IMPORTANT SNAPS BY TEAM PIS CLASS- IX TH

Subject: MATHEMATICS

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(chapter 1 to 10)

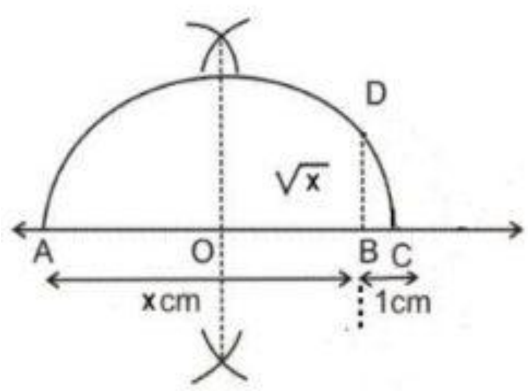
HIGHLIGHTS OF CHAPTER 1 (NUMBER SYSTEM)

- Rational Numbers
- A number 'r' is called a rational number if it can be written in the form p/q, where p and q are integers and $q \neq 0$.
- Irrational Numbers
- Any number that cannot be expressed in the form of p/q, where p and q are integers and $q \neq 0$, is an irrational number. Examples: $\sqrt{2}$, 1.010024563…, e, π
- Real Numbers
- Any number which can be represented on the number line is a Real Number**(R).** It includes both rational and irrational numbers. Every point on the number line represents a unique real number.

Irrational Numbers

Representation of Irrational numbers on the Number line

- Let √x be an irrational number. To represent it on the number line we will follow the following steps:
- Take any point A. Draw a line AB = x units.
- Extend AB to point C such that BC = 1 unit.
- Find out the mid-point of AC and name it 'O'. With 'O' as the centre draw a semi-circle with radius OC.
- Draw a straight line from B which is perpendicular to AC, such that it intersects the semi-circle at point D.
- Length of BD=√x.
- **Constructions to Find the root of *x*.** With BD as the radius and origin as the centre, cut the positive side of the number line to get √x



Arithmetic operations between:

- rational and irrational will give an irrational number.
- irrational and irrational will give a rational or irrational number.
- Example : $2 \times \surd 3 = 2\surd 3$ i.e. irrational. $\surd 3 \times \surd 3 = 3$ which is rational.

Identities for irrational numbers

- If a and b are real numbers then:
  - $\surd ab = \surd a \surd b$
  - $\surd ab = \surd a \surd b$
  - $(\surd a+\surd b)\ (\surd a-\surd b) = a - b$
  - $(a+\surd b)(a-\surd b) = a^2-b$
  - $(\surd a+\surd b)(\surd c+\surd d) = \surd ac+\surd ad+\surd bc+\surd bd$
  - $(\surd a+\surd b)(\surd c-\surd d) = \surd ac-\surd ad+\surd bc-\surd bd$
  - $(\surd a+\surd b)^2 = a+2\surd(ab)+b$
- **Rationalisation**
- Rationalisation is converting an irrational number into a rational number. Suppose if we have to rationalise $1/\surd a$.
  $1/\surd a \times 1/\surd a = 1/a$
- Rationalisation of $1/\surd a+b$:
- $(1/\surd a+b) \times (1/\surd a-b) = (1/a-b^2)$

Laws of Exponents for Real Numbers

- If a, b, m and n are real numbers then:
- $a^m \times a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $a^m / a^n = a^{m-n}$
- $a^m b^m = (ab)^m$
- Here, *a* and *b* are the bases and *m* and *n* are exponents.

Exponential representation of irrational numbers

- If a > 0 and n is a positive integer, then: n√a=a1n Let a > 0 be a real number and p and q be rational numbers, then:
- $a^p \times a^q = a^{p+q}$
- $(a^p)^q = a^{pq}$
- $a^p / a^q = a^{p-q}$
- $a^p b^p = (ab)^p$

Decimal Representation of Rational Numbers and irrational numbers

- The decimal expansion of a rational number is either terminating or non- terminating and recurring.
- Example: 1/2 = 0.5 , 1/3 = 3.33…….
The decimal expansion of an irrational number is non terminating and non-recurring.
Examples: √2 = 1.41421356..

IMPORTANT QUESTIONS

- Q.1: Find five rational numbers between 1 and 2. [Answer: 7/6, 8/6, 9/6, 10/6, 11/6]
- Q.2: Find five rational numbers between 3/5 and 4/5. [Answer: 19/30, 20/30, 21/30, 22/30, 23/30]
- Q.3: Locate √3 on the number line.
- Q.4: Are the square roots of all positive integers irrational? If not, give an example of the square root of a number that is a rational number.
- Q.5: Find the decimal expansions of 10/3, 7/8 and 1/7.
- Q.6: Show that 0.3333...=$0.\overline{3}$ can be expressed in the form p/q, where p and q are integers and q ≠ 0.
- Q.7: What can the maximum number of digits be in the repeating block of digits in the decimal expansion of 1/17? Perform the division to check your answer.
- Q.8: Find three different irrational numbers between the rational numbers 5/7 and 9/11. [Answer: 0.720720072000.., 0.730730073000..., 0.808008000...]
- Q.9: Visualise 3.765 on the number line, using successive magnification
- Q.10: Add 2√2+ 5√3 and √2 – 3√3. [Answer: 3√2 + 2√3]
- Q.11: Simplify: (√3+√7) (√3-√7). [Answer: -4]
- Q.12: Rationalise the denominator of 1/[7+3√3].

HIGHLIGHTS OF CHAPTER 2 (POLYNOMIALS)

- Polynomials are expressions with one or more terms with a non-zero coefficient.
- In the polynomial, each expression in it is called a **term**.
- Suppose $x^2 + 5x + 2$ is polynomial, then the expressions x^2, $5x$, and 2 are the terms of the polynomial.
- Each term of the polynomial has a **coefficient**. For example, if $2x + 1$ is the polynomial, then the coefficient of x is 2.
- The real numbers can also be expressed as polynomials. Like 3, 6, 7, are also polynomials without any variables. These are called **constant polynomials**.
- The constant polynomial 0 is called zero polynomial.
- The exponent of the polynomial should be a whole number. For example, $x^{-2} + 5x + 2$, cannot be considered as a polynomial, since the exponent of x is -2, which is not a whole number.
- The highest power of the polynomial is called the **degree of the polynomial**. For example, in $x^3 + y^3 + 3xy(x + y)$, the degree of the polynomial is 3.
- For a non zero constant polynomial, the degree is zero.
- Apart from these, there are other types of polynomials such as:
- Linear polynomial – of degree one
- Quadratic Polynomial- of degree two
- Cubic Polynomial – of degree three

- A polynomial of degree 1 is called as a **linear polynomial.**
- A polynomial of degree 2 is called a **quadratic polynomial.**
- A polynomial of degree 3 is called a **cubic polynomial.**
- A polynomial of 1 term is called a **monomial.**
- A polynomial of 2 terms is called **binomial.**
- A polynomial of 3 terms is called a **trinomial.**
- A real number 'a' is a zero of a polynomial p(x) if p(a) = 0, where a is also known as root of the equation p(x) = 0.
- A linear polynomial in one variable has a unique zero, a polynomial of a non-zero constant has no zero, and each real number is a zero of the zero polynomial.
- **Remainder Theorem:** If p(x) is any polynomial having degree greater than or equal to 1 and if it is divided by the linear polynomial x – a, then the remainder is p(a).
- **Factor Theorem** : x – c is a factor of the polynomial p(x), if p(c) = 0. Also, if x – c is a factor of p(x), then p(c) = 0.
- The degree of the zero polynomial is not defined.
- $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$
- $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
- $(x - y)^3 = x^3 - y^3 - 3xy(x - y)$

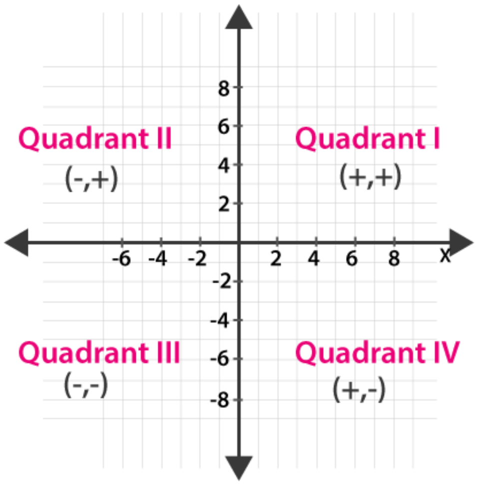
IMPORTANT QUESTIONS

- **Q. Compute the value of $9x^2 + 4y^2$ if $xy = 6$ and $3x + 2y = 12$.**
- **Solution:**
- Consider the equation $3x + 2y = 12$
- Now, square both sides:
- $(3x + 2y)^2 = 12^2$
- $\Rightarrow 9x^2 + 12xy + 4y^2 = 144$
- $\Rightarrow 9x^2 + 4y^2 = 144 - 12xy$
- From the questions, $xy = 6$
- So,
- $9x^2 + 4y^2 = 144 - 72$
- Thus, the value of $9x^2 + 4y^2 = 72$
- **Q Find the value of the polynomial $5x - 4x^2 + 3$ at $x = 2$ and $x = -1$**
- **Solution:**
- Let the polynomial be $f(x) = 5x - 4x^2 + 3$
- Now, for $x = 2$,
- $f(2) = 5(2) - 4(2)^2 + 3$
- $\Rightarrow f(2) = 10 - 16 + 3 = -3$
- Or, the value of the polynomial $5x - 4x^2 + 3$ at $x = 2$ is -3.
- Similarly, for $x = -1$,
- $f(-1) = 5(-1) - 4(-1)^2 + 3$
- $\Rightarrow f(-1) = -5 -4 + 3 = -6$
- The value of the polynomial $5x - 4x^2 + 3$ at $x = -1$ is -6.

