

Subject: MATHEMATICS
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(CHAPTER 1 TO 7)

IMPORTANT SNAPS BY TEAM PIS
CLASS- X TH

HIGHLIGHTS OF CHAPTER 1

NUMBER SYSTEM

Real Numbers

- Real numbers constitute the union of all rational and irrational numbers.
- Any real number can be plotted on the number line.

Fundamental Theorem of Arithmetic

- The Fundamental Theorem of Arithmetic states that the prime factorisation for a given number is unique if the arrangement of the prime factors is ignored.
- Example: $36=2 \times 2 \times 3 \times 3$ OR, $36=2 \times 3 \times 2 \times 3$
- Therefore, 36 is represented as a product of prime factors (Two 2s and two 3s) ignoring the arrangement of the factors.

Method of Finding LCM

- Example: To find the Least Common Multiple (L.C.M) of 36 and 56,

$$36=2 \times 2 \times 3 \times 3$$

$$56=2 \times 2 \times 2 \times 7$$

The common prime factors are 2×2

The uncommon prime factors are 3×3 for 36 and 2×7 for 56.

LCM of 36 and 56 = $2 \times 2 \times 3 \times 3 \times 2 \times 7$ which is 504

Method of Finding HCF

- H.C.F can be found using two methods - Prime factorisation and Euclid's division algorithm.
- Prime Factorisation:
 - Given two numbers, we express both of them as products of their respective prime factors. Then, we select the prime factors that are common to both the numbers
 - Example - To find the H.C.F of 20 and 24
 $20=2 \times 2 \times 5$ and $24=2 \times 2 \times 2 \times 3$
 - The factor common to 20 and 24 is 2×2 , which is 4, which in turn is the H.C.F of 20 and 24.

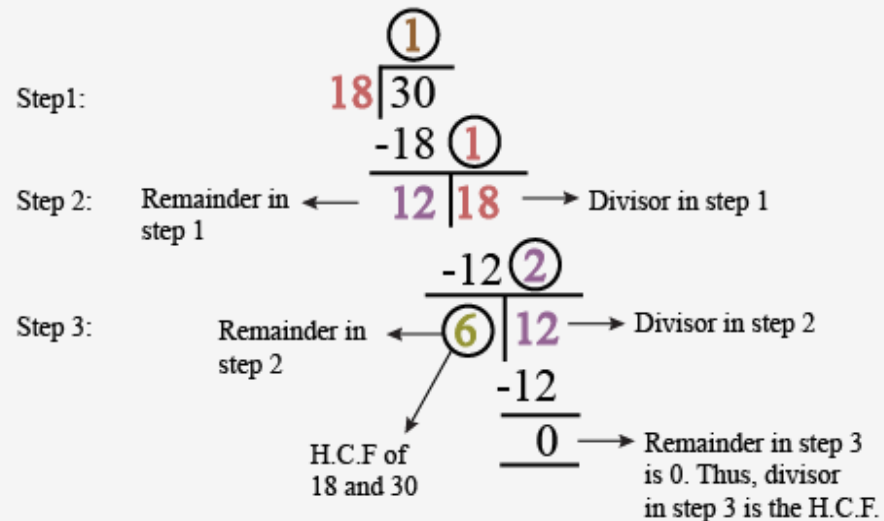
❖ Euclid's Division Algorithm:

It is the repeated use of Euclid's division lemma to find the H.C.F of two numbers.

Example: To find the HCF of 18 and 30

Finding the HCF of 18 and 30

The required HCF is 6.



Product of Two Numbers = HCF X LCM of the Two Numbers

- For any two positive integers a and b,
 $a \times b = \text{H.C.F} \times \text{L.C.M}$.
- Example - For 36 and 56, the H.C.F is 4 and the L.C.M is 504
 $36 \times 56 = 2016$
 $4 \times 504 = 2016$
Thus, $36 \times 56 = 4 \times 504$

The above relationship, however, doesn't hold true for 3 or more numbers

Applications of HCF & LCM in Real-World Problems

- L.C.M can be used to find the points of common occurrence. For example, ringing of bells that ring with different frequencies, the time at which two persons running at different speeds meet, and so on.

Revisiting Irrational Numbers

Irrational Numbers

- Any number that cannot be expressed in the form of p/q (where p and q are integers and $q \neq 0$.) is an irrational number. Examples $\sqrt{2}$, π , e and so on.

Number theory: Interesting results

- If a number p (a prime number) divides a^2 , then p divides a .
Example: 3 divides 6^2 i.e 36, which implies that 3 divides 6.
- The sum or difference of a rational and an irrational number is irrational
- The product and quotient of a non-zero rational and irrational number are irrational.
- \sqrt{p} is irrational when ' p ' is a prime. For example, 7 is a prime number and $\sqrt{7}$ is irrational. The above statement can be proved by the method of "Proof by contradiction".

Proof by Contradiction

- ◉ In the method of contradiction, to check whether a statement is TRUE

(i) We assume that the given statement is TRUE.

(ii) We arrive at some result which contradicts our assumption, thereby proving the contrary.

Eg: Prove that $\sqrt{7}$ is irrational.

Assumption: $\sqrt{7}$ is rational.

Since it is rational $\sqrt{7}$ can be expressed as

$\sqrt{7} = a/b$, where a and b are co-prime Integers, $b \neq 0$.

On squaring, $a^2/b^2=7$

$\Rightarrow a^2=7b^2$.

Hence, 7 divides a . Then, there exists a number c such that $a=7c$.

Then, $a^2=49c^2$. Hence, $7b^2=49c^2$ or $b^2=7c^2$.

Hence 7 divides b . Since 7 is a common factor for both a and b , it contradicts our assumption that a and b are coprime integers.

Hence, our initial assumption that $\sqrt{7}$ is rational is wrong.

Therefore, $\sqrt{7}$ is irrational.

Rational Numbers

- Rational numbers are numbers that can be written in the form p/q , where p and q are integers and $q \neq 0$.
Examples $-1/2$, $4/5$, 1 , 0 , -3 and so on.

Terminating and non-terminating decimals

- *Terminating* decimals are decimals that end at a certain point.
Example: 0.2 , 2.56 and so on.
Non-terminating decimals are decimals where the digits after the decimal point don't terminate. Example: $0.333333\dots$,
 $0.13135235343\dots$
Non-terminating decimals can be :
 - a) Recurring - a part of the decimal repeats indefinitely
($0.142857142857\dots$)
 - b) Non-recurring - no part of the decimal repeats indefinitely.
Example: $\pi = 3.1415926535\dots$

EXTRA QUESTIONS

- **Q: Given that HCF (306, 657) = 9, find LCM (306, 657).**
- Solution: As we know that,
- $\text{HCF} \times \text{LCM} = \text{Product of the two given numbers}$
- Therefore,
- $9 \times \text{LCM} = 306 \times 657$
- $\text{LCM} = (306 \times 657)/9 = 22338$
- Hence, $\text{LCM}(306,657) = 22338$
- **Q.: Prove that $3 + 2\sqrt{5}$ is irrational.**
- Solution: Let $3 + 2\sqrt{5}$ be a rational number.
- Then the co-primes x and y of the given rational number where ($y \neq 0$) is such that:
- $3 + 2\sqrt{5} = x/y$
- Rearranging, we get,
- $2\sqrt{5} = (x/y) - 3$
- $\sqrt{5} = 1/2[(x/y) - 3]$
- Since x and y are integers, thus, $1/2[(x/y) - 3]$ is a rational number.
- Therefore, $\sqrt{5}$ is also a rational number. But this confronts the fact that $\sqrt{5}$ is irrational.
- Thus, our assumption that $3 + 2\sqrt{5}$ is a rational number is wrong.
- Hence, $3 + 2\sqrt{5}$ is irrational.

