5) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2-y^2}} \frac{dxdydz}{\sqrt{a^2-x^2-y^2-z^2}}$.

6) Evaluate $\iiint_R xyzdxdydz$ over the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by changing it to spherical polar co-ordinates.

V. Answer **any two** questions. (2×5=10)

1) State and prove Green's theorem in the plane.

2) Evaluate $\iint_S \left(x\hat{i} + y\hat{j} + z^2\hat{k} \right) \cdot \hat{n} ds$, where S is the closed surface bounded by the cone $x^2 + y^2 = z^2$ and the plane $z = 1$, using divergence theorem.

3) Evaluate by Stoke's theorem $\oint_C (\sin z dx - \cos x dy + \sin y dz)$. Where C is the boundary of the rectangle $0 \le x \le \pi,\ 0 \le y \le 1, z = 3$.

VI. Answer **any two** questions. (2×5=10)

1) Prove that the union of any number of open subsets of R^2 is open.

2) Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {a,b}} then show that τ is a topology on X.

3) Let A and B be any two subsets of the topological space X, then prove that

i) If $A \subset B \Rightarrow \overline{A} \subset \overline{B}$

ii) $\left(\overline{A \cup B}\right) = \overline{A} \cup \overline{B}$.

4) Let X = {a, b, c} and τ = {X, ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}} be a topology for X. If β = {{a}, {b}, {c}} then show that β is a base of τ.

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3) Show that $f(z) = e^z$ is analytic and hence show that $f'(z) = e^z$.

4) Find the analytic function whose imaginary part is $e^{-y}(x\sin x + y\cos x)$.

5) Discuss the transformation $w = \sin z$.

6) Find the bilinear transformation which maps $z = 0, -i, -1$ onto $w = i, 1, 0$.

III. Answer **any two** questions. **(2×5=10)**

1) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where $c : |z| = 3$.

2) State and prove Cauchy's integral formula.

3) If a in any positive real number and c is the circle $|z| = 3$, show that $\int_C \frac{e^{2z}}{(z^2+1)^2} dz = \pi i(\sin a - a\cos a)$.

IV. Answer **any four** questions. **(4×5=20)**

1) Evaluate $\int_C \left[3x^2 dx + (2xz - y)dy + zdz\right]$ along the line joining (0, 0, 0) and (2, 1, 3).

2) Evaluate $\iint_R ydxdy$ where R is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

3) Evaluate $\int_0^1 \int_{\sqrt{y}}^{2-y} xydxdy$ by changing the order of integration.

4) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by double integration.