- **Q. Calculate the perimeter of a rectangle whose area is $25x^2 - 35x + 12$.**
- **Solution:**
- Given,
- Area of rectangle = $25x^2 – 35x + 12$
- We know, area of rectangle = length × breadth
- So, by factoring $25x^2 – 35x + 12$, the length and breadth can be obtained.
- $25x^2 – 35x + 12 = 25x^2 – 15x – 20x + 12$
- => $25x^2 – 35x + 12 = 5x(5x – 3) – 4(5x – 3)$
- => $25x^2 – 35x + 12 = (5x – 3)(5x – 4)$
- So, the length and breadth are $(5x – 3)(5x – 4)$.
- Now, perimeter = 2(length + breadth)
- So, perimeter of the rectangle = $2[(5x – 3)+(5x – 4)]$
- $= 2(5x – 3 + 5x – 4) = 2(10x – 7) = 20x – 14$
- So, the perimeter = $20x – 14$

HIGHLIGHTS OF CHAPTER 3 (COORDINATE GEOMETRY)

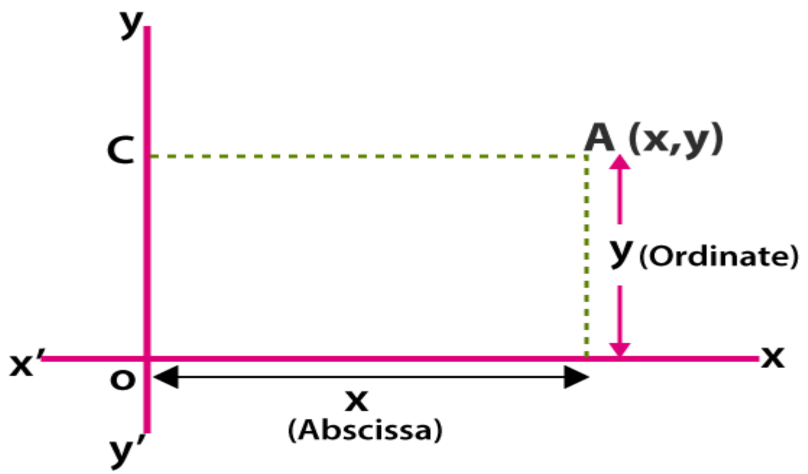
- **Cartesian Plane**: A cartesian plane is defined by **two perpendicular number lines**, A horizontal line(x−axis) and a vertical line (y−axis).
- These lines are called coordinate axes. The Cartesian plane extends infinitely in all directions.
- **Origin**: The coordinate axes intersect each other at right angles, **The point of intersection** of these two axes is called Origin.
- The cartesian plane is divided into four equal parts, called **quadrants.** These are named in the order as I, II, III and IV starting with the upper right and going around in anticlockwise direction.



- Points in different Quadrants.
- Signs of coordinates of points in different quadrants:
- **I Quadrant:** '+' x – coordinate and '+' y – coordinate. E.g. (2, 3)
- **II Quadrant:** '-' x – coordinate and '+' y – coordinate. E.g. (-1, 4)
- **III Quadrant:** '-' x – coordinate and '-' y – coordinate. E.g. (-3, -5)
- **IV Quadrant:** '+' x – coordinate and '-' y – coordinate. E.g. (6, -1)

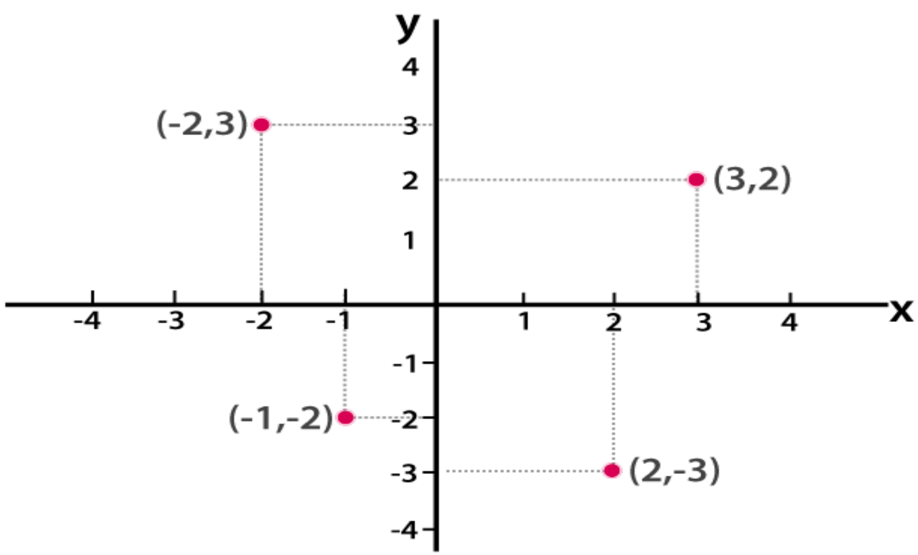
Plotting on a Graph

- Representation of a point on the Cartesian plane
- Using the co-ordinate axes, we can describe any point in the plane using an ordered pair of numbers. A point **A** is represented by an ordered pair (x, y) where x is the **abscissa** and y is the **ordinate** of the point.



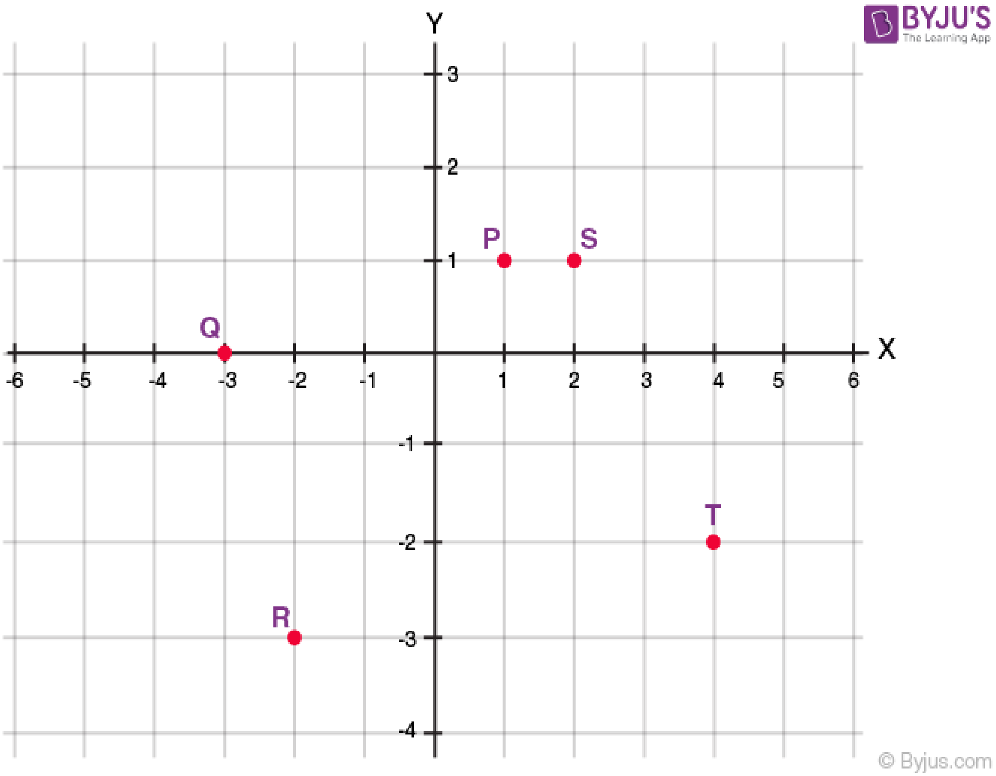
Plotting a point

- The coordinate points will define the location in the cartesian plane. The first point (x) in the coordinates represents the horizontal axis, and the second point in the coordinates (y) represents the vertical axis. Consider an example, Point (3, 2) is 3 units away from the positive y-axis and 2 units away from the positive x-axis. Therefore, point (3, 2) can be plotted, as shown below. Similarly, (-2, 3), (-1, -2) and (2, -3) are plotted.