HIGHLIGHTS OF CHAPTER 2 (POLYNOMIALS)

- ◉ An algebraic expression is an expression made up of variables and constants along with mathematical operators.
- ◉ An algebraic expression is a sum of terms, which are considered to be building blocks for expressions.
- ◉ A term is a product of variables and constants. A term can be an algebraic expression in itself.
- ◉ An algebraic expression can have exponents that are rational numbers. However, a polynomial is an algebraic expression in which the exponent on any variable is a whole number.
- ◉ $5x^3+3x+1$ is an example of a polynomial. It is an algebraic expression as well.
- ◉ $2x+3\sqrt{x}$ is an algebraic expression, but not a polynomial. - since the exponent on x is $1/2$ which is not a whole number.

Degree of a Polynomial

- ◉ For a polynomial in one variable - the highest exponent on the variable in a polynomial is the degree of the polynomial.
- ◉ Example: The degree of the polynomial x^2+2x+3 is 2, as the highest power of x in the given expression is x^2 .

Types of polynomials based on the number of terms

- ◉ a) Monomial - A polynomial with just one term. Example: $2x$, $6x^2$, $9xy$
- ◉ b) Binomial - A polynomial with two terms. Example: $4x^2+x$, $5x+4$
- ◉ a) Trinomial - A polynomial with three terms. Example: x^2+3x+4

Types of Polynomials based on Degree

◉ **Linear Polynomial**

- ◉ A polynomial whose degree is one is called a linear polynomial. For example, $2x+1$ is a linear polynomial.

◉ **Quadratic Polynomial**

- ◉ A polynomial of degree two is called a quadratic polynomial. For example, $3x^2+8x+5$ is a quadratic polynomial.

◉ **Cubic Polynomial**

- ◉ A polynomial of degree three is called a ***cubic polynomial***. For example, $2x^3+5x^2+9x+15$ is a cubic polynomial.

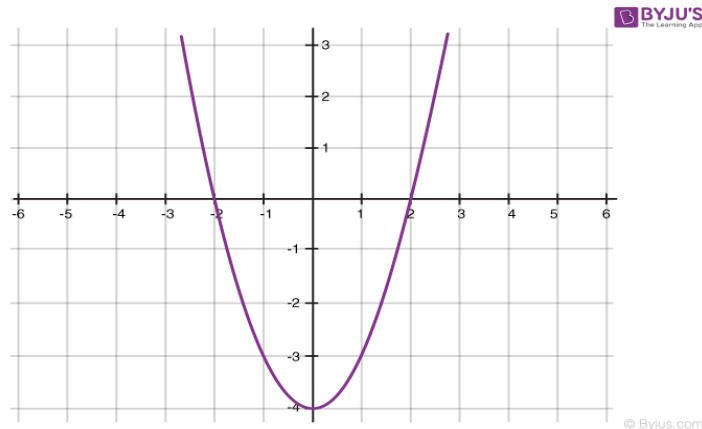
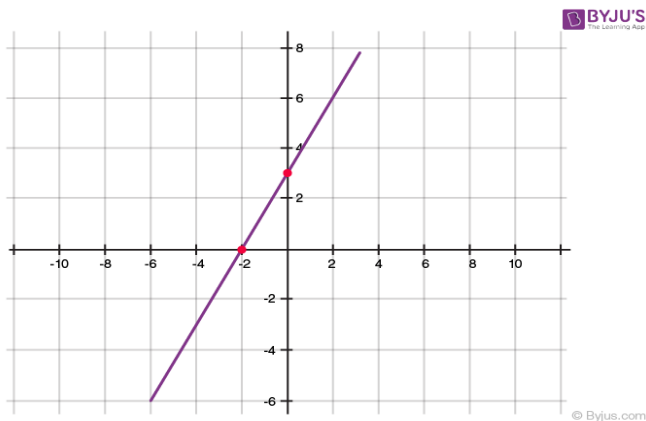
Geometrical Representation of a Linear Polynomial

- The graph of a linear polynomial is a straight line. It cuts the X-axis at exactly one point.

Geometrical Representation of a Quadratic Polynomial

- The graph of a quadratic polynomial is a parabola. It looks like a U which either opens upwards or opens downwards depending on the value of 'a' in ax^2+bx+c if 'a' is positive, then parabola opens upwards and if 'a' is negative then it opens downwards. It can cut the x-axis at 0, 1 or two points.

○



Zeroes of a Polynomial

- ◉ A zero of a polynomial $p(x)$ is the value of x for which the value of $p(x)$ is 0. If k is a zero of $p(x)$, then $p(k)=0$.

For example, consider a polynomial $p(x)=x^2-3x+2$.

When $x=1$, the value of $p(x)$ will be equal to

$$p(1)=1^2-3\times 1+2$$

$$=1-3+2$$

$$=0$$

- ◉ Since $p(x)=0$ at $x=1$, we say that 1 is a zero of the polynomial x^2-3x+2
- ◉ **Geometrically, zeroes of a polynomial are the points where its graph cuts the X-axis.**

Number of Zeroes

- ◉ In general, a polynomial of degree n has at most n zeros.
- ◉ A linear polynomial has one zero,
- ◉ A quadratic polynomial has at most two zeros.
- ◉ A cubic polynomial has at most 3 zeros.

