6510^{-10}\text{C})^2(2 \times 10^{-2}\text{T})^2(23 \times 10^{-3}\text{m})^2}{(9.1 \times 10^{-31}\text{kg})} = 2.97 \times 10^{-15} \text{J}$$

or $K = (2.97 \times 10^{-15} \text{J}) \frac{1\text{keV}}{1.6 \times 10^{-16} \text{J}} = 18.36 \text{ eV}$

The energy of the incident photon is $E_\nu = \frac{hc}{\lambda} = \frac{12.4\text{keV.A}}{0.50\text{Å}} = 24.8 \text{ eV}$

The binding energy is the difference between these two values:

$$BE = E_\nu - K = 24 \text{ keV} - 18.6 \text{ keV} = 6.2 \text{ keV}$$

Q.11 Calculate the wavelength of the emitted characteristic X-ray from a tungsten ($Z = 74$) target when an electron drops from an M shell to a vacancy in the K shell.

Sol. Tungsten is a multiel atom. Due to the shielding of the nuclear charge by the negative charge of the inner core electrons, each electron is subject to an effective nuclear charge $Z_{\text{eff}}$, which is different for different shells.

Thus, the energy of an electron in the $n$th level of a multielectron atom is given by

$$E_n = \frac{13.6Z_{\text{eff}}^2}{n^2} \text{ eV}$$

For an electron in the K shell ($n = 1$), $Z_{\text{eff}} = (Z - 1)$.

Thus, the energy of the electron in the K shell is:

$$E_k = \frac{(74 - 1)^2 \times 13.6}{1} \approx -72500 \text{ eV}$$

For an electron in the M shell ($n = 3$), the nucleus is shielded by one electron of the $n = 1$ state and eight electrons of the $n = 2$ state, a total of nine electrons, so that $Z_{\text{eff}} = Z - 9$. Thus the energy of an electron in the M shell is:

$$E_m = \frac{(74 - 9)^2 \times 13.6}{3^2} \approx -6380 \text{ eV}$$

Therefore, the emitted X-ray photon has an energy given by

$$h\nu = E_m - E_k = -6380 \text{ eV} - (-72500 \text{ eV}) = 66100 \text{ eV}$$
or \[
\frac{hc}{\lambda} = 66100 \times 1.6 \times 10^{-19} \text{ J}
\]

\[
\therefore \quad \lambda = \frac{hc}{66100 \times 1.6 \times 10^{-19} \text{ m}}
\]

\[
= \frac{(6.63 \times 10^{-34}) \times (3 \times 10^8)}{66100 \times 1.6 \times 10^{-19} \text{ m}}
\]

\[
= 0.0188 \times 10^{-4} \text{ m}.
\]

Q.12 If the short series limit of the Balmer series for hydrogen is 3646Å, calculate the atomic no. of the element which gives X-ray wavelength down to 1.0Å. Identify the element.

Sol. The short limit of the Balmer series is given by

\[
\bar{\nu} = \frac{1}{\lambda} = R \left( \frac{1}{2^2} - \frac{1}{n^2} \right) = R/4
\]

\[
\therefore R = 4/\lambda = (4 / 3646) \times 10^{10} \text{ m}^{-1}
\]

Further the wavelengths of the \( k_n \) series are given by the relation

\[
\frac{1}{\lambda} = R (Z - 1)^2
\]

or \[(Z - 1)^2 = \frac{1}{R\lambda} = \frac{3646 \times 10^{10}}{4 \times 1 \times 10^{-16}} = 911.5\]

\[
\therefore (Z - 1) = \sqrt{911.5}
\]

\[
= 30.2
\]

or \[Z = 30.2 \approx 31\]

Thus the atomic number of the element concerned is 31.

The element having atomic number \( Z = 31 \) is Gallium.

Q.13 In a nuclear reactor, fission is produced in 1 gm for \( ^{235} \text{U} \) (235.0439 a.m.u.) in 24 hours by a slow neutron (1.0087 a.m.u.). Assuming that \( ^{35} \text{Kr}^{92} \) (91.8973 a.m.u.) and \( ^{141} \text{Ba} \) (140.9139 a.m.u.) are produced in all reactions and no energy is lost, write the complete reaction and calculate the total energy produced in kilowatt hour. Given 1 a.m.u. = 931 MeV.