IMPORTANT QUESTIONS

- **1. Write the coordinates of each of the points P, Q, R, S, T and O from the figure given.**
- Solution:
- The coordinates of the points P, Q, R, S, T and O are as follows:
- P = (1, 1)
- Q = (-3, 0)
- R = (-2, -3)
- S = (2, 1)
- T = (4, -2)
- O = (0, 0)



- **Q.: Without plotting the points indicate the quadrant in which they will lie, if**
- **(i) the ordinate is 5 and abscissa is - 3**
- **(ii) the abscissa is - 5 and ordinate is - 3**
- **(iii) the abscissa is - 5 and ordinate is 3**
- **(iv) the ordinate is 5 and abscissa is 3**
- Solution:
- (i) The point is (-3,5).
- Hence, the point lies in the II quadrant.
- (ii) The point is (-5,-3).
- Hence, the point lies in the III quadrant.
- (iii) The point is (-5,3).
- Hence, the point lies in the II quadrant.
- (iv) The point is (3,5).
- Hence, the point lies in the I quadrant.
- **Q.: Write the answer to each of the following questions:**
- **(i) What is the name of the horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?**
- **(ii) What is the name of each part of the plane formed by these two lines?**
- **(iii) Write the name of the point where these two lines intersect**
- Solution:
- (i) The name of horizontal and vertical lines drawn to determine the position of any point in the Cartesian plane is x-axis and y-axis respectively.
- (ii) The name of each part of the plane formed by these two lines x-axis and the y-axis is called quadrants.
- (iii) The point where these two lines intersect is called the origin

HIGHLIGHTS OF CHAPTER 4 (LINEAR EQUATIONS IN TWO VARIABLES)

- Linear equation in 2 variables
- When an equation has two variables both of degree one, then that equation is known as linear equation in two variables.
- Standard form: **ax+by+c=0**, where a, b, c ϵ R & a, b ≠ 0
  Examples of linear equations in two variables are:
  - 7x+y=8
  - 6p-4q+12=0
- The solution of linear equation in 2 variables
- A linear equation in two variables has a pair of numbers that can satisfy the equation. This pair of numbers is called as the solution of the linear equation in two variables.
- The solution can be found by assuming the value of one of the variable and then proceed to find the other solution.
- There are infinitely many solutions for a single linear equation in two variables.

- Graphical representation of a linear equation in 2 variables
- Any linear equation in the standard form ax+by+c=0 has a pair of solutions in the form (x,y), that can be represented in the coordinate plane.
- When an equation is represented graphically, it is a straight line that may or may not cut the coordinate axes.
- Solutions of Linear equation in 2 variables on a graph
- A linear equation ax+by+c=0 is represented graphically as a straight line.
- Every point on the line is a solution for the linear equation.
- Every solution of the linear equation is a point on the line.
- Lines parallel to coordinate axes
- Linear equations of the form y=a, when represented graphically are lines parallel to the x-axis and a is the y-coordinate of the points in that line.
- Linear equations of the form x=a, when represented graphically are lines parallel to the y-axis and a is the x-coordinate of the points in that line.

IMPORTANT QUESTIONS

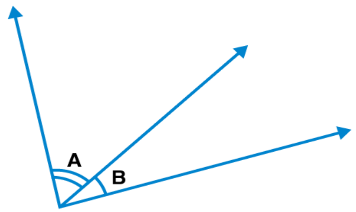
- **Q.: Express the following linear equations in the form ax + by + c = 0 and indicate the values of a, b and c in each case:**
- **(i) x - y/5 - 10 = 0**
- **So**lution:
- (i) The equation x-y/5-10 = 0 can be written as:
- (1)x + (-1/5) y + (-10) = 0
- Now compare the above equation with ax + by + c = 0
- Thus, we get;
- a = 1
- b = $-\frac{1}{5}$
- c = -10
- **Q: Find the value of k, if x = 2, y = 1 is a solution of the equation 2x + 3y = k.**
- Solution:
- The given equation is
- 2x + 3y = k
- According to the question, x = 2 and y = 1.
- Now, Substituting the values of x and y in the equation 2x + 3y = k,
- We get,
- $\Rightarrow (2 \times 2) + (3 \times 1) = k$
- $\Rightarrow 4+3 = k$
- $\Rightarrow 7 = k$
- $\Rightarrow k = 7$
- The value of k, if x = 2, y = 1 is a solution of the equation 2x + 3y = k, is 7.

- **Q. Show that the points A (1, 2), B (- 1, - 16) and C (0, - 7) lie on the graph of the linear equation y = 9x - 7.**
- Solution:
- We have the equation,
- y = 9x – 7
- For A (1, 2),
- Substituting (x,y) = (1, 2),
- We get,
- 2 = 9(1) – 7
- 2 = 9 – 7
- 2 = 2
- For B (–1, –16),
- Substituting (x,y) = (-1, -16),
- We get,
- –16 = 9(–1) – 7
- -16 = – 9 – 7
- -16 = - 16
- For C (0, –7),
- Substituting (x,y) = (0, –7),
- We get,
- – 7 = 9(0) – 7
- -7 = 0 – 7
- -7 = – 7
- Hence, the points A (1, 2), B (–1, –16) and C (0, –7) satisfy the line y = 9x – 7.
- Thus, A (1, 2), B (–1, –16) and C (0, –7) are solutions of the linear equation y = 9x – 7
- Therefore, the points A (1, 2), B (–1, –16), C (0, –7) lie on the graph of linear equation y = 9x – 7.

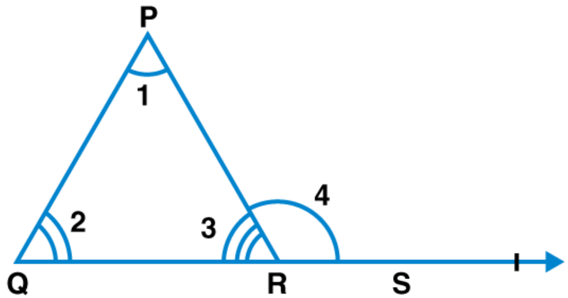
HIGHLIGHTS OF CHAPTER 6 (LINES AND ANGLES)

- **<u>Angles and types of angles</u>**
- When 2 rays originate from the same point at different directions, they form an angle.
- – The rays are called arms and the common point is called the vertex
  – Types of angles : (i) Acute angle $0^\circ < a < 90^\circ$
  (ii) Right angle $a = 90^\circ$
  (iii) Obtuse angle : $90^\circ < a < 180^\circ$
  (iv) Straight angle $= 180^\circ$
  (v) Reflex Angle $180^\circ < a < 360^\circ$
  (vi) Angles that add up to 90° are complementary angles
  (vii) Angles that add up to 180° are called supplementary angles.
- When 2 lines meet at a point they are called intersecting
- When 2 lines never meet at a point, they are called non-intersecting or parallel lines.

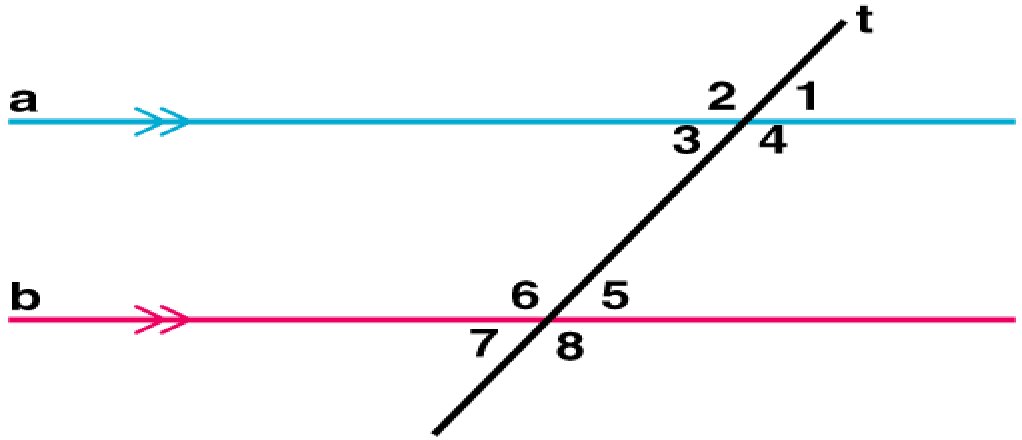
- <u>Adjacent angles</u>
- 2 angles are adjacent if they have the same vertex and one common point.