Factorisation of Polynomials

- ◉ Quadratic polynomials can be factorized by splitting the middle term.
- ◉ For example, consider the polynomial $2x^2-5x+3$

Splitting the middle term:

- ◉ The middle term in the polynomial $2x^2-5x+3$ is $-5x$. This must be expressed as a sum of two terms such that the product of their coefficients is equal to the product of 2 and 3 (coefficient of x^2 and the constant term)
- ◉ -5 can be expressed as $(-2)+(-3)$, as $-2 \times -3 = 6 = 2 \times 3$

Thus, $2x^2-5x+3=2x^2-2x-3x+3$

Now, identify the common factors in individual groups

$$2x^2-2x-3x+3=2x(x-1)-3(x-1)$$

Taking $(x-1)$ as the common factor, this can be expressed as:

$$2x(x-1)-3(x-1)=(x-1)(2x-3)$$

Relationship between Zeroes and Coefficients of a Polynomial

○ For Quadratic Polynomial:

If α and β are the roots of a quadratic polynomial ax^2+bx+c , then,

- $\alpha + \beta = -b/a$
- Sum of zeroes = -coefficient of x / coefficient of x^2
- $\alpha\beta = c/a$
- Product of zeroes = constant term / coefficient of x^2

○ For Cubic Polynomial

○ If α, β and γ are the roots of a cubic polynomial ax^3+bx^2+cx+d , then

- $\alpha+\beta+\gamma = -b/a$
- $\alpha\beta + \beta\gamma + \gamma\alpha = c/a$
- $\alpha\beta\gamma = -d/a$

Algebraic Identities

- 1. $(a+b)^2=a^2+2ab+b^2$
- 2. $(a-b)^2=a^2-2ab+b^2$
- 3. $(x+a)(x+b)=x^2+(a+b)x+ab$
- 4. $a^2-b^2=(a+b)(a-b)$
- 5. $a^3-b^3=(a-b)(a^2+ab+b^2)$
- 6. $a^3+b^3=(a+b)(a^2-ab+b^2)$
- 7. $(a+b)^3=a^3+3a^2b+3ab^2+b^3$
- 8. $(a-b)^3=a^3-3a^2b+3ab^2-b^3$

EXTRA QUESTIONS

- 1. Find the value of “p” from the polynomial $x^2 + 3x + p$, if one of the zeroes of the polynomial is 2.

- Solution:**

- As 2 is the zero of the polynomial.

- We know that if α is a zero of the polynomial $p(x)$, then $p(\alpha) = 0$

- Substituting $x = 2$ in $x^2 + 3x + p$,

- $\Rightarrow 2^2 + 3(2) + p = 0$

- $\Rightarrow 4 + 6 + p = 0$

- $\Rightarrow 10 + p = 0$

- $\Rightarrow p = -10$

- 2. Compute the zeroes of the polynomial $4x^2 - 4x - 8$. Also, establish a relationship between the zeroes and coefficients.

- Solution:**

- Let the given polynomial be $p(x) = 4x^2 - 4x - 8$

- To find the zeroes, take $p(x) = 0$

- Now, factorise the equation $4x^2 - 4x - 8 = 0$

- $4x^2 - 4x - 8 = 0$

- $4(x^2 - x - 2) = 0$

- $x^2 - x - 2 = 0$

- $x^2 - 2x + x - 2 = 0$

- $x(x - 2) + 1(x - 2) = 0$

- $(x - 2)(x + 1) = 0$

- $x = 2, x = -1$

- So, the roots of $4x^2 - 4x - 8$ are -1 and 2.

- Relation between the sum of zeroes and coefficients:

- $-1 + 2 = 1 = -(-4)/4$ i.e. (- coefficient of x / coefficient of x^2)

- Relation between the product of zeroes and coefficients:

- $(-1) \times 2 = -2 = -8/4$ i.e (constant/coefficient of x^2)

- ⦿ **Q3. Find the quadratic polynomial if its zeroes are 0, $\sqrt{5}$.**
- ⦿ **Solution:**
- ⦿ A quadratic polynomial can be written using the sum and product of its zeroes is:
- ⦿ $x^2 - (\alpha + \beta)x + \alpha\beta$
- ⦿ Where α and β are the roots of the polynomial.
- ⦿ Here, $\alpha = 0$ and $\beta = \sqrt{5}$
- ⦿ So, the polynomial will be:
- ⦿ $x^2 - (0 + \sqrt{5})x + 0(\sqrt{5})$
- ⦿ $\Rightarrow x^2 - \sqrt{5}x$
- ⦿ **Q4. Find the value of “x” in the polynomial $2a^2 + 2xa + 5a + 10$ if $(a + x)$ is one of its factors.**
- ⦿ **Solution:**
- ⦿ Say, $f(a) = 2a^2 + 2xa + 5a + 10$
- ⦿ Since, $(a + x)$ is a factor of $2a^2 + 2xa + 5a + 10$, $f(-x) = 0$
- ⦿ So, $f(-x) = 2x^2 - 2x^2 - 5x + 10 = 0$
- ⦿ Or, $-5x + 10 = 0$
- ⦿ Thus, $x = 2$

HIGHLIGHTS OF CHAPTER 3 (PAIR OF LINEAR EQUATIONS IN TWO VARIABLES)

Linear Equation

- Equations in which the powers of all the variables involved are one are called linear equations. The degree of a linear equation is always one.

General form of a Linear Equation in Two Variables

- The general form of a linear equation in two variables is $ax + by + c = 0$, where a and b cannot be zero simultaneously.

General form of a pair of linear equations in 2 variables

- A pair of linear equations in two variables can be represented as follows

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

The coefficients of x and y cannot be zero simultaneously for an equation.

For a pair of straight lines on a plane, there are three possibilities

- i) They intersect at exactly one point
- ii) They are parallel
- iii) They are coincident

Graphical Method of finding the solution to a pair of linear equations is as follows:

- Plot both the equations (two straight lines)
- Find the point of intersection of the lines.
- The point of intersection is the solution.

Comparing the ratios of coefficients of a Linear Equation

- i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations are said to be **consistent**. Graphs of the two equations intersect at a unique point. The pair of linear equations have **exactly one solution**.
- ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the equations are said to be **dependent**. One equation can be obtained by multiplying the other equation with a non-zero constant. In this case, graphs of both the equations coincide. Dependent equations are consistent. The pair linear equations have **infinitely many solutions**.
- iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the equations are said to be **inconsistent**. The graphs of the equations are parallel to each other. The pair of linear equations have **no solution**.

Algebraic Solution

- ⦿ Finding solution for consistent pair of Linear Equations
 - ⦿ The solution of a pair of linear equations is of the form (x,y) which satisfies both the equations simultaneously. Solution for a consistent pair of linear equations can be found out using
 - ⦿ i) Elimination method
 - ⦿ ii) Substitution Method
 - ⦿ Substitution method:
 - ⦿ $y - 2x = 1$
 - ⦿ $x + 2y = 12$
 - (i) express one variable in terms of the other using one of the equations. In this case, $y = 2x + 1$.
 - (ii) substitute for this variable (y) in the second equation to get a linear equation in one variable,
 x . $x + 2 \times (2x + 1) = 12$
 $\Rightarrow 5x + 2 = 12$
 - (iii) Solve the linear equation in one variable to find the value of that variable.
 $5x + 2 = 12$
 $\Rightarrow x = 2$
 - (iv) Substitute this value in one of the equations to get the value of the other variable.
 $y = 2 \times 2 + 1$
 $\Rightarrow y = 5$
- So, $(2,5)$ is the required solution of the pair of linear equations $y - 2x = 1$ and $x + 2y = 12$.

Elimination method

Consider $x + 2y = 8$ and $2x - 3y = 2$

- Step 1: Make the coefficients of any variable the same by multiplying the equations with constants. Multiplying the first equation by 2, we get,

- $2x + 4y = 16$

- Step 2: Add or subtract the equations to eliminate one variable, giving a single variable equation.

