

B.C.S. GOVT P.G. COLLEGE, DHAMTARI (C.G.)

ASSIGNMENT -2019-20

B.Sc. II Year (REGULAR)

MATHEMATICS

PAPER FIRST

(ADVANCED CALCULUS)

DATE: 03-10-2020

M.M.: 50

Note: Answer any five questions. Each question carries equal marks.

1. a) Define convergence of a sequence with an example. [4 + 6 = 10]

b) Test the convergence of the series:

$$1 + \frac{x}{2} + \frac{12}{3^2}x^2 + \frac{13}{4^3}x^3 + \frac{14}{x^4}x^4 + \dots \dots \dots, x > 0.$$

2. a) State Lagrange's mean value theorem. [4 + 6 = 10]

b) Verify Rolle's theorem for the function $f(x) = \sqrt{x^2 - 4}$ in the interval $[0, 1]$.

3. a) State Euler's theorem for homogeneous function of two variables. [4 + 6 = 10]

b) If $x^x y^y z^z = c$, then show that: $\frac{\partial^2 z}{\partial x \partial y} = -(x \ln_e x)^{-1}$, where $x = y = z$.

4. a) Define Envelope with an example. [4 + 6 = 10]

b) Find the envelop of the curves $\left(\frac{x}{a}\right)^m + \left(\frac{y}{b}\right)^m = 1$, where the parameters a and b are connected by the relation $a^p + b^p = c^p$.

5. a) Define Beta and gamma functions. [4 + 6 = 10]

b) Express $\int_0^1 x^m (1 - x^n)^p dx$ in terms of the beta function and hence evaluate $\int_0^1 x^5 (1 - x^3)^1 dx$.

6. a) Define Jacobian. [4 + 6 = 10]

b) If $x + y + z = u$; $y + z = v$; $z = w$, then prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$.