- <u>Linear Pair</u>
- When 2 adjacent angles are supplementary, i.e they form a straight line (add up to 180∘), they are called a linear pair.
- Vertically opposite angles
- When two lines intersect at a point, they form equal angles that are vertically opposite to each other.
- **Basic Properties of a Triangle**
- Triangle and sum of its internal angles
- Sum of all angles of a triangle add up to 180∘
- **An exterior angle of a triangle = sum of opposite internal angles**
- – If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles

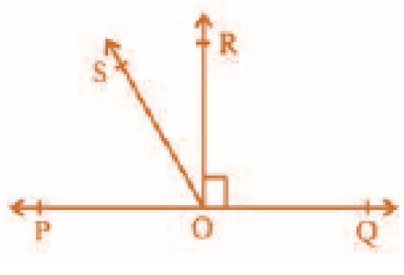


- **Parallel lines with a transversal**
- ∠1=∠5,∠2=∠6,∠4=∠8 and ∠3=∠7(Corresponding angles)
- ∠3=∠5,∠4=∠6 (Alternate interior angles)
- ∠1=∠7,∠2=∠8 (Alternate exterior angles)
- Lines parallel to the same line
- Lines that are parallel to the same line are also parallel to each other.

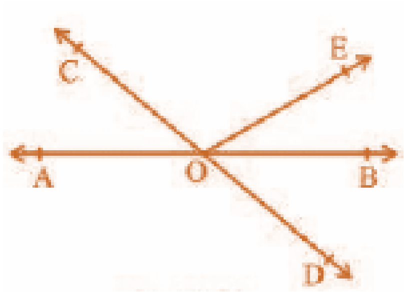


IMPORTANT QUESTIONS

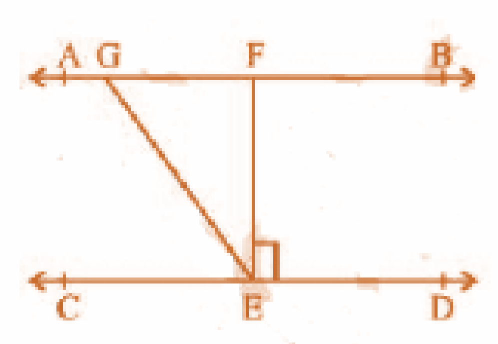
- **Q.: In the Figure, POQ is a line. Ray OR is perpendicular to line PQ. OS is another ray lying between rays OP and OR. Prove that ∠ROS = 1/2(∠QOS - ∠POS).**
- Solution:
- In the question, it is given that (OR ⊥ PQ) and ∠POQ = 180°
- So, ∠POS + ∠ROS + ∠ROQ = 180° (Linear pair of angles)
- Now, ∠POS + ∠ROS = 180° - 90° (Since ∠POR = ∠ROQ = 90°)
- ∴ ∠POS + ∠ROS = 90°
- Now, ∠QOS = ∠ROQ + ∠ROS
- It is given that ∠ROQ = 90°,
- ∴ ∠QOS = 90° + ∠ROS
- Or, ∠QOS – ∠ROS = 90°
- As ∠POS + ∠ROS = 90° and ∠QOS – ∠ROS = 90°, we get
- ∠POS + ∠ROS = ∠QOS – ∠ROS
- =>2 ∠ROS + ∠POS = ∠QOS
- Or, ∠ROS = ½ (∠QOS – ∠POS) (Hence proved).



- **Q.: In the figure, lines AB and CD intersect at O. If ∠AOC + ∠BOE = 70° and ∠BOD = 40°, find ∠BOE and reflex ∠COE.**
- Solution:
- From the given figure, we can see;
- ∠AOC, ∠BOE, ∠COE and ∠COE, ∠BOD, ∠BOE form a straight line each.
- So, ∠AOC + ∠BOE +∠COE = ∠COE +∠BOD + ∠BOE = 180°
- Now, by substituting the values of ∠AOC + ∠BOE = 70° and ∠BOD = 40° we get:
- 70° +∠COE = 180°
- ∠COE = 110°
- Similarly,
- 110° + 40° + ∠BOE = 180°
- ∠BOE = 30°



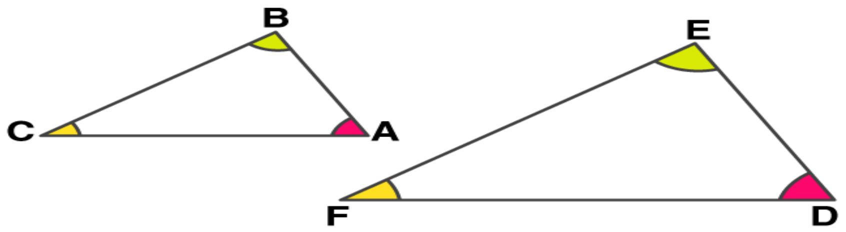
- **Q.: In the Figure, if AB || CD, EF ⊥ CD and ∠GED = 126°, find ∠AGE, ∠GEF and ∠FGE.**
- Solution:
- Since AB || CD GE is a transversal.
- It is given that ∠GED = 126°
- So, ∠GED = ∠AGE = 126° (alternate interior angles)
- Also,
- ∠GED = ∠GEF + ∠FED
- As
- EF ⊥ CD, ∠FED = 90°
- ∴ ∠GED = ∠GEF + 90°
- Or, ∠GEF = 126° – 90° = 36°
- Again, ∠FGE + ∠GED = 180° (Transversal)
- Substituting the value of ∠GED = 126° we get,
- ∠FGE = 54°
- So,
- ∠AGE = 126°
- ∠GEF = 36° and
- ∠FGE = 54°



HIGHLIGHTS OF CHAPTER 7 (TRIANGLES)

- **Congruent Triangles**
- In a pair of triangles if all three corresponding sides and three corresponding angles are exactly equal, then the triangles are said to be congruent.
- In congruent triangles, the corresponding parts are equal and are written as **CPCT** (Corresponding part of the congruent triangle).
- **Criteria for Congruency**
- The following are the criteria for the congruency of the triangles.
- **SSS Criteria for Congruency**
- If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.
- If all sides are exactly the same, then their corresponding angles must also be exactly the same.
- **SAS Criteria for Congruency**
- – Axiom: Two triangles are congruent if two sides and the **included** angle of one triangle are equal to the corresponding sides and the included angle of the other triangle.
- **ASA Criteria for Congruency**
- – Two triangles are congruent if two angles and the **included** side of one triangle are equal to the corresponding two angles and the included side of the other triangle
- **AAS Criteria for Congruency**
- – Two triangles are said to be congruent to each other if two angles and one side of one triangle are equal to two angles and one side of the other triangle.

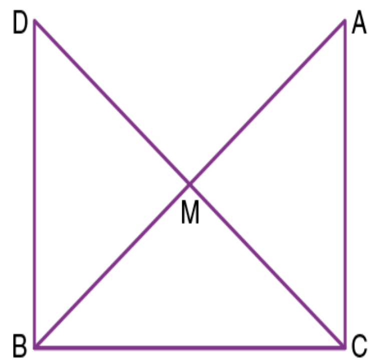
- **Why SSA and AAA congruency rules are not valid?**
- SSA or ASS test is not a valid test for congruency as the angle is not included between the pairs of equal sides.-
- The AAA test also is not a valid test as even though 2 triangles can have all three same angles, the sides can be of differing lengths. This becomes a test for similarity (AA).



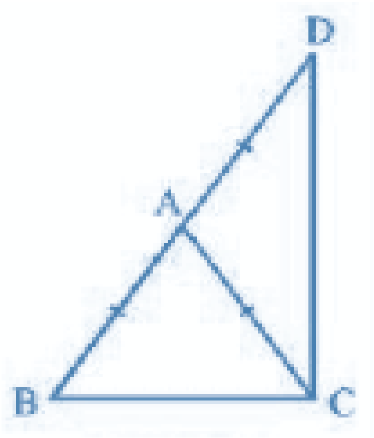
- **RHS Criteria for Congruency**
- If in two right triangles the hypotenuse and one side of one triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent.
- **RHS stands for Right angle – Hypotenuse – Side.**
- **Properties of Isosceles triangle**
- – If 2 sides of the triangle are equal, the angles opposite those sides are also equal and vice versa.