Subtract second equation from the previous equation

$$2x + 4y = 16$$

$$2x - 3y = 2$$

$$\begin{array}{r} - \quad + \quad - \\ \hline \end{array}$$

$$0(x) + 7y = 14$$

Step 3: Solve for one variable and substitute this in any equation to get the other variable.

- $y = 2,$
- $x = 8 - 2y$
- $\Rightarrow x = 8 - 4$
- $\Rightarrow x = 4$
- $(4, 2)$ is the solution.

Equations reducible to a pair of Linear Equations in 2 variables

- Some equations may be in a form which can be reduced to a linear equation through substitution.

$$2/x + 3/y = 4$$

$$5/x - 4/y = 9$$

In this case, we may make the substitution

$$1/x = u \text{ and } 1/y = v$$

The pair of equations reduces to

$$2u + 3v = 4$$

$$5u - 4v = 9$$

- The above pair of equations may be solved. After solving, back substitute to get the values of x and y .

EXTRA QUESTIONS

- Q1 : Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.
- Solution:
- Given, half the perimeter of a rectangular garden = 36 m
- so, $2(l + b)/2 = 36$
- $(l + b) = 36$ (1)
Given, the length is 4 m more than its width.
- Let width = x
- And length = x + 4
- Substituting this in eq(1), we get;
- $x + x + 4 = 36$
 $2x + 4 = 36$
 $2x = 32$
 $x = 16$
- Therefore, the width is 16 m and the length is $16 + 4 = 20$ m.

- Q.2: On comparing the ratios a_1/a_2 , b_1/b_2 , and c_1/c_2 , find out whether the following pair of linear equations are consistent, or inconsistent.
- $3x + 2y = 5$; $2x - 3y = 7$
- Solution:
- Given : $3x + 2y = 5$ or $3x + 2y - 5 = 0$
and $2x - 3y = 7$ or $2x - 3y - 7 = 0$
- Comparing the above equations with $a_1x + b_1y + c_1 = 0$
And $a_2x + b_2y + c_2 = 0$
We get,
 $a_1 = 3$, $b_1 = 2$, $c_1 = -5$
 $a_2 = 2$, $b_2 = -3$, $c_2 = -7$
- $a_1/a_2 = 3/2$, $b_1/b_2 = 2/-3$, $c_1/c_2 = -5/-7 = 5/7$
Since, $a_1/a_2 \neq b_1/b_2$ the lines intersect each other at a point and have only one possible solution.
- Hence, the equations are consistent.

⊙ Q.3: Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

⊙ Solution:

⊙ $2x + 3y = 11$(i)

$2x - 4y = -24$ (ii)

From equation (ii), we get;

$x = (11 - 3y)/2$ (iii)

⊙ Putting the value of x in equation (ii), we get

⊙ $2[(11 - 3y)/2] - 4y = -24$

$11 - 3y - 4y = -24$

$-7y = -35$

$y = 5$(iv)

Putting the value of y in equation (iii), we get;

⊙ $x = (11 - 15)/2 = -4/2 = -2$

Hence, $x = -2, y = 5$

Also,

$y = mx + 3$

$5 = -2m + 3$

$-2m = 2$

$m = -1$

- Q.4: Formulate the following problems as a pair of equations, and hence find their solutions:
- Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.
- Solution:
- Let us consider,
 Speed of Ritu is still water = x km/hr
 Speed of Stream = y km/hr
 Now, speed of Ritu, during,
 Downstream = $x + y$ km/hr
 Upstream = $x - y$ km/hr
 As per the question given,
 $2(x + y) = 20$
 Or $x + y = 10$(1)
 And, $2(x - y) = 4$
 Or $x - y = 2$(2)
 Adding both the eq.1 and 2, we get,
 $2x = 12$
 $x = 6$
 Putting the value of x in eq.1, we get,
 $y = 4$
 Therefore,
- Speed of Ritu is still water = 6 km/hr
 Speed of Stream = 4 km/hr

HIGHLIGHTS OF CHAPTER 4 (QUADRATIC EQUATIONS)

Quadratic Polynomial

- ⦿ A polynomial of the form ax^2+bx+c , where a , b and c are real numbers and $a \neq 0$ is called a quadratic polynomial.

Quadratic Equation

- ⦿ When we equate a quadratic polynomial to a constant, we get a quadratic equation.
- ⦿ Any equation of the form $p(x)=c$, where $p(x)$ is a polynomial of degree 2 and c is a constant, is a quadratic equation.
- ⦿ The standard form of a quadratic equation is $ax^2+bx+c=0$, where a , b and c are real numbers and $a \neq 0$.
' a ' is the coefficient of x^2 . It is called the quadratic coefficient.
' b ' is the coefficient of x . It is called the linear coefficient. ' c ' is the constant term.

Roots of a Quadratic equation

- ⦿ The values of x for which a quadratic equation is satisfied are called the roots of the quadratic equation.
- ⦿ If α is a root of the quadratic equation $ax^2+bx+c=0$, then $a\alpha^2+b\alpha+c=0$.
- ⦿ A quadratic equation can have two distinct real roots, two equal roots or real roots may not exist.
- ⦿ Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x -axis.

Solving a Quadratic Equation by Factorization method

- Consider a quadratic equation $2x^2 - 5x + 3 = 0$

$$\Rightarrow 2x^2 - 2x - 3x + 3 = 0$$

This step is splitting the middle term

We split the middle term by finding two numbers (-2 and -3) such that their sum is equal to the coefficient of x and their product is equal to the product of the coefficient of x^2 and the constant.

$$(-2) + (-3) = (-5)$$

$$\text{And } (-2) \times (-3) = 6$$

$$2x^2 - 2x - 3x + 3 = 0$$

$$2x(x-1) - 3(x-1) = 0$$

$$(x-1)(2x-3) = 0$$

In this step, we have expressed the quadratic polynomial as a product of its factors.

Thus, $x = 1$ and $x = 3/2$ are the roots of the given quadratic equation.

This method of solving a quadratic equation is called the factorisation method.

Solving QE Using Quadratic Formula

- ◉ Quadratic Formula is used to directly obtain the roots of a quadratic equation from the standard form of the equation.
- ◉ For the quadratic equation $ax^2+bx+c=0$,
- ◉ $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$
- ◉ By substituting the values of a, b and c, we can directly get the roots of the equation.

Discriminant

- ◉ For a quadratic equation of the form $ax^2+bx+c=0$, the expression b^2-4ac is called the **discriminant**, (denoted by **D**), of the quadratic equation.
The **discriminant** determines the **nature of roots** of the quadratic equation based on the **coefficients** of the quadratic equation.

