BRAIN INTERNATIONAL SCHOOL

SUBJECT : MATHEMATICS **CLASS : XII** **JULY 2021**

CHAPTER : INVERSE TRIGONOMETRIC FUNCTIONS

Q1. Show that $sin^{-1}\left(\sqrt{\frac{a-x}{2a}}\right) = \frac{1}{2}\ cos^{-1}\frac{x}{a}$.

Q2. Write the principle value :

(i) $cosec^{-1}(2)$

(ii) $cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

(iii) $tan^{-1}(-\sqrt{3})$

(iv) $tan^{-1}\left(\tan\frac{3\,\pi}{4}\right)$

Q3. Write the value in

(i) $cosec^{-1}(\sqrt{2}) + sec^{-1}(\sqrt{2})$

(ii) $cos^{-1}\left(cos\ \frac{2\,\pi}{3}\right) + sin^{-1}\left(cos\ \frac{2\,\pi}{3}\right)$

(iii) $tan^{-1}(\sqrt{3}) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Q4. What is the domain of the function $cosec^{-1}x$?

Q5. Write one branch of $tan^{-1}x$ other than the principle branch.

Q6. Evaluate in

(i) $sin^{-1}\left\{\cos\left(sin^{-1}\frac{3}{2}\right)\right\}$

(ii) $cosec^{-1}\left\{\text{cosec}\left(-\frac{\pi}{4}\right)\right\}$

(iii) $\cos\left\{\frac{\pi}{3} - cos^{-1}\left(\frac{1}{2}\right)\right\}$

(iv) $sec^{2}\ (tan^{-1}\ 2)$

(v) $cos^{-1}\left(\cos\frac{5\,\pi}{3}\right)$

(vi) $sec^{-1}\left(\frac{x-3}{x+3}\right) + sin^{-1}\left(\frac{x+3}{x-3}\right)$

(vii) $tan^{-1}\{\cos\pi\}$

Q7. Prove that in

(i) $2\ tan^{-1}x = sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(ii) $2\ cos^{-1}x = sec^{-1}\left(\frac{1}{2x^2-1}\right)$

(iii) $sin^{-1}x = cot^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

(iv) $cos^{-1}x = 2\ cos^{-1}\sqrt{\frac{1+x}{2}}$

Q8. Find the value of $cosec\left(cot^{-1}\frac{y}{2}\right)$ in terms of y alone.

Q9. Prove that

(i) $2\ tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{8} = tan^{-1}\frac{4}{7}$

(ii) $tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{3} + tan^{-1}\frac{1}{7} + tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

(iii) $cos^{-1}\frac{4}{5} + cos^{-1}\frac{12}{13} = cos^{-1}\frac{33}{65}$

(iv) $cos^{-1}\frac{4}{5} + tan^{-1}\frac{3}{5} = tan^{-1}\frac{27}{11}$

(v) $sin^{-1}\frac{4}{5} + 2\ tan^{-1}\frac{1}{3} = \frac{\pi}{2}$

(vi) $tan^{-1}\left[\frac{3a^2x-x^3}{a\ (a^2-3x^2)}\right] = 3\ tan^{-1}\left(\frac{x}{q}\right)$

(vii) $sec^2\ (tan^{-1}\ 3) + cosec^2(cot^{-1}\ 4) = 27$

(viii) $sin^{-1}\left(\frac{x+\sqrt{1-x^2}}{\sqrt{2}}\right) = \frac{\pi}{4} + sin^{-1}x, -1 \leq x \leq 1.$

Q10. Write in the simplest form

(i) $cos^{-1}\left(\frac{3}{5}\cos x + \frac{4}{5}\sin x\right)$

(ii) $tan^{-1}\left(\frac{8x}{1+20x^2}\right)$

(iii) $cot^{-1}\sqrt{\frac{1+\cos 5x}{1-\cos 5x}}$

(iv) $sin^{-1}\left(x^2\sqrt{1-x^2} + x\sqrt{1-x^4}\right)$

Q11. Solve for x

(i) $cos^{-1}x + sin^{-1}\frac{x}{2} = \frac{\pi}{6}$

(ii) $tan(cos^{-1}x) = \sin\left(cot^{-1}\frac{1}{2}\right)$

(iii) $cot^{-1}\left(\frac{2}{x}\right) + cot^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{4}$

(iv) $tan^{-1}\left(\frac{x-1}{x-2}\right) + tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

(v) $2\,cot^{-1}x + tan^{-1}x = \frac{2\pi}{3}$

(vi) $sin^{-1}\frac{3x}{5} + sin^{-1}\frac{4x}{5} = sin^{-1}x$

Q12. If $cos^{-1}a + cos^{-1}b + cos^{-1}c = \pi$, prove that $a^2 + b^2 + c^2 + 2abc = 1$.

Q13. If $tan^{-1}a + tan^{-1}b + tan^{-1}c = \frac{\pi}{2}$, prove that $ab + bc + ca = 1$.

Q14. Show that $2\,tan^{-1}\left\{tan\frac{\alpha}{2}\,tan\frac{\beta}{2}\right\} = cos^{-1}\left(\frac{\cos\alpha+\cos\beta}{1+\cos\alpha\cos\beta}\right)$.

Q15. If $sin^{-1}x + sin^{-1}y + sin^{-1}z = \pi$, then prove that

$$x\sqrt{1-x^2} + y\sqrt{1-y^2} + z\sqrt{1-z^2} = 2xyz.$$

Q16. If $sin^{-1}\frac{2a}{1-a^2} - cos^{-1}\frac{1-b^2}{1+b^2} = tan^{-1}\frac{2x}{1-x^2}$, prove that $x = \frac{a-b}{1+ab}$.

Q17. Find the value of $sin\left\{2\,cot^{-1}\left(-\frac{5}{12}\right)\right\}$.