



US – 353

IV Semester B.A./B.Sc. Examination, May 2017
(CBCS) (Fresh + Repeaters) (2015 – 16 and Onwards)
MATHEMATICS (Paper – IV)

Time : 3 Hours

Max. Marks : 70

Instruction : Answer all Parts.

PART – A

Answer any five questions.

(5×2=10)

1. a) Define normal subgroup of a group.
- b) If $f: (G, o) \rightarrow (G', *)$ is a homomorphism then prove that $f(e) = e'$ where e and e' are the identity elements of G and G' respectively.
- c) Calculate a_0 in the Fourier series of $f(x) = e^x$ in $(-\pi, \pi)$.
- d) Write Taylor's series of the function $f(x, y)$ about the point (a, b) .
- e) Find $L[1 - 2e^{3t}]$.
- f) Find $L^{-1}\left[\frac{s-1}{(s-1)^2+9}\right]$.
- g) Find the particular integral of $(D^2 + 1)y = \sin 3x$.
- h) Reduce the equation $y_2 - 2 \tan x y_1 + 5y = 0$ to normal form.

PART – B

Answer one full question.

(1×15=15)

2. a) If $f: (Z, +) \rightarrow (2Z, +)$ is defined by $f(x) = 2x, \forall x \in Z$, then show that f is an isomorphism.
- b) If $f: G \rightarrow G'$ be an isomorphism of a group G onto G' then prove that $\text{Ker} f = \{e\}$ if and only if f is one-one.

P.T.O.



- c) If H is a subgroup of G and K is a normal subgroup of G then prove that $H \cap K$ is a normal subgroup of G .

OR

3. a) If H is a normal subgroup of G then prove that G/H is a group w.r.t. the binary operation defined by

$$H_a \cdot H_b = H_{ab}, \forall H_a, H_b \in G/H.$$
- b) If $f: G \rightarrow G'$ be a homomorphism of a group G onto G' with Kernel K , then prove that K is a normal subgroup of G .
- c) Show that the mapping $f: (R, +) \rightarrow (R^+, \bullet)$ defined by $f(x) = e^x, \forall x \in R$, is an isomorphism. (R = set of reals and R^+ = set of positive reals).

PART – C

Answer **any two** full questions.

(2×15=30)

4. a) Obtain the Fourier series of $f(x) = x^2$ in $(-\pi, \pi)$.
- b) Find the half range cosine series of $f(x) = 2x - 1$ in $(0, 2)$.
- c) Expand e^x in powers of x and y upto second degree terms.

OR

5. a) Find the extreme values of the function $f(x, y) = xy(1 - x - y)$.
- b) A rectangular box open at the top is to have a volume of 32 cubic feet. Find the dimension of the box requiring least material for its construction.
- c) Find the Fourier series of $f(x) = 1 - x^2$ in $(-1, 1)$.

6. a) i) Prove that $L[e^{at}] = \frac{1}{s - a}$.

ii) Find $L[\cosh(t) \cdot \cos(2t)]$.

- b) Express $f(t) = \begin{cases} t^2, & 0 < t < 2 \\ 6, & t > 2 \end{cases}$ in terms of unit step function and find $L[f(t)]$.



c) Find $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$.

OR

7. a) Find $L\left[\frac{e^{-at} - e^{-bt}}{t}\right]$.

b) Using convolution theorem find $L^{-1}\left[\frac{1}{(s^2+1)(s-1)}\right]$.

c) Find $L^{-1}\left[\frac{s^2+3}{(s-1)^2(s+2)}\right]$.

PART - D

Answer **one full** question.

(1×15=15)

8. a) Solve : $(D^2 - 2D + 1)y = \sinh(x)$.

b) Solve : $x^2y_2 - 2x(x+1)y_1 - 2(x+1)y = x^3$ given that x is a part of complementary function.

c) Solve : $(D^2 + 2D + 4)y = e^x \sin x$.

OR

9. a) Solve : $x^2 \frac{d^2y}{dx^2} + 2x \frac{dy}{dx} = \sin(\log x)$.

b) Solve : $\frac{dx}{dt} = 3x - 4y, \frac{dy}{dt} = x - y$.

c) Solve : $\frac{d^2y}{dx^2} + 9y = \sec 3x$ by the method of variation of parameters.