Solving using Quadratic Formula when $D > 0$

- Solve $2x^2 - 7x + 3 = 0$ using the quadratic formula.
 - (i) Identify the coefficients of the quadratic equation. $a = 2, b = -7, c = 3$
 - (ii) Calculate the discriminant, $b^2 - 4ac$
 $D = (-7)^2 - 4 \times 2 \times 3 = 49 - 24 = 25$
 $D > 0$, therefore, the roots are distinct.
 - (iii) Substitute the coefficients in the quadratic formula to find the roots
- $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(3)}}{2(2)}$
- $x = \frac{7 \pm 5}{4}$
- $x = 3$ and $x = \frac{1}{2}$ are the roots.

Nature of Roots

- Based on the value of the discriminant, $D = b^2 - 4ac$, the roots of a quadratic equation can be of three types.
- Case 1: If $D > 0$, the equation has two **distinct real roots**.
- Case 2: If $D = 0$, the equation has two **equal real roots**.
- Case 3: If $D < 0$, the equation has **no real roots**.

EXTRA QUESTIONS

- ◉ .1: Represent the following situations in the form of quadratic equations:
- ◉ (i) The area of a rectangular plot is 528 m^2 . The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.
- ◉ (ii) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. What is the speed of the train?
- ◉ Solution:
- ◉ (i) Let us consider,
The breadth of the rectangular plot is $x \text{ m}$.
- ◉ Thus, the length of the plot = $(2x + 1) \text{ m}$
- ◉ As we know,
- ◉ Area of rectangle = length \times breadth = 528 m^2
Putting the value of length and breadth of the plot in the formula, we get,
- ◉ $(2x + 1) \times x = 528$
- ◉ $\Rightarrow 2x^2 + x = 528$
- ◉ $\Rightarrow 2x^2 + x - 528 = 0$
- ◉ Hence, $2x^2 + x - 528 = 0$, is the required equation which represents the given situation.
- ◉ (ii) Let us consider,
speed of train = $x \text{ km/h}$
And
Time taken to travel $480 \text{ km} = 480 (x) \text{ km/h}$
As per second situation, the speed of train = $(x - 8) \text{ km/h}$
- ◉ As given, the train will take 3 hours more to cover the same distance.
Therefore, time taken to travel $480 \text{ km} = (480/x) + 3 \text{ km/h}$
As we know,
Speed \times Time = Distance
Therefore,
 $(x - 8)[(480/x) + 3] = 480$
 $\Rightarrow 480 + 3x - (3840/x) - 24 = 480$
 $\Rightarrow 3x - (3840/x) = 24$
 $\Rightarrow 3x^2 - 24x - 3840 = 0$
- ◉ $\Rightarrow x^2 - 8x - 1280 = 0$
- ◉ Hence, $x^2 - 8x - 1280 = 0$ is the required representation of the problem mathematically.

○ **Q.3: Find two consecutive positive integers, sum of whose squares is 365.**

○ Solution:

○ Let us say, the two consecutive positive integers be x and $x + 1$.

Therefore, as per the given statement,

$$x^2 + (x + 1)^2 = 365$$

$$\Rightarrow x^2 + x^2 + 1 + 2x = 365$$

$$\Rightarrow 2x^2 + 2x - 364 = 0$$

$$\Rightarrow x^2 + x - 182 = 0$$

$$\Rightarrow x^2 + 14x - 13x - 182 = 0$$

$$\Rightarrow x(x + 14) - 13(x + 14) = 0$$

$$\Rightarrow (x + 14)(x - 13) = 0$$

Thus, either, $x + 14 = 0$ or $x - 13 = 0$,

$$\Rightarrow x = -14 \text{ or } x = 13$$

since, the integers are positive, so x can be 13, only.

○ So, $x + 1 = 13 + 1 = 14$

Therefore, the two consecutive positive integers will be 13 and 14.

○ **Q.4: Solve the quadratic equation $2x^2 - 7x + 3 = 0$ by using quadratic formula.**

○ Solution: $2x^2 - 7x + 3 = 0$

○ On comparing the given equation with $ax^2 + bx + c = 0$, we get,

○ $a = 2$, $b = -7$ and $c = 3$

By using quadratic formula, we get,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

○ Therefore,

$$\Rightarrow x = \frac{7+5}{4} \text{ or } x = \frac{7-5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

HIGHLIGHTS OF CHAPTER 5

(ARITHMETIC PROGRESSIONS)

- ◉ An arithmetic progression (A.P) is a progression in which the **difference** between two **consecutive** terms is constant. Example: 2, 5, 8, 11, 14... is an arithmetic progression.

Common Difference

- ◉ The difference between two consecutive terms in an AP, (*which is constant*) is the “**common difference**“(d) of an A.P. In the progression: 2, 5, 8, 11, 14 ...the common difference is 3. As it is the difference between any two consecutive terms, for any A.P, if the common difference is:
 - ◉ **positive**, the AP is **increasing**.
 - ◉ **zero**, the AP is **constant**.
 - ◉ **negative**, the A.P is **decreasing**.

Finite and Infinite AP

- ◉ A finite AP is an A.P in which the number of terms is finite. For example: the A.P: 2, 5, 8.....32, 35, 38
- ◉ An infinite A.P is an A.P in which the number of terms is infinite. For example: 2, 5, 8, 11.....
- ◉ A finite A.P will have the last term, whereas an infinite A.P won't.

General Term of AP

- ◉ The nth term of an AP
- ◉ The nth term of an A.P is given by $T_n = a + (n-1)d$, where a is the first term, d is a common difference and n is the number of terms.
- ◉ The general form of an AP
- ◉ The general form of an A.P is: $(a, a+d, a+2d, a+3d, \dots)$ where a is the first term and d is a common difference. Here, $d=0$, OR $d>0$, OR $d<0$

The sum to n terms of an A.P is given by:

- ◉ $S_n = n/2(2a + (n-1)d)$
- ◉ Where a is the first term, d is the common difference and n is the number of terms.

The sum of n terms of an A.P is also given by

- ◉ $S_n = n/2(a+l)$
- ◉ Where a is the first term, l is the last term of the A.P. and n is the number of terms.
- ◉ The sum of first n natural numbers is given by:
- ◉ $S_n = n(n+1)/2$

EXTRA QUESTIONS

- ◉ **Q1: How many multiples of 4 lie between 10 and 250?**
- ◉ Solution: The first multiple of 4 that is greater than 10 is 12.
- ◉ Next multiple will be 16.
Therefore, the series formed as;
- ◉ 12, 16, 20, 24, ...
- ◉ All these are divisible by 4 and thus, all these are terms of an A.P. with the first term as 12 and common difference as 4.
- ◉ When we divide 250 by 4, the remainder will be 2. Therefore, $250 - 2 = 248$ is divisible by 4.
- ◉ The series is as follows, now;
12, 16, 20, 24, ..., 248
- ◉ Let 248 be the n th term of this A.P.
first term, $a = 12$
common difference, $d = 4$
 $a_n = 248$
- ◉ As we know,
 $a_n = a + (n - 1) d$
 $248 = 12 + (n - 1) \times 4$
 $236/4 = n - 1$
 $59 = n - 1$
 $n = 60$
- ◉ **Q.2: Which term of the AP: 21, 18, 15, . . . is - 81? Also, is any term 0? Give reason for your answer.**
- ◉ Solution : Here, $a = 21$,
- ◉ $d = 18 - 21 = - 3$
- ◉ and
- ◉ $a_n = - 81$, and we have to find n .
As $a_n = a + (n - 1) d$,
we have;
- ◉ $- 81 = 21 + (n - 1)(- 3)$
 $- 81 = 24 - 3n$
 $- 105 = - 3n$
So, $n = 35$
Therefore, the 35th term of the given AP is - 81.
Next, we want to know if there is any n for which $a_n = 0$.
- ◉ If such an n is there, then;
 $21 + (n - 1) (-3) = 0$,
 $3(n - 1) = 21$
 $n = 8$
- ◉ Therefore, the eighth term is 0.