IMPORTANT QUESTIONS

- **Q.: In right triangle ABC, right-angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see the figure). Show that:**
- **(i) $\Delta AMC \cong \Delta BMD$**
- **(ii) $\angle DBC$ is a right angle.**
- **(iii) $\Delta DBC \cong \Delta ACB$**
- **(iv) CM = 1/2 AB**
- Solution:
- It is given that M is the mid-point of the line segment AB, $\angle C = 90^\circ$, and DM = CM
- (i) Consider the triangles ΔAMC and ΔBMD:
- AM = BM (Since M is the mid-point)
- CM = DM (Given)
- $\angle CMA = \angle DMB$ (Vertically opposite angles)
- So, by SAS congruency criterion, $\Delta AMC \cong \Delta BMD$.
- (ii) $\angle ACM = \angle BDM$ (by CPCT)
- $\therefore$ AC ∥ BD as alternate interior angles are equal.
- Now, $\angle ACB + \angle DBC = 180^\circ$ (Since they are co-interiors angles)
- $\Rightarrow 90^\circ + \angle B = 180^\circ$
- $\therefore \angle DBC = 90^\circ$
- (iii) In ΔDBC and ΔACB,
- BC = CB (Common side)
- $\angle ACB = \angle DBC$ (Both are right angles)
- DB = AC (by CPCT)
- So, $\Delta DBC \cong \Delta ACB$ by SAS congruency.
- (iv) DC = AB (Since $\Delta DBC \cong \Delta ACB$)
- $\Rightarrow$ DM = CM = AM = BM (Since M the is mid-point)
- So, DM + CM = BM + AM
- Hence, CM + CM = AB
- $\Rightarrow$ CM = (½) AB

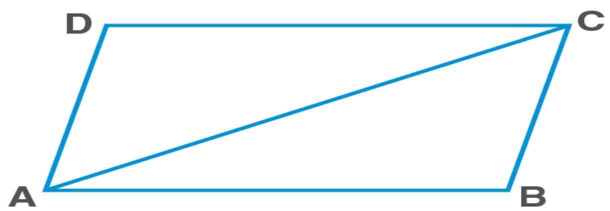


- **Q.: ΔABC is an isosceles triangle in which AB = AC. Side BA is produced to D such that AD = AB. Show that $\angle BCD$ is a right angle.**
- Solution:
- Given, AB = AC and AD = AB
- To prove: $\angle BCD$ is a right angle.
- Proof:
- Consider ΔABC,
- AB = AC (Given)
- Also, $\angle ACB = \angle ABC$ (Angles opposite to equal sides)
- Now, consider ΔACD,
- AD = AC
- Also, $\angle ADC = \angle ACD$ (Angles opposite to equal sides)
- Now,
- In ΔABC,
- $\angle CAB + \angle ACB + \angle ABC = 180^\circ$
- So, $\angle CAB + 2\angle ACB = 180^\circ$
- $\Rightarrow \angle CAB = 180^\circ - 2\angle ACB$ — (i)
- Similarly in ΔADC,
- $\angle CAD = 180^\circ - 2\angle ACD$ — (ii)
- Also,
- $\angle CAB + \angle CAD = 180^\circ$ (BD is a straight line.)
- Adding (i) and (ii) we get,
- $\angle CAB + \angle CAD = 180^\circ - 2\angle ACB + 180^\circ - 2\angle ACD$
- $\Rightarrow 180^\circ = 360^\circ - 2\angle ACB - 2\angle ACD$
- $\Rightarrow 2(\angle ACB + \angle ACD) = 180^\circ$
- $\Rightarrow \angle BCD = 90^\circ$



HIGHLIGHTS OF CHAPTER 8 (QUADRILATERALS)

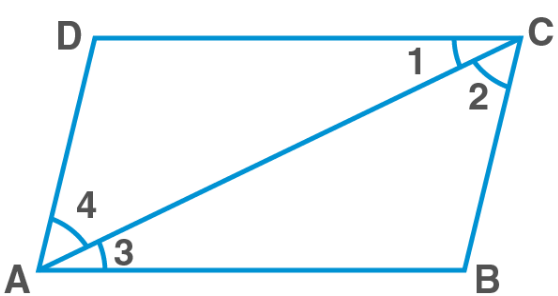
- **<u>Parallelogram: Opposite sides of a parallelogram are equal</u>**
- In ΔABC and ΔCDA



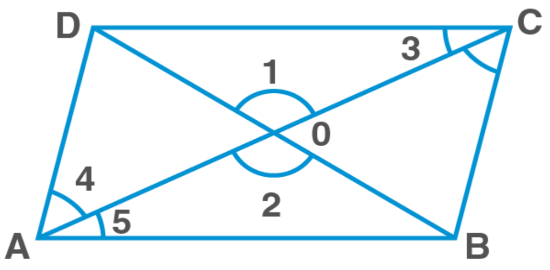
- AC=AC [Common / transversal]
- $\angle BCA=\angle DAC$ [alternate angles]
- $\angle BAC=\angle DCA$ [alternate angles]
- $\Delta ABC \cong \Delta CDA$ [ASA rule]
- Hence,
- AB=DC and AD=BC [C.P.C.T.C]

<u>Opposite angles in a parallelogram are equal</u>

- In parallelogram ABCD
- $AB \parallel CD$; and AC is the transversal
- Hence, $\angle 1=\angle 3$….(1) (alternate interior angles)
- $BC \parallel DA$; and AC is the transversal
- Hence, $\angle 2=\angle 4$….(2) (alternate interior angles)
- Adding (1) and (2)
- $\angle 1+\angle 2=\angle 3+\angle 4$
- $\angle BAD=\angle BCD$
- Similarly, $\angle ADC=\angle ABC$



- **Properties of diagonal of a parallelogram**
- – **Diagonals of a parallelogram bisect each other.**
- In ΔAOB and ΔCOD,



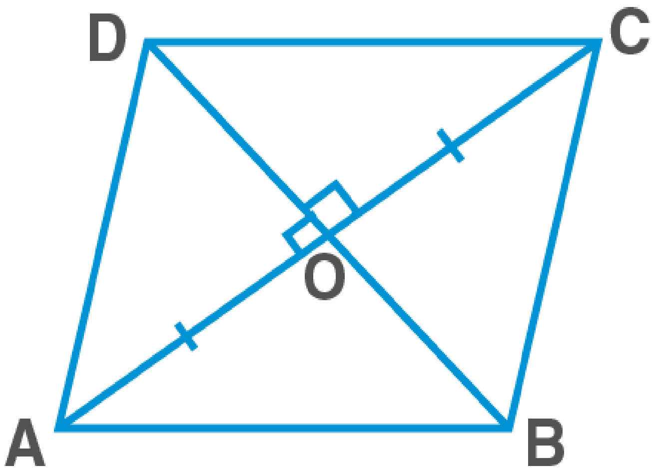
- ∠3=∠5 [alternate interior angles]
- ∠1=∠2 [vertically opposite angles]
- AB=CD [opp. Sides of parallelogram]
- ΔAOB≅ΔCOD [AAS rule]
- OB=OD and OA=OC [C.P.C.T]
- Hence, proved
- Conversely,
  – **If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.**
- – **Diagonal of a parallelogram divides it into two congruent triangles.**
- In ΔABC and ΔCDA,

- AB=CD [Opposite sides of parallelogram]
- BC=AD [Opposite sides of parallelogram]
- AC=AC [Common side]
- ΔABC≅ΔCDA [by SSS rule]
- Hence, proved.

Diagonals of a rhombus bisect each – other at right angles

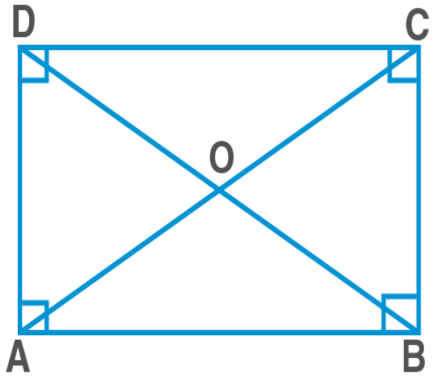
- In ΔAOD and ΔCOD,

- OA=OC [Diagonals of parallelogram bisect each other]
- OD=OD [Common side]
- AD=CD [Adjacent sides of a rhombus]
- ΔAOD≅ΔCOD [SSS rule]
- ∠AOD=∠DOC [C.P.C.T]
- ∠AOD+∠DOC=180 [∵ AOC is a straight line]
- Hence, ∠AOD=∠DOC=90
- Hence proved.



