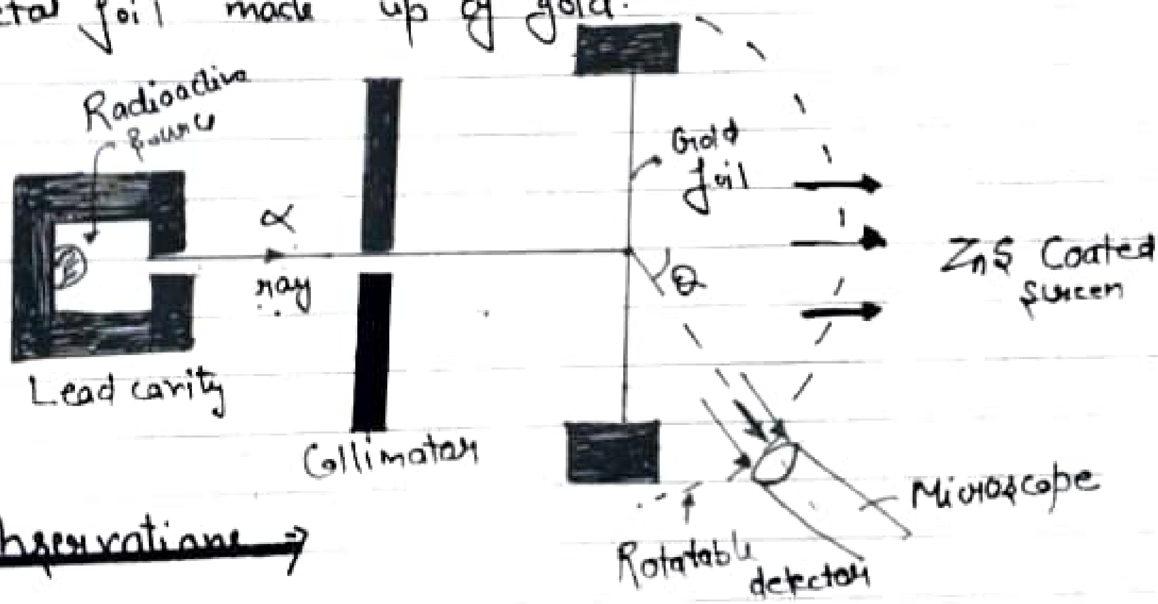
ATOMS - 12

Rutherford α - Particle Scattering Experiment ⇒

At the suggestion of Ernst Rutherford in (1911), H Geiger and E. Marsden performed α particle scattering experiment

They directed a beam of α particle of 5.5 MeV energy emitted from a $^{214}_{83}Bi$ radioactive source at a thin metal foil made up of gold.

Collimator → Device that produces parallel beam of light



Observations ⇒

(1) Most of the α particle passed through gold foil undeflected

(2) Some of the α particles were deflected through small angle

(3) A few α particles were deflected through large angles.

(4) Some of them even retraced its path that is they deflected at an angle of 180.

Rutherford Nuclear Model OF ATOM

1) Most of the mass and all the +ve charge of an atom are concentrated in a very small region known as atomic nucleus.

2) The size of nucleus is very small as compared to the size of atom, so most of the space is empty.

3) The negatively charged particle known as electron revolve around the nucleus in a circular orbits.

4) The number of revolving electron is equal to the number of +ve charges in the nucleus. hence the atom is electrically neutral.

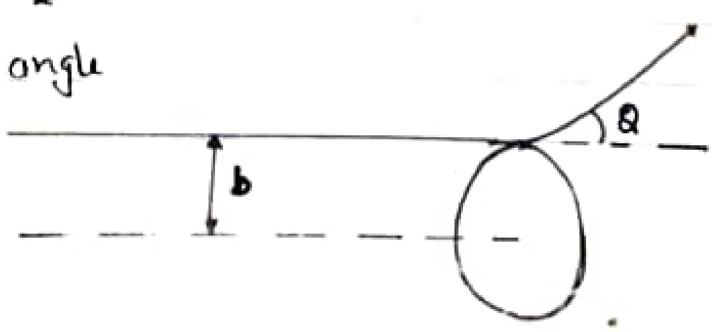
Impact Parameter (b)

It is the perpendicular distance of the initial velocity vector of the α particle from the center of nucleus.

