VI Semester B.A./B.Sc. Examination, May/June 2014
(Semester Scheme) (2013-14 and Onwards) (N.S.)
MATHEMATICS (Paper – VII)

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Find the locus of the point z satisfying $|z-1| \leq 4$.

2) Evaluate $\lim_{z \to e^{i\frac{\pi}{4}}} \left(\dfrac{z^2}{z^4 + z^2 + 1} \right)$.

3) Show that f(z) = cos z is an analytic function.

4) Prove that $u = x^3 - 3xy^2$ is a harmonic function.

5) Define bilinear transformation.

6) Evaluate $\int_C (\bar{z})^2 \, dz$ around the circle $|z| = 1$.

7) Evaluate $\int_C \dfrac{\cos \frac{\pi}{3} z}{z-1} \, dz$ where $C : |z| = \dfrac{3}{2}$.

8) State Liouville's theorem.

9) Evaluate $\int_C \left[\left(x^2 - y\right) dx + \left(y^2 + x\right) dy \right]$ where C is the curve given by $x = t,\ y = t^2 + 1,\ 0 \leq t \leq 1$.

10) Show that $\int_0^1 \int_0^{\sqrt{3}} \dfrac{dx\, dy}{\left(1+x^2\right)\left(1+y^2\right)} = \dfrac{\pi^2}{12}$.

11) Evaluate $\int_0^1 \int_0^{\pi/2} r^3 \sin^2 \theta \, d\theta \, dr$.

12) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-\left(x^2+y^2\right)} dx\, dy$ by changing into polar coordinates.

13) Evaluate $\int_0^1 \int_0^2 \int_1^2 x^2yz\, dx\, dy\, dz$.

14) State Green's theorem in the plane.

15) Show that $\iiint_V \text{div}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) dv = 3v$.

16) Using Stokes theorem prove that $\text{div}\left(\text{curl}\, \vec{F}\right) = 0$.

17) Define interior point on topology.

18) State Bolzano-Weierstrass theorem on R.

19) Write all possible topologies for X = {3, 4}.

20) Define sub base for a topology.

II. Answer **any four** questions. **(4×5=20)**

1) Show that the locus of a point z satisfying amp $\left(\frac{z-1}{z+2}\right) = \frac{\pi}{3}$ is a circle. Find its centre and radius.

2) If f(z) = u + iv be an analytic function in the domain D of a complex plane then $u = c_1$ and $v = c_2$. Where c_1 and c_2 are constants represents orthogonal family of curves.

3) Find the analytic function whose imaginary part is $\tan^{-1}\frac{y}{x}$ and hence find its real part.

4) Find the analytic function f(z) = u + iv given that $u - v = e^x(\cos y - \sin y)$.

5) Discuss the transformation $w = z^2$.

6) Find the bilinear transformation which maps the points 1, –i, –1 on to the points 0, i, ∞.

III. Answer **any two** questions. (2×5=10)

1) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$ where C is the circle $|z| = 4$.

2) Show that $\oint_C \frac{e^{2z}}{(z-2)^3} dz = 4\pi i e^4$ where C is the circle $|z| = 3$.

3) State and prove the fundamental theorem of algebra in complex variables.

IV. Answer **any four** questions : (4×5=20)

1) Evaluate $\iint_R xy(x+y)\, dx\, dy$ over the domain D between $y^2 = x$ and $y = x$.

2) Evaluate $\int_0^1 \int_y^1 (x^2 + y^2)\, dx\, dy$ by changing the order of integration.

3) Show that $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dy\, dx = \frac{3\pi a^4}{4}$ by changing into polar coordinates.

4) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \int_0^{\sqrt{a^2 - x^2 - y^2}} \frac{dx\, dy\, dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$.

5) Find the surface area of the sphere $x^2 + y^2 + z^2 = a^2$ by using double integration.

6) Evaluate $\iiint_R xyz\, dx\, dy\, dz$ where R is the positive octant of the sphere $x^2 + y^2 + z^2 = a^2$ by transforming in to cylindrical polar coordinates.