Diagonals of a rectangle bisect each other and are equal

- Rectangle ABCD
- In ΔABC and ΔBAD,
- AB=BA [Common side]
- BC=AD [Opposite sides of a rectangle]
- ∠ABC=∠BAD [Each = 900 ∵ ABCD is a Rectangle]
- ΔABC≅ΔBAD [SAS rule]
- ∴AC=BD [C.P.C.T]
- Consider ΔOAD and ΔOCB,
- AD=CB [Opposite sides of a rectangle]
- ∠OAD=∠OCB [∵ AD||BC and transversal AC intersects them]
- ∠ODA=∠OBC [∵ AD||BC and transversal BD intersects them]
- ΔOAD≅ΔOCB [ASA rule]
- ∴OA=OC [C.P.C.T]
- Similarly we can prove OB=OD



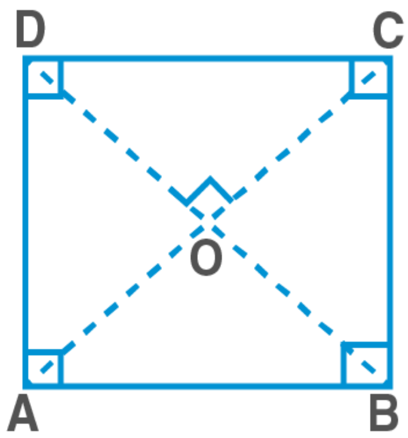
Diagonals of a square bisect each other at right angles and are equal

- Square ABCD
- In ΔABC and ΔBAD,

- AB=BA [Common side]
- BC=AD [Opposite sides of a Square]
- ∠ABC=∠BAD [Each = 900 ∵ ABCD is a Square]
- ΔABC≅ΔBAD [SAS rule]
- ∴AC=BD [C.P.C.T]
- Consider ΔOAD and ΔOCB,

- AD=CB [Opposite sides of a Square]
- ∠OAD=∠OCB [∵ AD||BC and transversal AC intersects them]
- ∠ODA=∠OBC [∵ AD||BC and transversal BD intersects them]
- ΔOAD≅ΔOCB [ASA rule]
- ∴OA=OC [C.P.C.T]
- Similarly we can prove OB=OD
- In ΔOBA and ΔODA,

- OB=OD   [proved above]
- BA=DA [Sides of a Square]
- OA=OA [Common side]
- ΔOBA≅ΔODA, [SSS rule]
- ∴∠AOB=∠AOD [C.P.C.T]
- But, ∠AOB+∠AOD=1800 [Linear pair]
- ∴∠AOB=∠AOD=900



- **Important results related to parallelograms**
- Parallelogram ABCD
- Opposite **sides** of a parallelogram are **parallel** and **equal**.

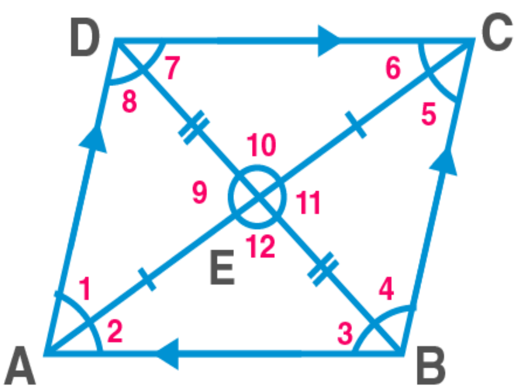
- $AB||CD, AD||BC, AB=CD, AD=BC$
- Opposite **angles** of a parallelogram are **equal** adjacent angels are **supplementary**.

- $\angle A=\angle C, \angle B=\angle D,$

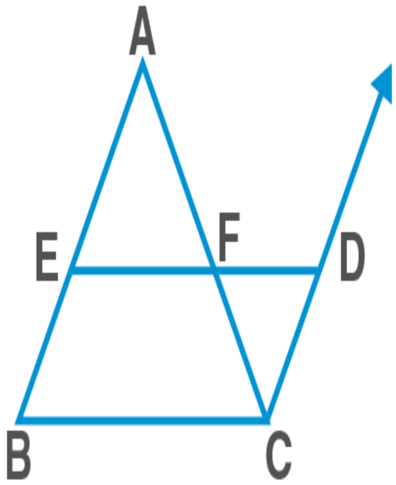
- $\angle A+\angle B=1800, \angle B+\angle C=1800, \angle C+\angle D=1800, \angle D+\angle A=1800$
- A **diagonal** of parallelogram divides it into **two congruent triangles**.

- $\Delta ABC \cong \Delta CDA$ [With respect to AC as diagonal]
- $\Delta ADB \cong \Delta CBD$ [With respect to BD as diagonal]
- The diagonals of a parallelogram **bisect** each other.

- $AE=CE, BE=DE$
- $\angle 1=\angle 5$ (alternate interior angles)
- $\angle 2=\angle 6$ (alternate interior angles)
- $\angle 3=\angle 7$ (alternate interior angles)
- $\angle 4=\angle 8$ (alternate interior angles)
- $\angle 9=\angle 11$ (vertically opp. angles)
- $\angle 10=\angle 12$ (vertically opp. angles)



- <u>**The Mid-Point Theorem**</u>
- **The line segment joining the midpoints of two sides of a triangle is parallel to the third side and is half of the third side**
- In ΔABC, E – the midpoint of AB; F – the midpoint of AC
- **Construction**: Produce EF to D such that EF=DF.
- In ΔAEF and ΔCDF,

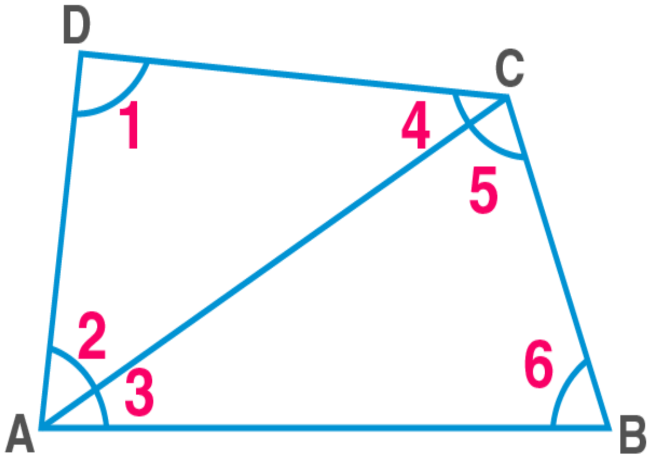


- AF=CF [F is the midpoint of AC]
- ∠AFE=∠CFD [V.O.A]
- EF=DF [Construction]
- ∴ΔAEF≅ΔCDF [SAS rule]
- Hence,

- ∠EAF=∠DCF….(1)

- DC=EA=EB [E is the midpoint of AB]
- DC∥EA∥AB [Since, (1), alternate interior angles]
- DC∥EB
- So EBCD is a parallelogram
- Therefore, BC=ED and BC∥ED
- Since, ED=EF+FD=2EF=BC [∵ EF=FD]
- We have,EF=12BC and EF||BC
- Hence proved.

- **Quadrilaterals**
- Any four points in a plane, of which three are non-collinear are joined in order results into a four-sided closed figure called **'quadrilateral'**
- **Angle sum property of a quadrilateral**
- Angle sum property – Sum of angles in a quadrilateral is 360
- In △ADC,
- $\angle 1+\angle 2+\angle 4=180$ (Angle sum property of triangle)…………….(1)
- In △ABC,
- $\angle 3+\angle 5+\angle 6=180$ (Angle sum property of triangle)………………(2)
- (1) + (2):
- $\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6=360$
- I.e, $\angle A+\angle B+\angle C+\angle D=360$
- Hence proved



IMPORTANT QUESTIONS

- **Q . Prove that the angle bisectors of a parallelogram form a rectangle.**
- **Solution:**
- LMNO is a parallelogram in which bisectors of the angles L, M, N, and O intersect at P, Q, R and S to form the quadrilateral PQRS.
  LM || NO (opposite sides of parallelogram LMNO)
  L + M = 180 (sum of consecutive interior angles is 180o)
  MLS + LMS = 90
  In LMS, MLS + LMS + LSM = 180
  90 + LSM = 180
  LSM = 90
  RSP = 90 (vertically opposite angles)
  SRQ = 90, RQP = 90 and SPQ = 90
  Therefore, PQRS is a rectangle.
- **Q. In a trapezium ABCD, AB∥CD. Calculate ∠C and ∠D if ∠A = 55° and ∠B = 70°**
- **Solution:**
- In a trapezium ABCD, ∠A + ∠D = 180° and ∠B + ∠C = 180°
- So, 55° + ∠D = 180°
- Or, ∠D = 125°
- Similarly,
- 70° + ∠C = 180°
- Or, ∠C = 110°