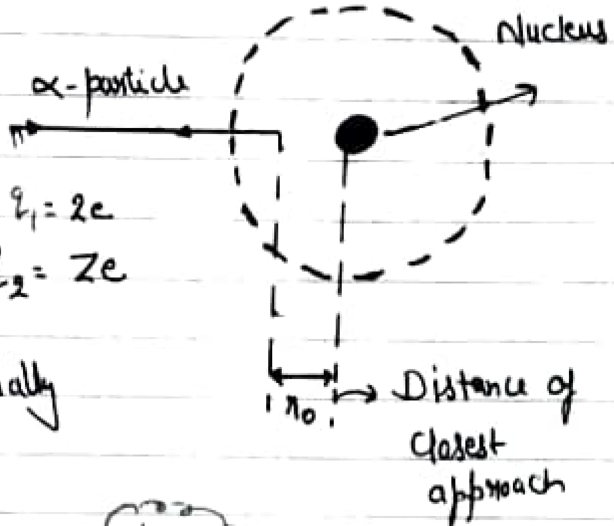
$$b = \frac{Ze^2 \cot(\theta/2)}{4\pi\epsilon_0\left(\frac{1}{2}mu^2\right)}$$

where

θ = Scattering angle



Distance Of closest approach → The minimum distance b/w the nucleus and the α particle, when the α particle travelling towards nucleus and momentarily comes to rest and Then began to retrace its path.



Net charge on α particle = $q_1 = 2e$
Charge on scattering nucleus = $q_2 = Ze$

If the α particle was initially moving with velocity v then

$$K.E_{initial} = \frac{1}{2}mv^2$$ $$K_f = 0$$

Electrostatic potential energy when α particle comes to rest at a distance r_0 from the nucleus.

$$U_f = \frac{Kq_1q_2}{r_0} = \frac{K(2e)(Ze)}{r_0} = \frac{2KZe^2}{r_0}$$

By conservation of energy

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = 0 + \frac{2KZe^2}{r_0}$$

$$\frac{1}{2}mv^2 = \frac{2KZe^2}{r_0}$$

$$r_0 = \frac{4KZe^2}{mv^2} \rightarrow r_0 = \frac{4KZe^2}{mv^2}$$

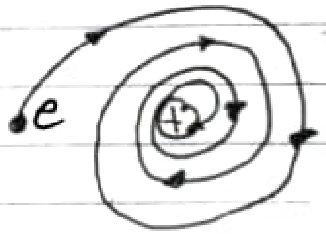
Limitations Of Rutherford's Atomic Model

According to electromagnetic theory an accelerated Charged particle must radiate electromagnetic energy.

An electron revolving around the nucleus is under continuous acceleration towards nucleus i.e centre.

It should continuously lose energy and move in a orbits of gradually decreasing radii.

The electron should follow spiral path and finally it should collapse into the nucleus.



Thus the Rutherford model can not explain the stability of an atom.

It failed to explain the spectrum of an atom

Bohr's Quantization condition

Consider the motion of an electron in a circular orbit of radius r. around the nucleus of an atom.

According to De Broglie Hypothesis this electron is also associated with wave character.

Circular orbit can be taken as stationary energy state only if it contains an integral number of de-Broglie wavelength i e

$$2\pi r = n\lambda \quad -① $$

But De Broglie wavelength is

$$\lambda = \frac{h}{mv} \quad -②$$

from ① and ②

$$\boxed{2\pi r = \frac{nh}{mv}} \Rightarrow mvr = \frac{nh}{2\pi} = \boxed{L = \frac{nh}{2\pi}}$$

Bohr's Quantisation Condition.

Bohr's Atomic Model : Postulates

1. **Bohr's first Postulate** ⇒ An electron in an atom could revolve in certain stable orbits without the emission of radiant energy contrary to the prediction of electromagnetic theory. The centripetal force required for this rotation is provided by the electrostatic attraction b/w the electrons and the nucleus.