Sol. The nuclear fission reaction is

\[
^{235} \text{U} + ^{1} \text{n} \rightarrow ^{141} \text{Ba}^{141} + ^{35} \text{Kr}^{92} + 3^{1} \text{n}
\]

The sum of the masses before reaction

\[
= 235.0439 + 1.0087 = 236.0526 \text{ a.m.u.}
\]

The sum of the masses after reaction

\[
= 140.9139 + 91.8973 + (1.0087) = 235.8375 \text{ a.m.u.}
\]

\[
\therefore \Delta m = 256.0526 - 235.8373 = 0.2153 \text{ a.m.u.}
\]
energy released in the fission of $^{235}\text{U}$ nucleus

$$E = 0.2153 \times 931 = 200 \text{ MeV}$$

Number of atoms in 1 gm

$$= \frac{6.02 \times 10^{23}}{235} = 2.56 \times 10^{21}$$

Energy released in fission of 1 gm of $^{235}\text{U}$

$$E = 200 \times 2.56 \times 10^{2} = 5.12 \times 10^{23} \text{ MeV}$$

$$= (5.12 \times 10^{23}) \times (1.6 \times 10^{-13}) = 8.2 \times 10^{10} \text{ joule}$$

$$= \frac{8.2 \times 10^{10}}{3.6 \times 10^{6}} \text{ kWh} = 2.28 \times 10^{4} \text{ kWh}$$

Q.14 A neutron collides elastically with an initially stationary deuteron. Find the fraction of the kinetic energy lost by the neutron (a) in a head-on collision; (b) in scattering at right angles.

Sol. (a) In a head on collision $\sqrt{2mK} = p_d + p_n$

$$K = \frac{p_d^2}{2M} + \frac{p_n^2}{2m}$$

where $p_d$ and $p_n$ are the momenta of deuteron and neutron after the collision. Squaring

$$p_d^2 + p_n^2 + 2p_dp_n = 2mK$$

$$p_n^2 + \frac{m}{M}p_d^2 = 2mK$$

or since $p_d \neq 0$ in a head on collision

$$p_n = -\frac{1}{2} \left( 1 - \frac{m}{M} \right) p_d$$

Going back to energy conservation

$$\frac{p_d^2}{2M} \left[ 1 + \frac{M}{4m} \left( 1 - \frac{m}{M} \right)^2 \right] = K$$

So

$$\frac{p_d^2}{2M} = \frac{4mM}{(m+M)^2} K$$

This is the energy lost by neutron. So the fraction of energy lost is

$$\eta = \frac{4mM}{(m+M)^2} = \frac{8}{9}$$

(b) In this case neutron is scattered by $90^\circ$. Then we have from the diagram

$$\vec{p}_d = p_n \hat{j} + \sqrt{2mK} \hat{i}$$
Then by energy conservation
\[
\frac{p_n^2}{2M} + \frac{2mK}{2M} = K
\]
or
\[
\frac{p_n^2}{2m} \left(1 + \frac{m}{M}\right) = K \left(1 - \frac{m}{M}\right)
\]
or
\[
\frac{p_n^2}{2m} \left(1 + \frac{m}{M}\right) = K \left(1 - \frac{m}{M}\right)
\]
The energy lost by neutron in then
\[
K - \frac{p_n^2}{2m} = \frac{2m}{M + m} K
\]
or fraction of energy lost is
\[
\eta = \frac{2m}{M + m} = \frac{2}{3}
\]

Q.15 A stationary Pb\(^{200}\) nucleus emits an α-particle with K.E., \(K = 5.77\) MeV. Find the recoil velocity of daughter nucleus. What fraction of the total energy liberated in this decay is accounted for the recoil energy of the daughter nucleus?