- **Q. Calculate all the angles of a parallelogram if one of its angles is twice its adjacent angle.**
- **Solution:**
- Let the angle of the parallelogram given in the question statement be “x”.
- Now, its adjacent angle will be 2x.
- It is known that the opposite angles of a parallelogram are equal.
- So, all the angles of a parallelogram will be x, 2x, x, and 2x
- As the sum of interior angles of a parallelogram = 360°,
- $x + 2x + x + 2x = 360^\circ$
- Or, $x = 60^\circ$
- Thus, all the angles will be 60°, 120°, 60°, and 120°.
- **Q. Calculate all the angles of a quadrilateral if they are in the ratio 2:5:4:1.**
- **Solution:**
- As the angles are in the ratio 2:5:4:1, they can be written as-
- 2x, 5x, 4x, and x
- Now, as the sum of the angles of a quadrilateral is 360°,
- $2x + 5x + 4x + x = 360^\circ$
- Or, $x = 30^\circ$
- Now, all the angles will be,
- $2x = 2 \times 30^\circ = 60^\circ$
- $5x = 5 \times 30^\circ = 150^\circ$
- $4x = 4 \times 30^\circ = 120^\circ$, and
- $x = 30^\circ$

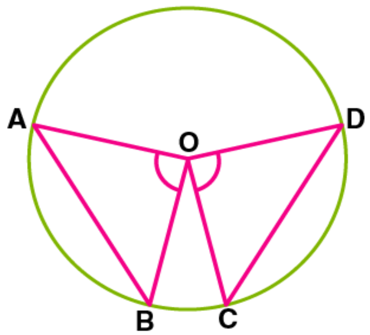
HIGHLIGHTS OF CHAPTER 10 (CIRCLES)

- **Circles**
- The **set of all the points** in a plane that is at a **fixed distance** from a **fixed point** makes a circle.
- A **Fixed point** from which the set of points are at fixed distance is called the **centre** of the circle.
- A circle divides the plane into 3 parts: **interior** (inside the circle), the **circle** itself and **exterior** (outside the circle)
- – The **distance** between the **centre** of the circle and any **point on its edge** is called the **radius.**
- Tangent and Secant
- A **line** that **touches** the circle at **exactly one point** is called it's **tangent.** A **line** that **cuts** a circle at **two points** is called a **secant.**
- **Chord**
- **-**The **line segment** within the circle joining any 2 points on the circle is called the chord.
- **Diameter**
- – A **Chord** passing through the centre of the circle is called the **diameter.** – The **Diameter is 2 times the radius** and it is the **longest chord.**

- Arc
- – The **portion** of a circle(curve) **between 2 points** is called an **arc**. – Among the two pieces made by an arc, the **longer** one is called a **major arc** and the **shorter** one is called a **minor arc.**
- Circumference
- The **perimeter** of a circle is the **distance** covered by going around its **boundary once**. The perimeter of a circle has a special name: **Circumference**, which is π times the diameter which is given by the formula 2πr
- Segment and Sector
- – A circular **segment** is a region of a circle which is "**cut off**" from the rest of the circle by a secant or a chord. – **Smaller region** cut off by a chord is called **minor segment** and the **bigger region** is called **major segment**. –
- -A **sector** is the portion of a circle **enclosed by two radii and an arc**, where the **smaller area** is known as the **minor sector** and the **larger** being the **major sector**.
- – For **2 equal** arcs or for semicircles – both the segment and sector is called the **semicircular region.**

Theorem of equal chords subtending angles at the centre.

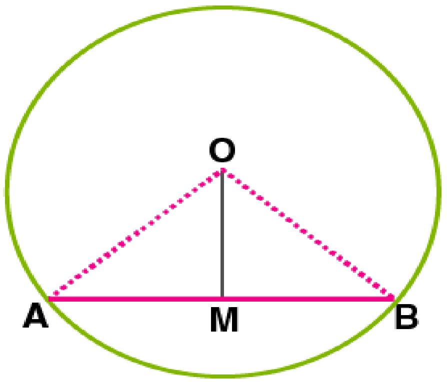
- – Equal **chords** subtend equal **angles at the centre**.
- **Proof**: AB and CD are the 2 equal chords.
- In Δ AOB and Δ COD
- OB = OC [Radii]
- OA = OD [Radii]
- AB = CD [Given]
- ΔAOB ≅ ΔCOD (SSS rule)
- Hence, ∠AOB = ∠COD [CPCT]



- Theorem of equal angles subtended by different chords.
- – If the **angles** subtended by the chords of a circle at the centre are **equal**, then the **chords are equal.**
- Proof: In ΔAOB and ΔCOD
- OB = OC [Radii] ∠AOB=∠COD [Given]
- OA = OD [Radii]
- ΔAOB ≅ ΔCOD (SAS rule)
- Hence, AB=CD [CPCT]

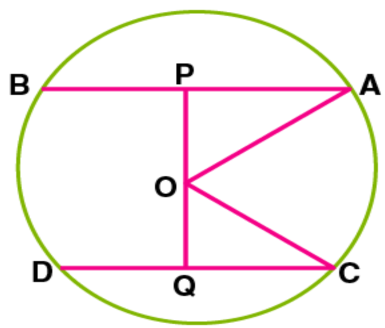
- <u>**Perpendicular from the centre to a chord bisects the chord.**</u>
- **Perpendicular** from the **centre** of a circle to a **chord bisects the chord**.
- Proof: AB is a chord and OM is the perpendicular drawn from the centre.
- From ΔOMB and ΔOMA,
- $\angle OMA = \angle OMB = 90^0$ OA = OB (radii)
- OM = OM (common)
- Hence, ΔOMB ≅ ΔOMA (RHS rule)
- Therefore AM = MB [CPCT]

- <u>**A Line through the centre that bisects the chord is perpendicular to the chord.**</u>
- – A **line drawn** through the centre of a circle to **bisect** a chord is **perpendicular** to the chord.
- **Proof:** OM drawn from the center to bisect chord AB.
- From ΔOMA and ΔOMB,
- OA = OB (Radii)
- OM = OM (common)
- AM = BM (Given)
- Therefore, ΔOMA ≅ ΔOMB (SSS rule)
- $\Rightarrow \angle OMA = \angle OMB$ (C.P.C.T)
- But, $\angle OMA + \angle OMB = 180^0$
- Hence, $\angle OMA = \angle OMB = 90^0 \Rightarrow OM \perp AB$

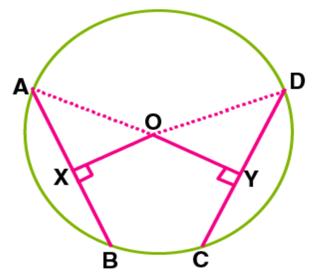


- <u>Circle through 3 points</u>
- – There is **one** and **only** one **circle** passing through **three given noncollinear points.** – A unique circle passes through 3 vertices of a triangle ABC called as the **circumcircle.** The **centre** and **radius** are called the **circumcenter** and **circumradius** of this triangle, respectively.

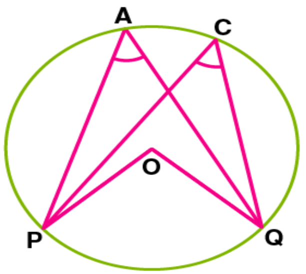
- **<u>Equal chords are at equal distances from the centre.</u>**
- **Equal chords** of a circle(or of congruent circles) are **equidistant from the centre** (or centres).
- **Proof:** Given, AB = CD, O is the centre. Join OA and OC.
- Draw, $OP \perp AB$, $OQ \perp CD$
- In ΔOAP and ΔOCQ,
- OA=OC (Radii)
- AP=CQ (AB = CD $\Rightarrow$(1/2)AB = (1/2)CD since OP and OQ bisects the chords AB and CD.)
- $\Delta OAP \cong \Delta OCQ$ (RHS rule)
- Hence, OP=OQ (C.P.C.T.C)