2. **Bohr's Second Postulate** ⇒ According to this postulate electrons are allowed to revolve in only those orbits in which their angular momentum is integral multiple of $\frac{h}{2\pi}$

$$\boxed{L = mvr = \frac{nh}{2\pi}} \qquad n = 1, 2, 3, 4 \cdots$$

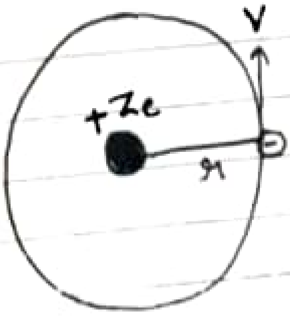
Bohr's Third Postulate $\Rightarrow$ It state that when an electrons jump from lower orbit to higher orbit then it absorbs energy.

And when it jump from higher to lower then it radiates energy.

$$\boxed{h\nu = E_i - E_f}$$

Bohr's Theory Of Hydrogen Atom

According to Bohr's theory a hydrogen atom consist of a nucleus with a positive charge Ze, and a single ~~char~~ electron with charge $-e$ which revolves around the nucleus in a circular orbit of radius r.



To keep the electron in a circular path centrepetal force must be equal to centrifugal force

$$F_{centripetal} = F_{centrifugal}$$

$$\frac{KZe^2}{r^2} = \frac{mv^2}{r}$$

$$\left\{ F = \frac{Kq_1 q_2}{r^2} \quad q_1 = Ze,\ q_2 = e \quad F = \frac{KZe^2}{r^2} \right.$$

$$r = \frac{KZe^2}{mv^2} \quad — (1)$$

Radii Of Bohr's Stationary orbits.

According to Bohr's Quantization condition of angular momentum

$$mvr = \frac{nh}{2\pi}$$

$$r = \frac{nh}{2\pi mv} \quad — (2)$$

from (1) and (2)

$$\frac{KZe^2}{mv^2} = \frac{nh}{2\pi mv}$$

$$v = \frac{2\pi KZe^2}{nh} \quad — (3)$$

Substitute the value of equation (3) in (2)

$$r = \frac{nh}{2\pi m} \cdot \frac{nh}{2\pi KZe^2} = \frac{n^2h^2}{4\pi^2 mKZe^2}$$

$$r = \frac{n^2h^2}{4\pi^2 mKZe^2}$$

$$r = \left(\frac{h^2}{4\pi^2 mKZe^2}\right)n^2 \Rightarrow \boxed{r = 0.53\,A^\circ}$$

when $n = 1$

$$r \propto n^2$$

II Velocity of an electron in hydrogen atom

$$V = \frac{2\pi Z K e^2}{nh} \qquad Z = 1$$

$$V = \frac{2\pi K e^2}{nh} \times \frac{c}{c} \Rightarrow V = \frac{c}{137n}$$

$n = 1$

$$V = \frac{c}{137}$$

III Frequency of Electron in Bohr's Stationary orbit

$$V = \frac{2\pi K Z e^2}{nh}$$

$$\nu = \frac{V}{2\pi r} = \frac{2\pi K Z e^2}{2\pi r}$$

$$T = \frac{2\pi r}{V}$$

$$\nu = \frac{1}{T} = \frac{V}{2\pi r}$$

Energy of the electron

Kinetic Energy of Electron in n^{th} orbit

$$K.E = \frac{1}{2}mv^2 = \frac{KZe^2}{2r}$$

$$\frac{mv^2}{\cancel{r}} = \frac{KZe^2}{r^2}$$

$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{KZe^2}{r}$$

$$K.E = \frac{KZe^2}{2r}$$

$$P.E = \frac{K q_1 q_2}{r} = \frac{K Ze(-e)}{r}$$

$$P.E = -\frac{KZe^2}{r}$$

Total Energy of Electron in n^{th} orbit

$$E_n = K.E + P.E$$

$$E_n = \frac{KZe^2}{2r} + \left[-\frac{KZe^2}{r}\right]$$

$$E_n = -\frac{KZe^2}{2r}$$

$$K.E = -T.E$$

☆☆☆☆

if $K.E = x$

Then $P.E = -2x$

$T.E = -x$

Spectral Series Of hydrogen Atom

From Bohr's Theory, the energy of an electron in the n^{th} orbit of hydrogen atom is given by