Sol. The momentum of the a-particle is given by,
\[
P_a = \sqrt{2m_aK}
\]
...(i)
Let the recoiled momentum of the daughte nucleus be \(P_d = m_d v_d\), where \(m_d\) and \(v_d\) are the mass and velocity of daughter nucleis. Using the principle of conservation of momentum we get,
\[
P_d = P_a = \sqrt{2m_aK}
\]
\[
\Rightarrow V_d = \frac{\sqrt{2m_aK}}{m_d}
\]
...(ii)
\[
\Rightarrow V_d = \frac{1}{196} \sqrt{\frac{2 \times 4 \times K}{m_p}} = \frac{2}{196} \sqrt{\frac{2K}{m_p}}
\]
Where \(m_p\) is the mass of the proton.
\[
\Rightarrow V_d = 3.39 \times 10^4 \text{ m/s}
\]
Let the K.E. of the daughter nucleus be \(K'\) then,
\[
\frac{K'}{K} = \frac{m_a}{m_d}
\]
As the momenta are same
\[ \frac{K'}{K_i} = \frac{m_a}{m_a + m_d} \]

\[ \Rightarrow \quad K' = \frac{m_a}{m_a + m_d} K_i = \frac{4}{196 + 4} K_i \]

\[ \Rightarrow \quad K' = 0.02 K_i \]

\[ \Rightarrow \quad \frac{K'}{K_i} = 0.02 \]

Q.16 A P^{32} radionuclide with half-life \( T = 14.3 \text{ days} \) is produced in a reactor at a constant rate \( q = 2.7 \times 10^9 \text{ nuclei per second.} \) How soon after the beginning of production of that radionuclide will its activity be equal to \( A = 1.0 \times 10^9 \text{ dps?} \)

Sol. Production of the nucleus is governed by the equation

\[ \frac{dN}{dt} = \frac{g}{\text{supply}} - \frac{\lambda}{\text{decay}} N \]

we see that \( N \) will approach a constant value \( N = \frac{g}{\lambda} \). This can also be proved directly. Multiply by \( e^{\lambda t} \) and write.

\[ \frac{dN}{dt} e^{\lambda t} + \lambda e^{\lambda t} N = ge^{\lambda t} \]

Then

\[ \frac{d}{dt} (N e^{\lambda t}) = ge^{\lambda t} \]

or

\[ N e^{\lambda t} = \frac{g}{\lambda} e^{\lambda t} + \text{const.} \]

At \( t = 0 \) when the production is started, \( N = 0 \)

\[ 0 = \frac{g}{\lambda} + \text{constant} \]

Hence

\[ N = \frac{g}{\lambda} (1 - e^{-\lambda t}) \]

Now the activity is

\[ A = \lambda N = g (1 - e^{-\lambda t}) \]

From the problem

\[ \frac{1}{2.7} = 1 - e^{-\lambda t} \]

This gives \( \lambda t = 0.463 \)
so \( t = \frac{0.463}{\lambda} = \frac{0.463 \times T}{0.693} = 9.5 \text{ days} \).

Algebraically \( t = -\frac{T}{\ln 2} \ln \left(1 - \frac{A}{g}\right) \).

Q.17 A dose of 5mCi of P³² (\( t_{1/2} = 14 \text{ days} \)) is administered intravenously to a patient whose blood volume is 3.5 lts. At the end of 1 hour, it is assumed that the phosphorous is uniformly distributed. What would be the count rate/mℓ of the withdrawn blood if the counter measuring the activity had an efficiency of 10%:
(a) 1 hour after injection
(b) 28 days after injection

Sol. Let \( A_0 = \text{initial activity} \quad A_t = \text{activity at time } t \)

According to question

\[ A_0 = \frac{5 \times 10^{-5}}{35 \times 10^3} = 0.143 \times 10^{-5} \text{ Ci/mℓ} \]

and

\[ \lambda = \frac{0.693}{14 \times 24} = 2.06 \times 10^{-3} / \text{Hr} \]

Now using

\[ \lambda t = 2.303 \log \frac{A_0}{A_t} \]

(a) After 1.0 Hr

\[ \frac{2.06 \times 10^{-3} \times 1}{2.303} = \log \frac{0.143 \times 10^{-5}}{A_t} \]

\[ \Rightarrow A_t = 1.42 \times 10^{-6} \text{ Ci/mℓ} \]

\[ \Rightarrow \text{Count rate} = (10/100) \times (1.42 \times 10^{-6}) \times 3.7 \times 10^{10} \text{ dps} = 5280 \text{ dps} \]

