V Semester B.A./B.Sc. Examination, November/December 2017
(Semester Scheme) (Repeaters – Prior to 2016 – 17)
(NS – 2013 – 14 and Onwards)
MATHEMATICS – VI

Time : 3 Hours Max. Marks : 100

Instruction : *Answer* ***all*** *questions.*

I. Answer **any fifteen** questions. **(15×2=30)**

1) Solve $(ydx + xdy)(a - z) + xy \cdot dz = 0$.

2) Verify the condition for integrability

$(yz + 2x)\,dx + (zx - 2z)\,dy + (xy - 2y)\,dz = 0$.

3) Eliminate the arbitrary constants a and b from the equation

$Z = (x - a)^2 + (y - b)^2$ and form the partial differential equation.

4) Solve $p\tan x + q\tan y = \tan z$.

5) Solve $p^2 - q^2 = x - y$.

6) Solve $(D^2 + DD^1 - 6{D^1}^2)\,Z = 0$.

7) Using Rodrigue's formula, obtain expression for $P_0(x)$ and $P_1(x)$.

8) Express the polynomial $2x - 3x^2$ in terms of Legendre polynomials.

9) Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$.

10) Show that $J_0'(x) = -J_1(x)$.

11) Express $J_4(x)$ in terms of J_0 and J_1.

12) Show that $\Delta \log f(x) = \log\left(1+\frac{\Delta f(x)}{f(x)}\right)$.

13) Express $f(x) = 3x^2 + 2x - 5$ in factorial form.

14) Construct Newton's divided difference table for the following :

x	1	3	6	11
f(x)	4	32	224	1344

15) Write the Newton's backward interpolation formula.

16) Evaluate $\int_0^6 \frac{dx}{1+x}$ by using Trapezoidal rule.

17) A population grows at the rate of 5% per year. How long does it take for the population to be doubled ?

18) Explain :

i) Deterministic

ii) Stochastic mathematical models.

19) How long does it take for a given amount of money to double at 10% per annum compounded annually ?

20) Define mathematical modelling and give example.

II. Answer **any four** questions. **(4×5=20)**

1) Verify the condition for integrability and solve
$3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})\, dz = 0$.

2) Solve $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$.

3) Form the partial differential equation given that $f(x + y + z, x^2 + y^2 - z^2) = 0$.

4) Find the complete integral of $p(1 + q^2) + (b - z)q = 0$.

5) Solve $(D^2 - 2DD^1 - D^{1^2})\, Z = e^{x+2y}$.

6) A lightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. If it is released from rest in this position, find the displacement $u(x, t)$.

OR

Solve $z^2(p^2 + q^2 + 1) = 1$.

III. Answer **any three** questions. **(3×5=15)**

1) Prove that $(n + 1)\, P_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x)$.

2) Prove that $\int_{-1}^{1} P_m(x).P_n(x)dx = \begin{cases} 0 & \text{if } m \neq n \\ \frac{2}{2n+1} & \text{if } m = n \end{cases}$ by Legendre polynomials.

3) Derive Rodrigue's formula for Legendre polynomial.

4) Show that

i) $\cos(x\cos\theta) = J_0 - 2\cos 2\theta\, J_2 + 2\cos 4\theta\, J_4 + ...$

ii) $\sin(x\cos\theta) = 2[J_1\cos\theta - J_3\cos 3\theta + J_5\cos 5\theta + ...]$.

5) Prove that $J_{\frac{-5}{2}}(x) = \sqrt{\frac{2}{\pi x}}\left[\frac{3}{2}\sin x + \frac{(3-x^2)}{x^2}\cos x\right]$.

IV. Answer **any four** questions. **(4×5=20)**

1) If $u_4 = 25$, $u_6 = 49$, $u_8 = 81$, find the value of u_5 and u_7.

2) By separation of symbols prove $u_x = u_{x-1} + \Delta u_{x-2} + \Delta^2 u_{x-3} + ... + \Delta^n u_{x-n}$.

3) Given that $\sin 45° = 0.7071$, $\sin 50° = 0.7660$, $\sin 55° = 0.8192$, $\sin 60° = 0.8660$, find $\sin 52°$ using Newton-Gregory forward interpolation formula.

4) The following table gives the normal weights of bodies during the first few months of life.

Age in months	2	5	8	10	12
Weight in kgs	4.4	6.2	6.7	7.5	8.7

Estimate by Lagrange's method, the normal weight of a baby of 7 months old.