$$E_n = -\frac{KZe^2}{2r}$$

$$r = \frac{n^2 h^2}{4\pi^2 mKZe^2}$$

$$E_n = -\frac{2\pi^2 mK^2 Z^2 e^4}{h^2} \cdot \frac{1}{n^2}$$

According to Bohr whenever an electron makes a transition from higher energy (n_2) level to lower energy level n_1, the difference of energy appears in the form of a photon.

The frequency ν of the emitted photon is given by

$$h\nu = E_{n_2} - E_{n_1}$$

$$h\nu = \left(-\frac{2\pi^2 m k^2 e^4}{n_2^2 h^2}\right) - \left(-\frac{2\pi^2 m k^2 e^4}{n_1^2 h^2}\right)$$

$$h\nu = \frac{2\pi^2 m k^2 e^4}{h^2}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\nu = \frac{2\pi^2 m k^2 e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$c = \nu\lambda \Rightarrow \nu = \frac{c}{\lambda}$$

$$\frac{c}{\lambda} = \frac{2\pi^2 m k^2 e^4}{h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\frac{1}{\lambda} = \frac{2\pi^2 m k^2 e^4}{c h^3}\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$\boxed{\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]}$$

n_1 = final

n_2 = Initial

R = Rydberg constant $= \frac{2\pi^2 m k^2 e^4}{c h^3} = 1.0973 \times 10^7 \, m^{-1}$

~~Series~~ Spectral Series of hydrogen

(i) Lyman Series ⇒ If an electron jumps from any higher energy level $n_2 = 2, 3, 4 \cdots$ to a lower energy level $n_1 = 1$ we get a set of spectral lines known as lyman series.

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_2^2}\right], \quad n_2 = 2, 3, 4, 5, \cdots$$

This series belong to ultraviolet region.

(ii) Balmer Series ⇒

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{n_2^2}\right]; \quad n_2 = 3, 4, 5, 6 \cdots$$

This series belongs to visible region.

(iii) Paschen Series ⇒

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right], \quad n_2 = 4, 5, 6 \cdots$$

Infrared

(iv) Brackett Series ⇒

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right], \quad n_2 = 5, 6, 7 \cdots$$

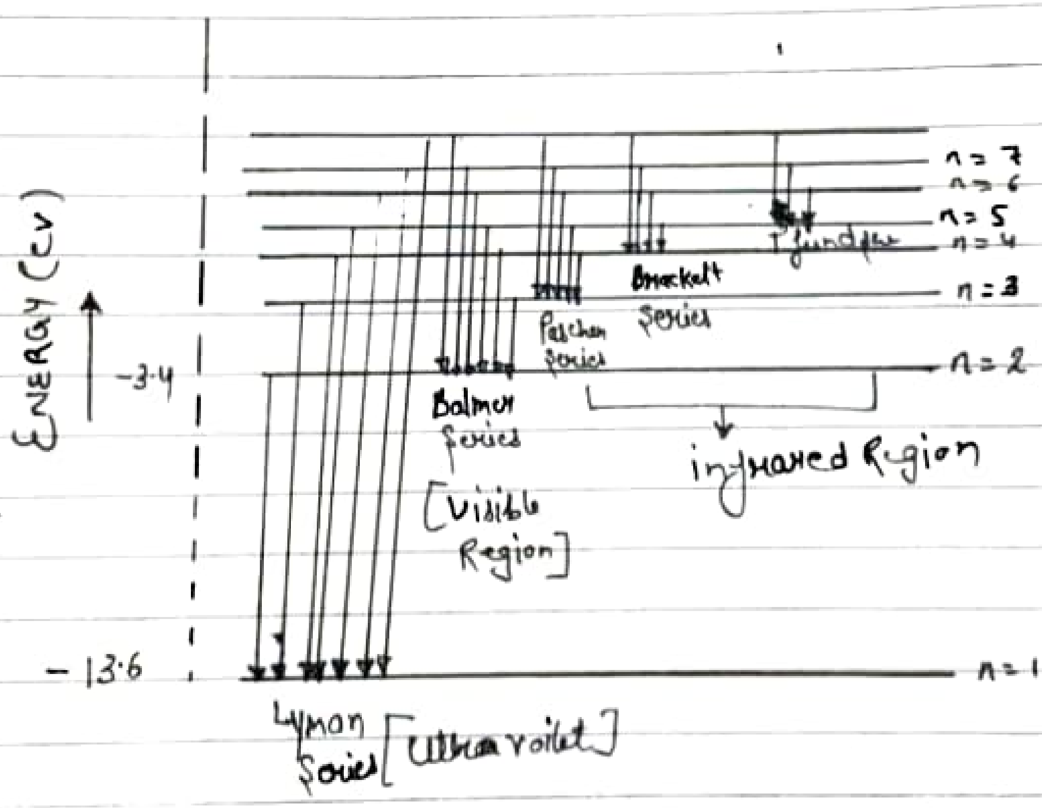
Infrared.