V. Answer **any two** questions. (2×5=10)

1) Using Green's theorem evaluate

$\int_C \left[e^{-x} \sin y\, dx + e^{-x} \cos y\, dy\right]$ where C is the rectangle with vertices (0, 0), $\left(0, \frac{\pi}{2}\right), \left(\pi, \frac{\pi}{2}\right), (\pi, 0)$.

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3) Show that $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\, dy\, dx = \frac{3\pi a^4}{4}$ by changing into polar coordinates.

4) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \int_0^{\sqrt{a^2 - x^2 - y^2}} \frac{dx\, dy\, dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$.

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3) Show that $\int_0^{2a} \int_0^{\sqrt{2ax - x^2}} (x^2 + y^2)\,dy\,dx = \frac{3\pi a^4}{4}$ by changing into polar coordinates.

4) Evaluate $\int_0^a \int_0^{\sqrt{a^2 - x^2}} \int_0^{\sqrt{a^2 - x^2 - y^2}} \frac{dx\,dy\,dz}{\sqrt{a^2 - x^2 - y^2 - z^2}}$.

5) Find the surface area of the sphere $x^2 + y^2 + z^2 = a^2$ by using double integration.

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V. Answer **any two** questions. **(2×5=10)**

1) Using Green's theorem evaluate

$\int_C \left[e^{-x} \sin y\,dx + e^{-x} \cos y\,dy\right]$ where C is the rectangle with vertices (0, 0), $\left(0, \frac{\pi}{2}\right)$, $\left(\pi, \frac{\pi}{2}\right)$, $(\pi, 0)$.

2) State and prove the Gauss divergence theorem.

3) Verify Stoke's theorem for the function $\vec{F} = y^2\hat{i} + xy\hat{j} - xz\hat{k}$, where S is the hemisphere $x^2 + y^2 + z^2 = a^2$, $z \geq 0$.

VI. Answer **any two** questions. **(2×5=10)**

1) Prove that the union of an arbitrary collection of open sets is open.

2) Define topological space. Let $X = \{m, n\}$ and $\tau = \{X, \phi, \{m\}, \{n\}\}$ then show that τ is a topology on X.

3) If (X, τ) be a topological space and $A, B \subset X$ then prove that

i) $A \subset B \Rightarrow A^\circ \subset B^\circ$

ii) $(A \cap B)^\circ = A^\circ \cap B^\circ$.

4) Show that every convergent sequence is a Cauchy sequence.

VI Semester B.A./B.Sc. Examination, May/June 2014
(Semester Scheme) (N.S.) (2013-14 & Onwards)
MATHEMATICS – VIII

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) A particle is moved by a force $3\hat{i} - 4\hat{j} - 6\hat{k}$ along a straight line from a point A to B with position vectors $2\hat{i} - 3\hat{j} + 4\hat{k}$ and $4\hat{i} + 5\hat{j} - 6\hat{k}$ – find the work done.

2) A particle starting and executing SHM with period 6 seconds travels 12 meters in 2 seconds. Find the amplitude of the particle.

3) A cricket ball is thrown with a velocity of 30 mts/sec, find the greatest range on horizontal plane.

4) Find the velocity of projection of a particle when the horizontal range 12 ft and elevation is 15°.

5) Mention the equation of motion when a particle moves inside a smooth verticle circle.

6) Define apsidal distance.

7) Derive the law of force for a particle describing the central orbit. Whose pedal equation is $pr = a^2$?

8) Write the formula for transverse velocity and transverse acceleration.

9) Define variational problem.

10) Obtain the differential equation of the variation problem $\int_{x_1}^{x_2} [y^1(1 + x^2 y^1] \, dx$.

P.T.O.

11) Show that the Eulers equation for the extremum of $\int_{x_1}^{x_2}[y^2+(y^1)^2+2y\,e^x]\,dx$ reduce to $y''-y=e^x$.

12) Show that the functional $\int_{x_1}^{x_2}[y^2+x^2y^1]\,dx$ assumes extreme values on the straight line $y = x$.

13) Find the positive root of the equation $x^3-3x-5=0$ which lies between 2 and 2.5 by bisection method (use one approximation).

14) Find the first approximation root of $x^3-2x-5=0$ lying between 2 and 3 by Regula-Falsi method.

15) Find the real root of the equation $x^3-x-2=0$ over interval (1.5, 2) upto two approximation by bisection method.

