Roll No. ....

D–3757

M. A./M. Sc. (Final)
EXAMINATION, 2020

MATHEMATICS

(Compulsory)

Paper Second

(Partial Differential Equations and Mechanics)

Time : Three Hours] [*Maximum Marks : 100*

Note : All questions are compulsory. Attempt any *two* parts from each question. All questions carry equal marks.

Unit—I

1. (a) Define u by :

$$u(x) = \int_{\mathrm{R}^n} \Phi(x-y)\, f(y)\, dy$$

where Φ is the fundamental solution. Then prove that :

(i) $u \in \mathbf{C}^2(\mathrm{R}^n)$

(ii) $-\Delta u = f$ in R^n

(b) Derive the Euler-Poisson-Darboux equation.

(c) State and prove the mean value property for the heat equation.

Unit—II

2. (a) State and prove the Euler-Lagrange equations.

(b) Explain Hodograph transform with an example.

(c) Solve the Bessel potential :

$$-\Delta u + u = f \text{ in } \mathbf{R}^n$$

where $f \in L^2(\mathbf{R}^n)$ using Fourier transform.

Unit—III

3. (a) State and prove Donkin's theorem.

(b) Prove the following property of Poisson bracket for any arbitrary functions u, v and w :

$$[u, [v, w]] + [v, [w, u]] + [w, [u, v]] = 0$$

(c) Explain and solve the Brachistochrone problem.

Unit—IV

4. (a) State and prove the principle of least action.

(b) Find the relation between Poisson bracket and Lagrange bracket.

(c) State and prove the necessary and sufficient condition for the transformations :

$$Q_i = Q_i(q_i, p_i, t)$$

$$P_i = P_i(q_i, p_i, t)$$

to represent canonical transformation.

Unit—V

5. (a) Find the attraction of a uniform circular disc of radius 'a' and small thickness 'k', at a point on the axis of the disc at a distance 'p' from its centre.

(b) Find the potential of a finite rod AB at an external point P.

(c) State and prove Gauss theorem.