HIGHLIGHTS OF CHAPTER 6

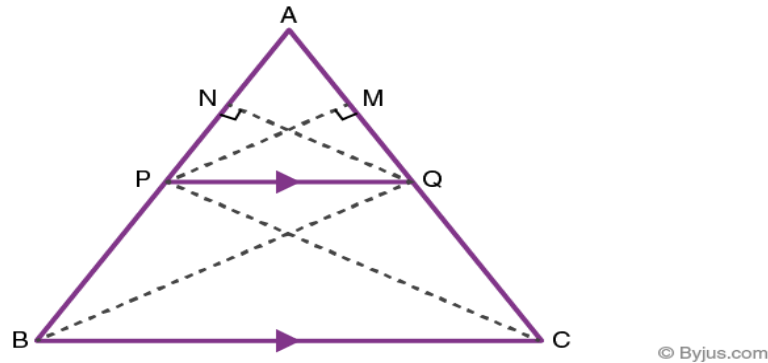
(TRIANGLES)

Similarity Criteria of Two Polygons Having the Same Number of Sides

- Any two polygons which have the same number of sides are similar if the following two criteria are met-
- Their corresponding angles are equal, and
- Their corresponding sides are in the same ratio (or proportion)
- Similarity Criteria of Triangles
- To find whether the given two triangles are similar or not, it has four criteria. They are:
- **Side-Side- Side (SSS) Similarity Criterion** - When the corresponding sides of any two triangles are in the same ratio, then their corresponding angles will be equal and the triangle will be considered as similar triangles.
- **Angle Angle Angle (AAA) Similarity Criterion** - When the corresponding angles of any two triangles are equal, then their corresponding side will be in the same ratio and the triangles are considered to be similar.
- **Angle-Angle (AA) Similarity Criterion** - When two angles of one triangle are respectively equal to the two angles of the other triangle, then the two triangles are considered as similar.
- **Side-Angle-Side (SAS) Similarity Criterion** - When one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio (proportional), then the triangles are said to be similar.

Basic Proportionality/ Thales Theorem Statement

- If a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio.



Proof:

- Consider a triangle ΔABC , as shown in the given figure. In this triangle, we draw a line PQ parallel to the side BC of ΔABC and intersecting the sides AB and AC in P and Q , respectively.
- According to the basic proportionality theorem as stated above, we need to prove:
- $AP/PB = AQ/QC$
- **Construction**
- Join the vertex B of ΔABC to Q and the vertex C to P to form the lines BQ and CP and then drop a perpendicular QN to the side AB and also draw $PM \perp AC$ as shown in the given figure.
- **Proof**
- Now the area of $\Delta APQ = 1/2 \times AP \times QN$ (Since, area of a triangle = $1/2 \times \text{Base} \times \text{Height}$)
- Similarly, area of $\Delta PBQ = 1/2 \times PB \times QN$
- area of $\Delta APQ = 1/2 \times AQ \times PM$

Also, area of $\Delta QCP = \frac{1}{2} \times QC \times PM \dots\dots\dots (1)$

Now, if we find the ratio of the area of triangles ΔAPQ and ΔPBQ , we have

$$\frac{\text{area of } \Delta APQ}{\text{area of } \Delta PBQ} = \frac{\frac{1}{2} \times AP \times QN}{\frac{1}{2} \times PB \times QN} = \frac{AP}{PB}$$

Similarly, $\frac{\text{area of } \Delta APQ}{\text{area of } \Delta QCP} = \frac{\frac{1}{2} \times AQ \times PM}{\frac{1}{2} \times QC \times PM} = \frac{AQ}{QC} \dots\dots\dots (2)$

According to the property of triangles, the triangles drawn between the same parallel lines and on the same base have equal areas.

Therefore, we can say that ΔPBQ and ΔQCP have the same area.

$$\text{area of } \Delta PBQ = \text{area of } \Delta QCP \dots\dots\dots (3)$$

Therefore, from the equations (1), (2) and (3) we can say that,

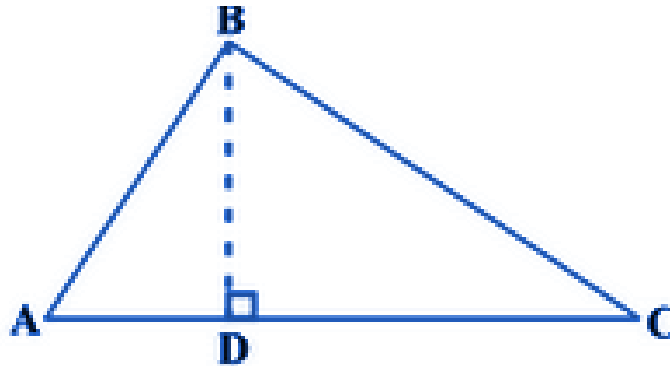
$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Also, ΔABC and ΔAPQ fulfil the conditions for similar triangles, as stated above.

Thus, we can say that $\Delta ABC \sim \Delta APQ$.

Proof of Pythagoras Theorem

- ◉ **Statement:** As per Pythagoras theorem, “In a right-angled triangle, the sum of squares of two sides of a right triangle is equal to the square of the hypotenuse of the triangle.”
- ◉ **Proof -**
- ◉ Consider the right triangle, right-angled at B.
- ◉ Construction-
- ◉ Draw $BD \perp AC$
- ◉ Now, $\triangle ADC \sim \triangle ABC$
- ◉ So, $AD/AB = AB/AC$
- ◉ or $AD \cdot AC = AB^2$ (i)
- ◉ Also, $\triangle BCD \sim \triangle ABC$
- ◉ So, $CD/BC = BC/AC$
- ◉ or, $CD \cdot AC = BC^2$ (ii)
- ◉ Adding (i) and (ii),
- ◉ $AD \cdot AC + CD \cdot AC = AB^2 + BC^2$
- ◉ $AC(AD + DC) = AB^2 + BC^2$
- ◉ $AC(AC) = AB^2 + BC^2$
- ◉ $\Rightarrow AC^2 = AB^2 + BC^2$
- ◉ Hence, proved.