(b) After 28 days, i.e., after two half lives (\( t_{1/2} = \text{of } P³² = 14 \text{ days} \));

\[ A_t = A_0 / 4 = 1.42 \times 10^{-6} / 4 \]

\[ \Rightarrow \text{Count rate} = (10/100) \times (1.42 \times 10^{-6} / 4) \times 3.7 \times 10^{10} \text{ dps} = 1322.75 \text{ dps} \]
Q. 18  In the chemical analysis of a rock, the mass ratio of two radioactive isotopes is found to be 100 : 1. The mean lives of the two isotopes are $4 \times 10^9$ and $2 \times 10^9$ years respectively. If it is assumed that at the time of formation the atoms of both the two isotopes were in equal proportion, calculate the age of the rock. Ratio of the atomic weights of two isotopes is $1.02 : 1$.

Sol.  Let two isotopes are A and B

$$\frac{m_A}{m_B} = 100; \quad \frac{A_A}{A_B} = \frac{1.02}{1}$$

$T_A = 4 \times 10^9$ years $T_B = 2 \times 10^9$ years

[Also $\lambda = 1/T$]

Let ratio of nuclei of two isotopes be:

$$\frac{N_{A0}}{N_{B0}} \text{ at } t = 0 \text{ and } \frac{N_{A1}}{N_{B1}} \text{ at } t = 1$$

For isotope A

$$\lambda_A t = 2.303 \log \frac{N_{A0}}{N_{A1}}$$

Similarly for isotope B

$$\lambda_B t = 2.303 \log \frac{N_{B0}}{N_{B1}}$$

On subtracting

$$\left(\lambda_A - \lambda_B\right) t = 2.303 \log \frac{N_{A0} / N_{B0}}{N_{A1} / N_{B1}}$$

$$\Rightarrow (\lambda_A - \lambda_B)t = 2.303 \log \frac{\text{initial ratio}}{\text{final ratio}}$$

$$\Rightarrow \left(\frac{1}{4 \times 10^9} - \frac{1}{2 \times 10^9}\right) t = 2.303 \log \frac{1}{100 / 1.02}$$

$$\Rightarrow \text{age } = t = 1.83 \times 10^{10} \text{ years}$$

Q. 19  A given sample contains two types of atoms A and B in the ratio 3 : 1. Atoms of type A undergo $\alpha$-decay with a half-life of 30 days to form 'B' while atoms of type B undergo $\alpha$-decay with a half-life of 45 days to form 'C', which is stable. Calculate the time after which the activities of A and that of B are in the ratio 9 : 22.

Sol.  The radioactive decay series is given

$A \xrightarrow{T_A = 30 \text{days}} B \xrightarrow{T_B = 45 \text{days}} C$

Initially $N_A(0) : N_B(0) = 3 : 1$
\[
\frac{dN_A}{dt} + \lambda_A N_A = 0
\]
\[
\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B
\]
\[
\frac{dN_C}{dt} = \lambda_B N_B
\]
\[
N_A = N_A(0) e^{-\lambda_A t}
\]
\[
N_B = c_i e^{-\lambda_B t} + \frac{\lambda_A N_A(0) e^{-\lambda_A t}}{-\lambda_A + \lambda_B}
\]

Then we get, \( c_i = \frac{5}{2} N_0 \)

\[
\therefore N_A(t) = \frac{3}{4} N_0 \left( \frac{1}{2} \right)^{t/10\text{days}} = \frac{3}{4} N_0 \left( \frac{1}{2} \right)^{1/30\text{days}}
\]

and \[
N_B(t) = \left[ \frac{5}{2} N_0 \left( \frac{1}{2} \right)^{45\text{days}} - \frac{9}{4} N_0 \left( \frac{1}{2} \right)^{1/30\text{days}} \right]
\]

Now, \[
\frac{\lambda_A N_A}{\lambda_B N_B} = \frac{9}{22} \text{ i.e. } \frac{N_A}{N_B} = \frac{3}{11}
\]

or, \[
\left( \frac{1}{2} \right)^{-1.90} = 2
\]

or, \( t = 90 \text{ days} \)