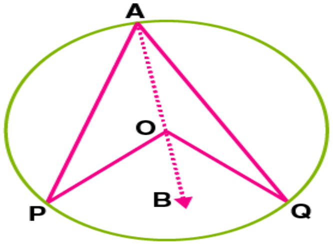
- <u>**Chords equidistant from the centre are equal**</u>
- Chords **equidistant** from the centre of a circle are **equal in length.**
- **Proof:** Given OX = OY (The chords AB and CD are at equidistant) OX⊥AB, OY⊥CD
- In ΔAOX and ΔDOY
- ∠OXA =∠OYD (Both 90^0)
- OA = OD (Radii)
- OX = OY (Given)
- ΔAOX ≅ ΔDOY (RHS rule)
- Therefore AX = DY (CPCT)
- Similarly XB = YC
- So, AB = CD

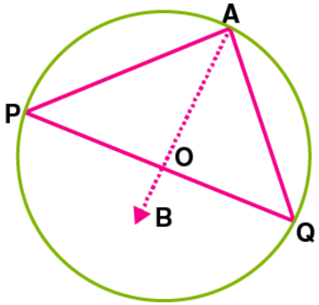


- <u>**Angles in the same segment of a circle.**</u>
- -**Angles** in the **same segment** of a circle are **equal.**
- Consider a circle with centre O.
- ∠PAQ and ∠PCQ are the angles formed in the major segment PACQ with respect to the arc PQ.
- Join OP and OQ
- ∠POQ = 2∠PAQ = 2∠PCQ ……….[Angle subtended by an arc at the centre is double the angle subtended by it in any part of the circle]
- ⇒∠PCQ = ∠PAQ
- Hence proved
- The angle subtend



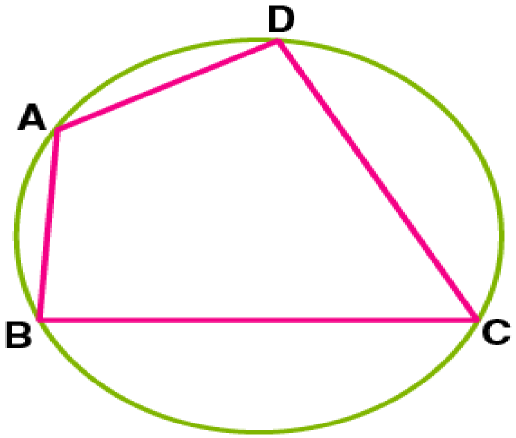
- **The angle subtended by an arc of a circle on the circle and at the centre**
- The **angle** subtended by **an arc** at the **centre is double** the angle subtended by it on any **part of the circle.**
- Here PQ is the arc of a circle with centre O, that subtends ∠POQ at the centre.
- Join AO and extend it to B.
- In ΔOAQ OA = OQ….. [Radii]
- Hence, ∠OAQ = ∠OQA ……..[Property of isosceles triangle]
- Implies ∠BOQ = 2∠OAQ …..[Exterior angle of triangle = Sum of 2 interior angles]
- Similarly, ∠BOP = 2∠OAP
- ⇒∠BOQ + ∠BOP = 2∠OAQ + 2∠OAP
- ⇒∠POQ = 2∠PAQ
- Hence proved.





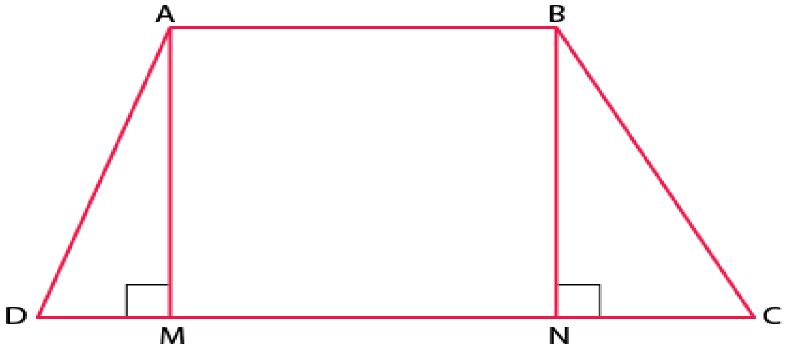
- **The angle subtended by diameter on the circle**
- - **Angle** subtended by **diameter** on a circle is a **right angle**. (Angle in a semicircle is a right angle)
- Consider a circle with centre O, POQ is the diameter of the circle.
- ∠PAQ is the angle subtended by diameter PQ at the circumference.
- ∠POQ is the angle subtended by diameter PQ at the centre.
- ∠PAQ = (1/2)∠POQ……..[Angle subtended by arc at the centre is double the angle at any other part]
- ∠PAQ = (1/2) × 180^0 = 90^0
- Hence proved

- <u>**Cyclic Quadrilateral**</u>
- – A **Quadrilateral** is called a **cyclic quadrilateral** if all the **four vertices lie on a circle**.
- In a circle, if all **four points** A, B, C and D lie **on the circle**, then quadrilateral ABCD is a **cyclic quadrilateral.**
- Sum of opposite angles of a cyclic quadrilateral
- – If the sum of a pair of opposite angles of a quadrilateral is 180 degree, the quadrilateral is cyclic.
- Sum of pair of opposite angles in a quadrilateral
- – The sum of either pair of opposite angles of a cyclic quadrilateral is 180 degree.

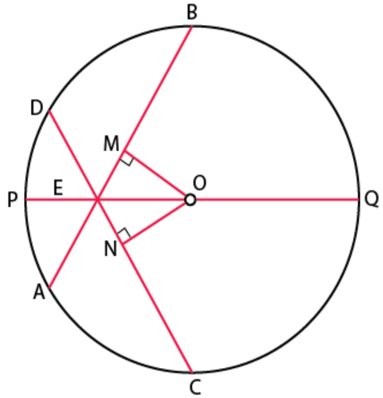


IMPORTANT QUESTIONS

- **Q.: If the non-parallel sides of a trapezium are equal, prove that it is cyclic.**
- Solution:
- Construction-Consider a trapezium ABCD with AB||CD and BC = AD.
- Draw AM ⊥CD and BN ⊥ CD
- In ΔAMD and ΔBNC;
- AD = BC (Given)
- ∠AMD = ∠BNC (90°)
- AM =BN (perpendiculars between parallel lines)
- ΔAMD = ΔBNC (By RHS congruency)
- ΔADC = ΔBCD (By CPCT rule) …….(i)
- ∠BAD and ∠ADC are on the same side of transversal AD.
- ∠BAD + ∠ADC = 180° ……(ii)
- ∠BAD + ∠BCD = 180° (by equation (i))
- Since, the opposite angles are supplementary, therefore, ABCD is a cyclic quadrilateral.



- **Q.: If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.**
- Solution:
- From the question we have the following conditions:
- (i) AB and CD are 2 chords which are intersecting at point E.
- (ii) PQ is the diameter of the circle.
- (iii) AB = CD.
- Now, we will have to prove that ∠BEQ = ∠CEQ
- For this, the following construction has to be done:
- Construction:
- Draw two perpendiculars are drawn as OM ⊥ AB and ON ⊥ CD. Now, join OE. The constructed diagram will look as follows:
- Now, consider the triangles ΔOEM and ΔOEN.
- Here,
- (i) OM = ON [Since the equal chords are always equidistant from the centre]
- (ii) OE = OE [It is the common side]
- (iii) ∠OME = ∠ONE [These are the perpendiculars]
- So, by RHS similarity criterion, ΔOEM ≅ ΔOEN.
- Hence, by CPCT rule, ∠MEO = ∠NEO
- ∴ ∠BEQ = ∠CEQ (Hence proved).



- **Q.: Two chords AB and CD of lengths 5 cm and 11 cm respectively of a circle are parallel to each other and are on opposite sides of its centre. If the distance between AB and CD is 6, find the radius of the circle.**
- Solution:
- Here, OM ⊥ AB and ON ⊥ CD. is drawn and OB and OD are joined.
- As we know, AB bisects BM as the perpendicular from the centre bisects the chord.
- Since AB = 5 so,
- BM = AB/2
- Similarly, ND = CD/2 = 11/2
- Now, let ON be x.
- So, OM = 6− x.
- Consider ΔMOB,
- $OB^2 = OM^2 + MB^2$
- Or,
- $OB^2 = 36 + x^2 - 12x + 25/4$ ……(1)
- Consider ΔNOD,
- $OD^2 = ON^2 + ND^2$
- Or,
- $OD^2 = x^2+121/4$ ……….(2)
- We know, OB = OD (radii)
- From eq. (1) and eq. (2) we have;
- $36 + x^2 -12x + 25/4 = x^2 + 121/4$
- $12x = 36 + 25/4 - 121/4$
- $12x = (144 + 25 -121)/4$
- $12x = 48/4 = 12$
- $x = 1$
- Now, from eq. (2) we have,
- OD2 = 11 + (121/4)
- Or OD = $(5/2) \times \sqrt{5}$