Question:

- In a right-angled triangle ABC, which is right-angled at A, where CM and BL are the medians of a triangle. Show that, $4(BL^2 + CM^2) = 5 BC^2$

Solution:

Given that,

Medians BL and CM, $\angle A = 90^\circ$

From the triangle ABC, we can write it as:

$$BC^2 = AB^2 + AC^2 \text{ (Using Pythagoras Theorem) ... (1)}$$

From the triangle, ABL,

$$BL^2 = AL^2 + AB^2$$

or we can write the above equation as:

$$BL^2 = (AC/2)^2 + AB^2 \text{ (Where L is the midpoint of AC)}$$

$$BL^2 = (AC^2/4) + AB^2$$

$$4BL^2 = AC^2 + 4 AB^2 \text{ (2)}$$

From triangle CMA,

$$CM^2 = AC^2 + AM^2$$

$$CM^2 = AC^2 + (AB/2)^2 \text{ (Where M is the midpoint of AB)}$$

$$CM^2 = AC^2 + AB^2/4$$

$$4CM^2 = 4 AC^2 + AB^2 \text{ (3)}$$

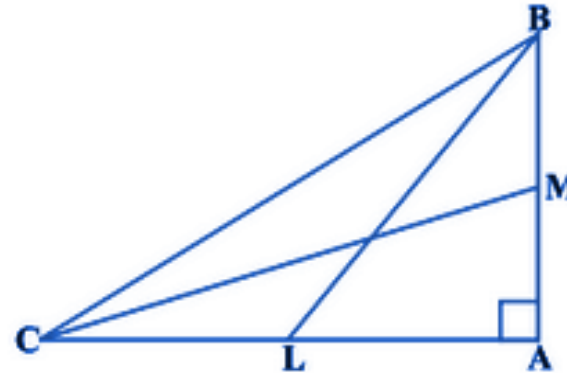
Now, by adding (2) and (3), we get,

$$4(BL^2 + CM^2) = 5(AC^2 + AB^2)$$

Using equation (1), we can write it as:

$$4(BL^2 + CM^2) = 5 BC^2$$

Hence, it is proved.



QUESTION : A vertical pole of length 6 m casts a shadow 4 m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower

Solution:

Given,

Length of the vertical pole = 6 m

Shadow of the pole = 4 m

Let the height of the tower be h m.

Length of the shadow of the tower = 28 m

In $\triangle ABC$ and $\triangle DFE$,

$\angle C = \angle E$ (angle of elevation)

$\angle B = \angle F = 90^\circ$

By AA similarity criterion,

$\triangle ABC \sim \triangle DFE$

We know that the corresponding sides of two similar triangles are proportional.

$AB/DF = BC/EF$

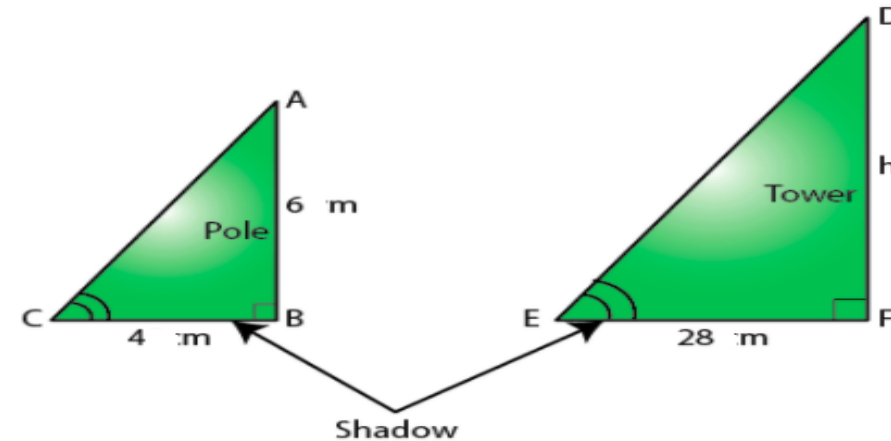
$6/h = 4/28$

$h = (6 \times 28)/4$

$h = 6 \times 7$

$h = 42$

Hence, the height of the tower = 42 m.



QUESTION: If the areas of two similar triangles are equal, prove that they are congruent.

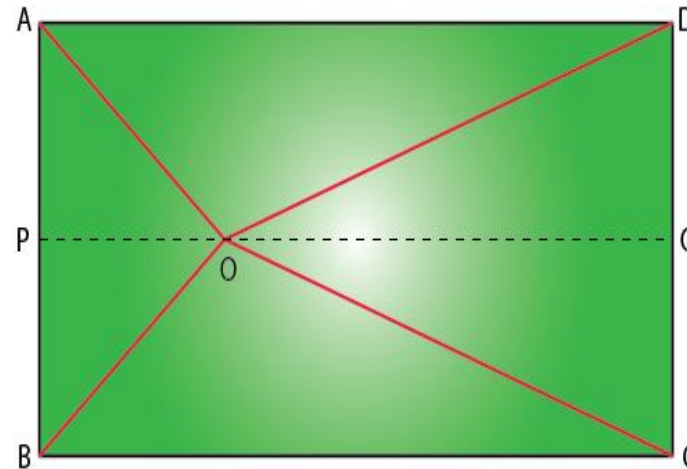
Solution:

- ⊙ Let $\triangle ABC$ and $\triangle PQR$ be the two similar triangles with equal area.
- ⊙ To prove $\triangle ABC \cong \triangle PQR$.
- ⊙ Proof:
- ⊙ $\triangle ABC \sim \triangle PQR$
- ⊙ $\therefore \text{Area of } (\triangle ABC) / \text{Area of } (\triangle PQR) = BC^2 / QR^2 = AB^2 / PQ^2 = AC^2 / PR^2$
 $\Rightarrow BC^2 / QR^2 = AB^2 / PQ^2 = AC^2 / PR^2 = 1$ [Since, $\text{ar } (\triangle ABC) = \text{ar } (\triangle PQR)$] $\Rightarrow BC^2 / QR^2 = 1$
 $\Rightarrow AB^2 / PQ^2 = 1$
 $\Rightarrow AC^2 / PR^2 = 1$
 $BC = QR$
 $AB = PQ$
 $AC = PR$
Therefore, $\triangle ABC \cong \triangle PQR$ [SSS criterion of congruence]

QUESTION: O is any point inside a rectangle ABCD as shown in the figure. Prove that $OB^2 + OD^2 = OA^2 + OC^2$.

