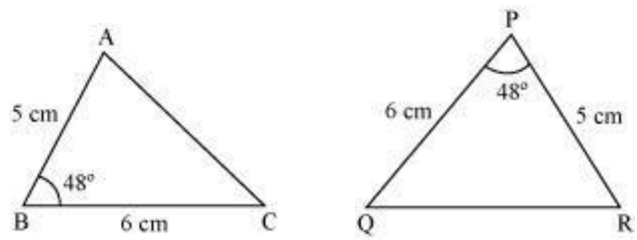
Triangles

- **SAS congruence rule**

  If two sides of a triangle and the angle included between them are equal to the corresponding two sides and included angle of another triangle, then the triangles are congruent by SAS congruence rule.

  Example:



  Are ΔABC and ΔRPQ congruent?

  Solution: In ΔABC and ΔRPQ,

  AB = RP

  ∠ABC = ∠RPQ

  BC = PQ

  ∴ ΔABC ≅ ΔRPQ (By SAS congruence rule)

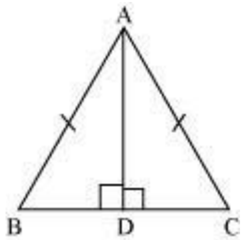
- **CPCT**

  CPCT stands for 'corresponding parts of congruent triangles'. 'Corresponding parts' means corresponding sides and angles of triangles. According to CPCT, if two or more triangles are congruent to one another, then all of their corresponding parts are equal.

- **ASA congruence rule**

  If two angles and included side of a triangle are equal to the two corresponding angles and the included side of another triangle, then the triangles are congruent by ASA congruence rule.

  Example: In the following figure, AD is the median of ΔABC.



Are ΔABD and ΔACD congruent?

Solution: In ΔABC,

AB = AC (Given)

∴∠ACB = ∠ABC (Base angles of an isosceles triangle have equal measures)

Now, in ΔABD and ΔACD,

∠ABD = ∠ACD

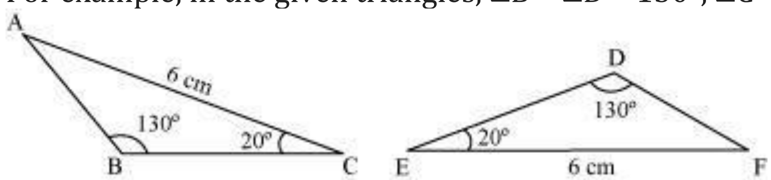
BD = CD (AD is the median)

∠ADB = ∠ADC = 90°

∴ ΔABD ≅ ΔACD (By ASA congruence rule)

- **AAS congruence rule**

  If two angles and one side of a triangle are equal to the corresponding angles and side of the other triangle then the two triangles are congruent to each other. This criterion is known as the **AAS congruence rule.**
  For example, in the given triangles, ∠B = ∠D = 130°, ∠C = ∠E = 20° and AC = EF = 6 cm.

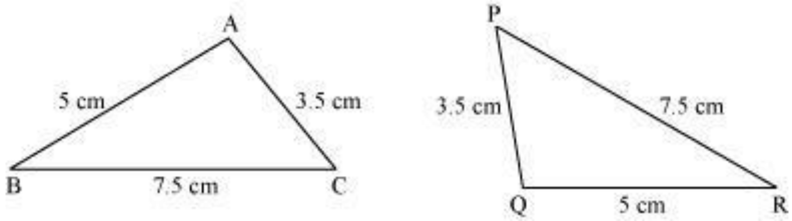


∴ By AAS congruence rule, ΔABC ≅ ΔFDE

- **SSS congruence rule**

  If three sides of a triangle are equal to the three sides of the other triangle, then the two triangles are congruent by SSS congruence rule.

Example:



Are ΔABC and ΔQRP congruent?

Solution: In ΔABC and ΔQRP

AB = QR = 5 cm

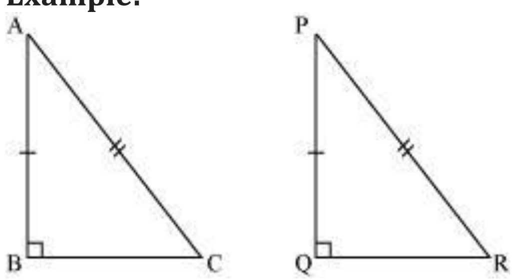
BC = PR = 7.5 cm

AC = PQ = 3.5 cm

∴ ΔABC ≅ ΔQRP (By SSS congruence rule)

- **RHS congruence rule**
  If the hypotenuse and one side of a right triangle are equal to the hypotenuse and one side of the other right triangle, then the two triangles are congruent to each other by RHS congruence rule.

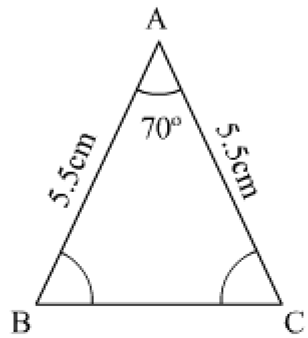
  Example:

  If in the given figure, ∠B = ∠Q = 90°, AC = PR, and AB = PQ,
  ∴ By RHS congruence rule, ΔABC ≅ ΔPQR

- Angles opposite to equal sides of a triangle are equal.

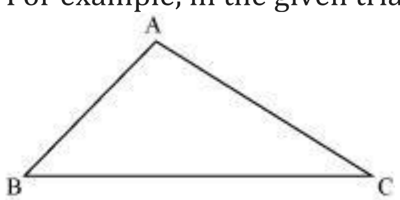
  Example: Find the missing angles in the following triangles.



Solution: We know that angles opposite to equal sides of a triangle are equal.
$\therefore \angle ABC = \angle ACB = x$ (say)
By angle sum property of triangles, we obtain
$\angle ABC + \angle BCA + \angle CAB = 180°$
$\Rightarrow x + x + 70° = 180°$
$\Rightarrow 2x = 110°$
$\Rightarrow x = 55°$
Thus, $\angle ABC = \angle BCA = 55°$

- Sides opposite to equal angles of a triangle are equal in length. Thus, we can say that if two angles of a triangle are equal then the sides opposite to them are also equal, therefore the triangle is isosceles.

- If two sides of a triangle are unequal then the longer side has the greater angle opposite it. Thus, we can say that angle opposite to the shorter side of a triangle is smaller.
  For example, in the given triangle, AC > AB, therefore $\angle ABC > \angle ACB$.



- If two angles of a triangle are unequal then the greater angle has the longer side opposite it.Thus, we can say that the smaller angle has the shorter side opposite it.

  For example, in the given figure, $\angle BAC > \angle ACB$, therefore BC > AB.