SN – 249

I Semester B.A./B.Sc. Examination, Nov./Dec. 2014
(2014-15 and Onwards) (Semester Scheme) (CBCS) (Fresh)
MATHEMATICS – I

Time : 3 Hours Max. Marks : 70

***Instruction**: Answer **all** questions.*

PART – A

1. Answer **any five** questions : **(5×2=10)**

a) Define :

i) Equivalent matrices

ii) Row reduced Echelon form of a matrix.

b) Find the value of λ, for which the system $3x + y - \lambda z = 0$; $4x - 2y - 3z = 0$; $2\lambda x + 4y + \lambda z = 0$ has a non-trivial solution.

c) Find the n^{th} derivative of $\sin^2 x$.

d) If $z = \cos^{-1}(xy)$ prove that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$.

e) Using reduction formula, evaluate $\int \sin^3 x dx$.

f) Evaluate $\int_0^{\pi} x \cos^4 x dx$.

g) Show that the planes $x + 2y - 3z + 4 = 0$ and $4x + 7y + 6z + 2 = 0$ are perpendicular.

h) Write the condition for co-planarity of two lines.

P.T.O.

PART – B

Answer **any one full** question. **(1×15=15)**

2. a) Find the rank of the matrix
$$\begin{bmatrix} 2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$$

b) Solve completely the system equations $x + 3y - 2z = 0;\ 2x - y + 4z = 0;\ x - 11y + 14z = 0.$

c) Verify Cayley-Hamilton's theorem and hence find inverse of the matrix
$$\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}.$$

OR

3. a) Reduce the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ into normal form and hence find the rank.

b) Show that the following system of equations is consistent and solve them
$x + 2y + 2z = 1;\ \ 2x + y + z = 2;\ 3x + 2y + 2z = 3;\ \ y + z = 0.$

c) Find the eigen values and the corresponding eigen vectors of the matrix
$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix}.$$

PART – C

Answer **any two full** questions. **(2×15=30)**

4. a) Find the n^{th} derivative of $\frac{4x}{(x+1)^2(x-1)}$.

b) State and prove Leibnitz's theorem.

c) If $y = \sin^{-1}x$ show that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2y_n = 0.$

OR

5. a) If $z = \sin(ax + y) + \cos(ax - y)$ prove that $\frac{\partial^2 z}{\partial x^2} = a^2 \frac{\partial^2 z}{\partial y^2}$.

b) State and prove Euler's theorem for homogeneous functions.

c) Find the total derivative of u w.r.t 't' where $u = e^x \sin y$ where $x = \log t$, $y = t^2$.

6. a) If $u = f(r)$ where $r^2 = x^2 + y^2 + z^2$ prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2}{r} f'(r)$.

b) If $u = z - x$, $v = y - z$ and $w = x + y + z$ find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$.

c) Obtain the reduction formula for $\int \sin^n x dx$ where n is a positive integer.

OR

7. a) Obtain the reduction formula for $\int \tan^n x dx$.

b) Evaluate :

i) $\int_0^2 x^{\frac{5}{2}} \sqrt{2 - x}\, dx$

ii) $\int_0^\pi x \sin^5 x . \cos^4 x dx$.

c) Evaluate $\int_0^1 \frac{x^a - 1}{\log x} dx$.

Where a is a parameter.

PART – D

Answer **any one full** question. **(1×15=15)**

8. a) Find the equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 4$, $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$.

b) Obtain the condition for co-planarity of two lines in both vector and Cartesian forms.

c) Find the equation of the right circular cone through the point (2, 1, 3) with vertex at the point (1, 1, 2) and axis parallel to the line

$$\frac{x-2}{2}=\frac{y-1}{-4}=\frac{z+2}{3}.$$

OR

9. a) Find the shortest distance between the skew lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-6}{4}=\frac{z-5}{5}$

b) Show that the plane $2x + 3y + 4z = 58$ touches the sphere $x^2 + y^2 + z^2 - 4x - 6y - 8z = 0$ and find the point of contact.

c) Find the equation of the right circular cylinder whose axis is the line $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$ and whose radius is 'r'.

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