Solution:

- ⊙ Through O, draw $PQ \parallel BC$ so that P lies on AB and Q lies on DC.
- ⊙ $PQ \parallel BC$
- ⊙ Therefore, $PQ \perp AB$ and $PQ \perp DC$ ($\angle B = 90^\circ$ and $\angle C = 90^\circ$)
- ⊙ So, $\angle BPQ = 90^\circ$ and $\angle CQP = 90^\circ$
- ⊙ Hence, BPQC and APQD are both rectangles.
- ⊙ By Pythagoras theorem,
- ⊙ In ΔOPB ,
- ⊙ $OB^2 = BP^2 + OP^2 \dots (1)$
- ⊙ Similarly,
- ⊙ In ΔOQD ,
- ⊙ $OD^2 = OQ^2 + DQ^2 \dots (2)$
- ⊙ In ΔOQC ,
- ⊙ $OC^2 = OQ^2 + CQ^2 \dots (3)$
- ⊙ In ΔOAP ,
- ⊙ $OA^2 = AP^2 + OP^2 \dots (4)$
- ⊙ Adding (1) and (2),
- ⊙ $OB^2 + OD^2 = BP^2 + OP^2 + OQ^2 + DQ^2$
- ⊙ $= CQ^2 + OP^2 + OQ^2 + AP^2$
- ⊙ (since $BP = CQ$ and $DQ = AP$)
- ⊙ $= CQ^2 + OQ^2 + OP^2 + AP^2$
- ⊙ $= OC^2 + OA^2$ [From (3) and (4)]
- ⊙ Hence proved that $OB^2 + OD^2 = OA^2 + OC^2$.

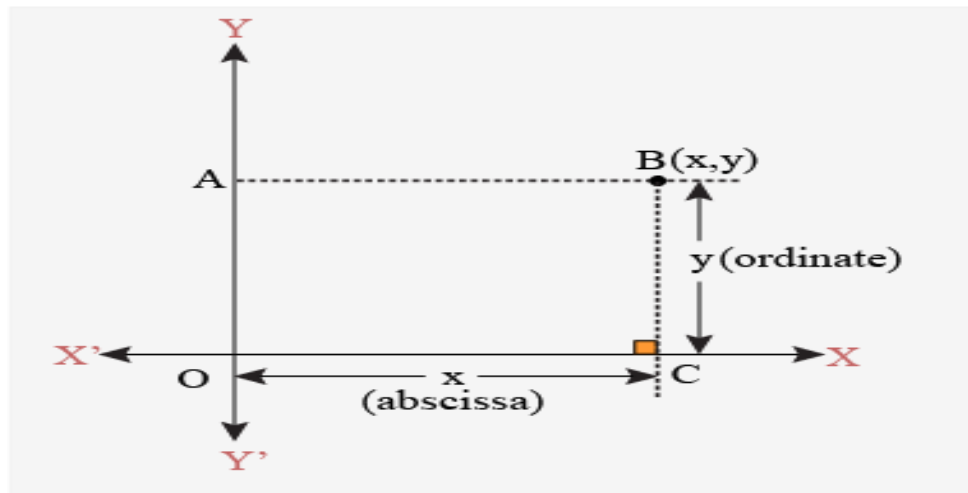


HIGHLIGHTS OF CHAPTER 7

COORDINATE GEOMETRY

Points on a Cartesian Plane

- ⦿ A pair of numbers locate points on a plane called the **coordinates**. The distance of a point from the y-axis is known as **abscissa** or x-coordinate. The distance of a point from the x-axis is called **ordinates** or y-coordinate.



- ◉ Distance Formula

- ◉ Distance between any two points (x_1, y_1) and (x_2, y_2) is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Where d is the distance between the points (x_1, y_1) and (x_2, y_2) .

- ◉ Section Formula

- ◉ If the point $P(x, y)$ **divides** the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ **internally** in the **ratio $m:n$** , then, the coordinates of P are given by the **section formula** as:

- ◉ $P(x, y) = \left[\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n} \right]$

Finding ratio for given points

- To find the ratio in which a given point $P(x, y)$ divides the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$,
- Assume that the ratio is $k : 1$
- Substitute the ratio in the section formula for any of the coordinates to get the value of k .
- $x = \frac{kx_2 + x_1}{k + 1}$ When x_1, x_2 and x are known, k can be calculated. The same can be calculated from the y -coordinate also.

The **midpoint** of any line segment divides it in the ratio **1 : 1**.

The coordinates of the midpoint(P) of line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$p(x, y) = \left[\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right]$$

Points of Trisection

- To find the points of trisection P and Q which divides the line segment joining A(x₁, y₁) and B(x₂, y₂) into three equal parts:
- i) **AP : PB = 1 : 2**
- $P = \left[\frac{x_2 + 2x_1}{3}, \frac{y_2 + 2y_1}{3} \right]$
- ii) **AQ : QB = 2 : 1**
- $Q = \left[\frac{2x_2 + x_1}{3}, \frac{2y_2 + y_1}{3} \right]$

Collinearity Condition

- If three points A, B and C are collinear and B lies between A and C, then,
- $AB + BC = AC$. AB, BC, and AC can be calculated using the distance formula.
- The ratio in which B divides AC, calculated using section formula for both the x and y coordinates separately will be equal.
- Area of a triangle formed by three collinear points is zero.

EXTRA QUESTIONS

- Q. 1: The distance of the point P (2, 3) from the x-axis is
- (A) 2 (B) 3 (C) 1 (D) 5
- Solution:**
- We know that,
- $(x, y) = (2, 3)$ is a point on the Cartesian plane in the first quadrant.
- x = Perpendicular distance from Y-axis
- y = Perpendicular distance from X-axis
- Therefore, the perpendicular distance from X-axis = y coordinate = 3
- Q. 2: Find a relation between x and y such that the point (x, y) is equidistant from the points (7, 1) and (3, 5).
- Solution:**
- Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5).
- Then, $AP = BP$
- $AP^2 = BP^2$
- Using distance formula,
- $(x - 7)^2 + (y - 1)^2 = (x - 3)^2 + (y - 5)^2$
- $x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$
- $x - y = 2$
- Hence, the relation between x and y is $x - y = 2$.

- ⦿ **Q. 3: Find the coordinates of the points of trisection (i.e., points dividing into three equal parts) of the line segment joining the points A(2, - 2) and B(- 7, 4).**
- ⦿ **Solution:**
- ⦿ Let P and Q be the points of trisection of AB, i.e., $AP = PQ = QB$.
- ⦿ Therefore, P divides AB internally in the ratio 1: 2.
- ⦿ Let $(x_1, y_1) = (2, -2)$
- ⦿ $(x_2, y_2) = (-7, 4)$
- ⦿ $m_1 : m_2 = 1 : 2$
- ⦿ Therefore, the coordinates of P, by applying the section formula,
- ⦿ $[(m_1x_2+m_2x_1)/m_1+m_2, (m_1y_2+m_2y_1)/m_1+m_2] = [1(-7)+2(2)]/1+2, [1(4)+2(-2)]/1+2 = (-3/3, 0/3)$
- ⦿ $= (-1, 0)$
- ⦿ Similarly, Q also divides AB internally in the ratio 2 : 1. and the coordinates of Q by applying the section formula,
- ⦿ $=[(2(-7)+1(2))/(2+1), (2(4)+1(-2))/(2+1)] = (-12/3, 6/3)$
- ⦿ $= (-4, 2)$
- ⦿ Hence, the coordinates of the points of trisection of the line segment joining A and B are (-1, 0) and (- 4, 2).