III. Answer **any two** of the following : **(2×5=10)**

1) Derive the expression for radial and transverse velocity.
2) A particle describes the curve $2r = a(1 + \cos\theta)$ under the central force towards the pole. Find the law of force.
3) Derive the expression for the velocity of the particle at any point on the central orbit in the form $v^2 = h^2\left[\left(\frac{du}{d\theta}\right)^2 + u^2\right]$.
4) The component of velocity of a particle moving along a plane curve along and perpendicular to the radius vector to a fixed point are λr^n and $\mu\theta^n$ respectively, where λ and μ are constants. Derive the equation of the curve.

IV. Answer **any three** of the following : **(3×5=15)**

1) Find the extremal of the functional $I = \int_1^2 \left[\frac{\sqrt{1+(y')^2}}{x}\right] dx$ given $y(1) = 0$ and $y(2) = 1$.
2) Show that the general solution of Euler's equation for the integral $\int_a^b \frac{1}{y}\left[\sqrt{1+(y')^2}\right] dx$ is $(x - B)^2 + y^2 = R^2$.
3) Show that the geodesics on a right circular cylinder is a circular helix.
4) Show that extremal of the functional $\int_0^2 \left[\sqrt{1+(y')^2}\right] dx$ subject to the constraint $\int_0^2 y\,dx = \frac{\pi}{2}$ and end conditions $y(0) = 0$, $y(2) = 2$ is a circular arc.

V. Answer **any three** of the following : **(3×5=15)**

1) Using bisection method find the real root of the equation $x^3 - 5x + 3 = 0$ lying between 0 and 1 correct to 3 places of decimals.
2) Solve the equation $x^3 - 4x + 1 = 0$ over (0, 1) by Regula-Falsi method.

3) Solve the equations $20x + y - 2z = 17$; $3x + 20y - z = -18$;

$2x - 3y + 20z = 25$ by Jacobi iteration method.

4) Find the largest eigen value of the matrix $\begin{pmatrix} -4 & -5 \\ 1 & 2 \end{pmatrix}$ by power method.

VI. Answer **any three** of the following : (3×5=15)

1) Use Taylor's series method to find y at $x = 0.1$ considering terms upto third degree terms given $\frac{dy}{dx} = x^2 + y^2$ and $y(0) = 1$.

2) Solve $\frac{dy}{dx} = x + y$, $y(0) = 1$ for $x = 0.025$ by Picard's method upto 3rd approximation.

3) Solve $\frac{dy}{dx} = xy$; $y(1) = 2$, find the approximate solution at $x = 1.2$ using Runge-Kutta method.

OR

Form the difference equation by eliminating the arbitrary constants 'a' and 'b' $y_x = a.2^x + b.3^x$.

4) Solve the difference equation $(E^2 - 8E + 15)\, y_n = 3^n + e^{3n}$.

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VI Semester B.A./B.Sc. Examination, May 2016
(Semester Scheme) (NS) (F + R)
(2013 – 14 and Onwards)
MATHEMATICS – VIII

Time : 3 Hours Max. Marks : 100

Instruction : *Answer **all** questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Find the kinetic energy of the particle of mass 10 units moving with velocity $2\hat{i}-3\hat{j}+4\hat{k}$.

2) In a simple harmonic motion, if 'f' is the acceleration, 'v' the velocity at any instant and 'T' is the period, show that $f^2T^2+4\pi^2v^2$ is a constant.

3) Find the maximum horizontal range when the velocity of projection is 14 mtrs/sec.

4) Write the condition for the particle to strike the inclined plane at right angles.

5) A point describes the cycloid $s = 4a\sin(\psi)$ with uniform speed, show that its acceleration at any point is $\dfrac{v^2}{\sqrt{16a^2-s^2}}$.

6) A particle moves along a curve so that its tangential and normal acceleration are equal. Find its velocity.

7) Write the expressions for radial acceleration and transverse acceleration.

8) Define apsidal distance.

9) If $I=\int_{x_1}^{x_2} f(x, y, y')dx$ then prove that $\delta\int_{x_1}^{x_2} f(x, y, y')dx=\int_{x_1}^{x_2}\delta f(x, y, y')dx$.

10) State Euler's equation.

11) Define geodesic on a curve.

12) Find the differential equation of the functional $I=\int_0^{\pi/2}\left[y^2-(y')^2-2y\sin x\right]dx$.

P.T.O.

13) Use bisection method in two stages to obtain the root of the equation $x^3 - 2x - 5 = 0$.

14) Find the real root of the equation $x^3 - 2x - 5 = 0$ by Regula-Falsi method (Only First approximate value).

15) Find the real root of the equation $x^4 - x - 10 = 0$ which is near to $x = 2$ by Newton-Raphson method (only two approximate values).

16) Solve the system of equations

$$x + y + 54z = 110$$
$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$

by Gauss-Seidal method (only one set of solution).

17) Using Euler's method, solve

$\frac{dy}{dx} = x + y$, $y(0) = 1$ for $x = 0.0\ (0.2)\ 1.0$ (only two approximate values).

18) Find the order and degree of the difference equation $y_{x+2} - 7y_{x+1} + 2y_x = 0$.

19) Form the difference equation by eliminating 'a' from the relation $y_n = a.3^n$.

20) Solve $y_{x+2} - 2y_{x+1} + y_x = 0$ by the method of differences.

II. Answer **any three** of the following : **(3x5=15)**

1) Show that $\vec{F} = (y^2 z^3 - 6xz^2)\,\hat{i} + 2xyz^3\,\hat{j} + (3xy^2z^2 - 6x^2z)\,\hat{k}$ is a conservative force field. Find the work done by $\vec{F}$ in moving a particle from $(-2, 1, 3)$ to $(1, -2, -1)$.

2) A particle executing simple harmonic motion along a line, travels a distance 'a' in the first second after starting from rest and in the next second it travels a distance 'b'. Show that the amplitude of the motion is $\frac{2a^2}{3a-b}$.

3) If 'R' be the range of a projectile on the horizontal plane and 'h' its maximum height for a given angle of projection. Show that the maximum horizontal range with the same velocity of projection is $\left[2h + \frac{R^2}{8h}\right]$.

4) A particle is projected with velocity 80 ft/sec. at an angle of 45° to the horizontal. Find its range on the plane inclined at an angle of 30° to the horizontal when projected upto the plane.