16) Find the largest eigen value of $\begin{pmatrix}-4 & -5\\ 1 & 2\end{pmatrix}$ by power method.

17) Solve $\frac{dy}{dx}=x+y^2$, $y(0)=1$ By Picards method upto first approximation. Find the value of $y(0.1)$.

18) Write the Tayler's series formula for the numerical solution of the differential equation $\frac{dy}{dx}=t(x,y)$ with intial condition $y(x_0)=y_0$.

19) Solve : $y_{x+2}-2y_{x+1}+y_x=0$ by the method of differences.

20) Solve : $y_{x+1}-y_x=x^2$.

II. Answer **any three** of the following : **(3×5=15)**

1) Show that $\vec{F}=(x^2y-2^3)\hat{i}+(3xyz+xz^2)\hat{j}+(2x^2yz+yz^4)\hat{k}$ is not conservative.

2) At the end of three consecutive seconds the distances of a partile moving with SHM from its mean position are x_1, x_2 and x_3. Show that the time for one complete oscillation is $\frac{2\pi}{\cos^{-1}\left(\frac{x_1 + x_3}{2x_2}\right)}$.

3) Show that the path traced by a projectile is a parabola.

4) A particle is thrown over a triangle form one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A and B be the base angles of the triangle and α the angle of projection show that $\tan \alpha = \tan A + \tan B$.

III. Answer **any two** of the following : **(2×5=10)**

1) Derive with usual notation for a central orbit $\frac{d^2u}{d\theta^2} + u = \frac{f}{h^2u^2}$.

2) If the central orbit is $r = a\tan\theta$ show that the magnitude of acceleration towards the centre of force is $h^2u^3(3 + 2a^2u^2)$ also find the velocity interms of r.

3) A particle describes the curve $r^2 = a^2\sin 2\theta$ under a force to the pole. Find the law of force.

4) A particle describes a cycloid with uniform speed. Show that the normal acceleration at any point varies inversly as the square root of the distance from the base of the cycloid.

IV. Answer **any three** of the following : **(3×5=15)**

1) Prove that necessary condition for the integras $I = \int_{x_1}^{x_2} f(x, y\, y^1)\, dx$ where $y(x_1) = y_1$ and $y(x_2) = y_2$ to be an extremum is that $\frac{\partial x}{\partial y} - \frac{d}{dx}\left(\frac{\partial x}{\partial y^1}\right) = 0$.

2) Find the extremal of the function $\int_1^2 [x^2(y^1)^2 + 2y(x+y)]\, dx = 0$ given that $y(1) = y(2) = 0$.

3) Show that geodesic of a sphere of radus a are its greatest circle.

4) Find the extremal of the functional $\int_0^1 [(y^1)^2 + x^2]\, dx$ subject to the constraint

$\int_0^1 y\, dx = 2$ and having end conditions $y(0) = 0$, $y(1) = 1$.

V. Answer **any three** of the following : **(3×5=15)**

1) Solve $x^3 + 4x + 1 = 0$ for the real root lying between 2 and 3 by Regula – Falsi method.

2) Use Newton – Raphson method to find a real root of the equation $x^3 - 2x - 5 = 0$ correct to three decimal places.

3) Solve the equations $10x + y + z = 12$; $2x + 10y + z = 13$; $2x + 2y + 10z = 1y$.

Using Zacobian method correct to three decimal places.

4) Find the greatest eigen value of the matrix $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ by power method.

VI. Answer **any three** of the following : **(3×5=15)**

1) Use Taylors series method to find $\frac{dy}{dx} = x^2 + y^2$ given $y(0) = 1$ for $x = 0.1, 0.2$ considering terms up to 3rd degree.

2) Solve : $\frac{dy}{dx} = y - x^2$, $y(0) = 1$ by Picards method upto 3rd approximation.

3) Solve using Runge – Kutta method $\frac{dy}{dx} = x + y$, $y(0) = 1$ for $x = 0(.2).4$.

OR

Form the difference equation by eliminating a and b from the relation $y_n = a\cos n\alpha + b\sin n\alpha$.

4) Solve the difference equation

$(E^3 - 5E^2 + 3E + 9)y_n = 2^n + 3^n$.

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