To convert Joule into eV we divide Joule by 1.6×10^{-19}

Pfund Series ⇒

$$\bar{\nu} = \frac{1}{\lambda} = \left[\frac{1}{5^2} - \frac{1}{n_2^2}\right] \qquad n_2 = 6, 7$$

Infrared Region



Energy Levels Of hydrogen

$$E_n = \frac{2\pi^2 m K^2 Z^2 e^4}{n^2 h^2}$$

$Z = 1$, $\pi = 3.14$ $m = 9.1 \times 10^{-31}$ kg

$n = 1$ $e = 1.6 \times 10^{-19}$ C $h = 6.63 \times 10^{-34}$

$\cancel{E} : \cancel{\frac{13.6}{n^2}}$ $\cancel{eV}$

$$E_n = -21.76 \times 10^{-19} \text{ J} = \frac{-21.76 \times 10^{-19}}{1.6 \times 10^{-19}} \text{ eV} = -13.6 \text{ eV}$$

☆☆☆

Ground state ⇒ The energy state corresponding to $n = 1$ has lowest energy equal to $-13.6\,eV$.

Excited state ⇒ The energy state corresponding to $n = 2, 3 \ldots$ are called excited state

$n = 2$ First Excited state
$n = 3$ Second Excited state
n . . . .

Excitation Energy = It is defined as the amount of energy required by the electron to jump from ground state to any one of the excited state.

First Excitation : $E_2 - E_1$
Second Excitation = $E_3 - E_1$

Ionisation Energy = It is the amount of energy required to knock an electron completely out of the atom.

$$\text{Ionisation energy} = E_\infty - E_1 = 0 - (-13.6)$$
$$= 13.6\,eV$$

Excitation potential ⇒ It is that accelerating potential which when applied to a bombarding electron gives sufficient energy to excite the target atom by raising one of its electrons from an inner to an outer orbit.

☆☆ First Excitation potential of hydrogen $= -\frac{13.6}{2^2} - \left(-\frac{13.6}{1^2}\right)$
$$= 10.2\,eV$$

Second Excitation potential $= -1.51 - (-13.6) = 12.09\,V$

Some Important points Regarding Numerical

1) Wave length of light emitted will be maximum when b/w two energy level which are having minimum difference and vice - versa.

(2) For Ground state $n = 1$
For excited state First $n = 2$
Second $n = 3$

3 For calculating shortest wave length of any series $n_2 = \infty$

4 For calculating the first member wave length of series
for → Lyman $n_1 = 1$ $n_2 = 2$
→ Balmer $n_1 = 2$ $n_2 = 3$
→ Paschen $n_3 = 3$ $n_3 = 4$ and so on

(5) Kinetic Energy = − T.E
Potential Energy = 2 (T.E)

(6) Total No. of spectral line

$$N = \frac{n(n-1)}{2}$$

7) Radius of orbit
$r = r_0 n^2$ $r_0 = 5.3 \times 10^{-11} m$

$2\pi r = n\lambda$